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Proceedings of the American Academy of Arts and Sciences.

Vol. XXXVIII. No. 1. — June, 1902.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF CASE SCHOOL OF APPLIED SCIENCE. — XLII.

AN APPARATUS FOR CONTINUOUS VACUUM DISTILLATION.

By Charles F. Mabery.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF CASE SCHOOL OF APPLIED SCIENCE. — XLII.

AN APPARATUS FOR CONTINUOUS VACUUM DISTILLATION.*

By Charles F. Mabery.

Received May 13, 1902.

The occasional contributions to methods for vacuum distillation seem to indicate that a method is still wanting that shall combine convenience and efficiency. It is quite true that the various attachments that have been suggested and that are described in dealers' catalogues fall short of efficiency in essential details. In the great amount of vacuum distillation carried on in this laboratory during the last fifteen years, probably much exceeding what has been done elsewhere in a single line of work, a durable apparatus has been gradually evolved in which this work can be carried on as expeditiously as distillations under ordinary pressures.

One of the most essential features is a regulator to maintain a constant tension, and the stopcock G with lever attachment formerly described † and constantly in use is very satisfactory.

The chief features to be provided for in a convenient apparatus are the following:

1. Exclusion of air from hot oil in still during change of receiver.
2. Admission of distillates into still without interruption.
3. Admission of air into receiver before removal of each fraction.
4. Exhaustion of receiver for new fraction without connection with still.

The complete apparatus in the form used at present is shown in the following figure: —

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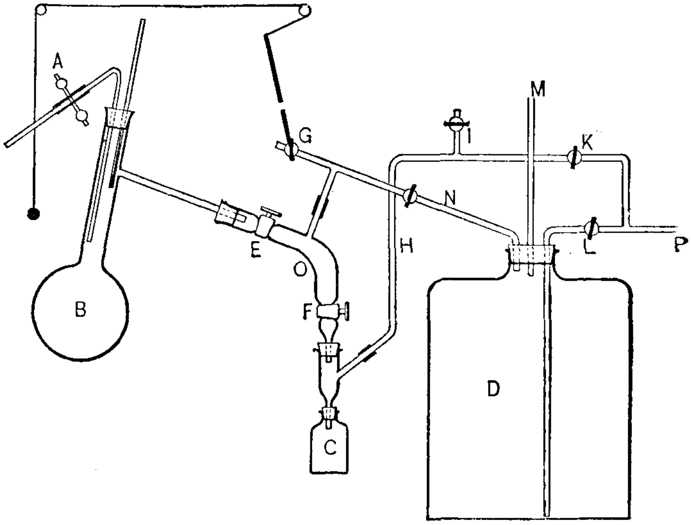
* This method is a part of the work that is carried on in this laboratory with aid granted by the Academy from the C. M. Warren fund for chemical research.

† These Proceedings, XXXI. p. 10.



The fractions are drawn into the still through the tube closed by the nipper tap A. The still is exhausted by the tube N connecting the tube O with the vacuum reservoir D. The reservoir C is exhausted by the tube H which connects with the water pump through the tube P. Air is let into the receiver C by means of the cock I, and kept from the still by the cocks E and F. The tube with the stopcocks E and F afford a convenient means for separating fractions without interruption, and without admission of air into the still. The tube M leads to the manometer. By means of a single efficient water pump the entire apparatus may be kept under a tension of 12mm. or less during continuous distillation. By means of common corks, the apparatus is readily set up and easily kept tight by the use of rubber lute.

The tubes H and N may be given less rigidity by putting them together in sections with connectors. Any water that may occasionally run back from the pump is readily drawn out if the pipe P extends to the bottom of the reservoir D.

The apparatus in this form is especially adapted for the separation of fractions with high boiling points. For very high temperatures the still must be packed in asbestos. For more volatile distillates a condenser should be inserted between the still and tube O, best by passing the exit tube of the still through the condenser. Our distillation flasks are made with a high exit tube to give a long neck, which is filled with broken glass resting on a piece of glass rod with a head as previously described. As is evident from the figure, this apparatus may readily be set up from supplies always at hand in the laboratory, except the tube O, which any glass-blower can make.

Suggestions as to details have been made by various assistants, especially by Mr. O. J. Sieplein, instructor in chemistry, who prepared the drawing.

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VOL. XXXVIII. NO. 2. — JUNE, 1902.

CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORY OF HARVARD UNIVERSITY. — L.

PRELIMINARY DIAGNOSES OF NEW SPECIES OF LABOULBENIACEAE. — V.

BY ROLAND THAXTER.

CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORY OF HARVARD UNIVERSITY. — L.

PRELIMINARY DIAGNOSES OF NEW SPECIES OF LABOULBENIACEAE. — V.

By Roland Thaxter.

Presented May 14, 1902. Received May 15, 1902.

Dimeromyces Forficulae nov. sp.

Male individual. Receptacle consisting of three superposed cells, the upper distinguished by a well-defined black septum from a short two- to three-celled terminal appendage, the subbasal septum of which is also blackened; the subbasal cell of the receptacle producing a suberect, short-stalked, rather long and narrow antheridium; the neck relatively broad, blunt, about as long as the stalk and venter. Total length to tip of antheridium 60 μ: the antheridium, including stalk, 28–30 × 7–8 μ.

Female individual more or less tinged with purplish-brown, especially the body of the perithecium. Receptacle consisting of usually five cells obliquely superposed, with the exception of the uppermost, successively smaller from below upward, the series more or less strongly curved outward from the male; the subbasal cell bearing a simple differentiated appendage, its basal cell more or less geniculate and separated from the basally inflated, tapering, brown, five- to six-celled distal portion by a blackish constriction; the cell next above it producing the single perithecium; the next a simple cylindrical slightly tapering appendage with a black subbasal constricted septum; the terminal cell bearing terminally a short, simple, few-celled primary appendage, distinguished by a constricted black basal and a pale subbasal septum, and laterally a similar appendage distinguished by a subbasal blackish constricted septum. Perithecium rather elongate, subclavate or subfusoid, the stalk not distinguished from the body of the perithecium, and sometimes showing irregular septa; the tip often somewhat abruptly distinguished, blunt, slightly asymmetrical. Spores about 35 × 3.5 μ. Perithecia, including stalk, 90–110 × 18–22 μ. Longest appendage 80 μ. Receptacle 60–70 μ. Total length to tip of perithecium 150–185 μ.

On all parts of *Forficula taeniata* Dohrn.; Mus. Comp. Zoöl., No. 1355; Guatemala.

Monoicomyces Oxypodae nov. sp.

Receptacle very small, the two cells subequal, the basal cell involved by the blackening of the foot and hardly distinguishable; the terminal appendage straight and tapering, its basal cell hyaline, nearly isodiametric; the subbasal cell brown, slightly inflated and twice as long; the two cells above inflated, brown, distinguished by constrictions at the dark septa. Receptacle giving rise to a branch on either side; one usually sterile, short, blunt, extending externally above the subbasal cell of the primary appendage, and wholly blackened to its base: the other fertile; consisting of a single cell which is brown, broadly blackened externally, the blackening involving its narrow base almost completely; its distal half becoming more than twice as broad, and giving rise to a single antheridium terminally and a stalked perithecium subterminally on its inner side; stalk of the antheridium brown, two-celled (?), equal in diameter to the branch cell which bears it, and like it externally blackened; the basal cells small and forming with the wall and antheridial cells a compact antheridium twice as long as broad, distally rounded and slightly sulcate; one only of the terminal cells growing out to form a rather short colorless appendage. Stalk-cell of the perithecium arising immediately below that of the antheridium on the inner side, its narrow base black and opaque, resembling a "foot," distally hyaline, broader, about as long as the antheridium; the basal cells hyaline, rather small; the perithecium faintly purplish, rather long and narrow, relatively large, the tip rather abruptly distinguished and usually slightly bent. Spores 45 × 4.5 μ. Perithecia 75–90 × 18–20 μ. Antheridia 25–35 × 107 μ, the appendage 40–50 μ. Receptacle 10 μ, its appendage 75 μ. Total length about 150–165 μ.

On the inferior tip of the abdomen of *Oxypoda* sp; Intervale, N. H., July 17, 1901. A species most nearly related to *D. furcillatus* and like that species placed only provisionally in the present genus.

Monoicomyces nigrescens nov. sp.

Receptacle minute, its basal cell hardly distinguishable above the foot, bearing distally a simple appendage the basal cell of which is hyaline, the subbasal somewhat longer than those above and deeply tinged with blackish brown below. Fertile branches two or more, usually four, each consisting of a single cell which bears an antheridium terminally and a perithecium subterminally: the primary branches normally two, lateral and symmetrical, edged externally with blackish brown, the blackening contrasting and continuous with a similar coloration which extends to the tip of the primary antheridium: the secondary fertile branches arising, when present, between the primary and resembling the latter, except for the absence of the black discoloration; the whole group of branches forming, with their closely crowded antheridia and perithecia, a compact fan-like usually symmetrical tuft. Antheridia relatively long, suffused with smoky brown, more deeply blackish externally, the secondary ones with a more or less conspicuous foot-like blackened base; the stalk clearly two-celled, shorter than the main body; two of the distal cells growing upward to form the two unequal terminal appendages, which are smoky brown, darker about their blackened basal septa, the cells immediately below them projecting upward very slightly on either side. Perithecia furnished with variably developed stalk-cells the bases of which are blackened, but which are otherwise hyaline, as is the rest of the rather short, often stout, subconical, bluntly pointed perithecium. Perithecia 60–75 × 22–25 μ; the stalk-cell 12–55 μ. Antheridia, including stalk, ·35 μ, the appendages 35–50 μ. Total length 100–160 μ.

At the tip of the abdomen of *Calodera* sp. and of *Tachyusa* sp.; Intervale, N. H., No. 1357. The hosts frequenting fleshy fungi.

HERPOMYCES nov. gen.

Sexual organs normally separated on different individuals. Antheridia simple.

Male individual consisting of several (four) superposed cells terminated by a characteristically modified spinous or small foot-like process or by both; the basal cell attached by a small normal foot: one or more of the distal cells giving rise to short branches which may bear from one to several antheridia terminally, or become more or less copiously branched; the branchlets terminated by antheridia, or in some cases sterile. Antheridia long, flask-shaped. The subbasal cell of the receptacle sometimes producing a fertile branch as in the female individual from which are produced secondary receptacles which give rise to antheridial branches.

Female individual consisting primarily, as in the male, of several superposed cells similarly modified at the tip, and attached by a small normal foot; the basal and subbasal cells constituting a "primary receptacle;" the latter giving rise to a variably developed fertile branch (sometimes apparently dividing to several cells each of which may produce a fertile branch) from which is developed a "secondary receptacle," or, as a result of branching, more than one. Secondary receptacles consisting of a partly double series of cells, variable in number, one or more of which may be fertile, the rest sometimes specially differentiated, or unmodified; those in contact with the host perforating the chitinous integument by means of fine haustoria. Trichogynes short filamentous. Perithecium borne on variably developed stalk-cells, the ascigerous portion including three tiers of wall-cells, more or less clearly distinguished from the distal portion, the wall-cells of which are more or less differentiated, four or five in each row. Spores minute, of the usual type, normally discharged in pairs the members of which produce male and female individuals. Asci apparently eight-spored.

The discovery of this very remarkable genus is due to Mr. Charles Bullard, who first observed it on *Ectobia* and *Periplaneta* in Cambridge. Though perhaps distantly related to *Moschomyces* it is in some respects unique, and with the exception of *Dimeromyces* is the only genus which contains species inhabiting orthopterous hosts. The peculiar cell relations of the perithecia and secondary receptacles seem unlike those of most genera, but a further description of them seems undesirable in the present connection.

Herpomyces chaetophilus nov. sp.

Male individual consisting of four superposed cells, hyaline, the distal cell terminated by a blackish projection similar to the small foot. The three distal cells, sometimes only the terminal one, usually producing slight outgrowths which bear the single, nearly erect, long, slender antheridia directly; or may separate a cell which bears one, or very rarely two such antheridia; the latter nearly as long as the four-celled individual. Total length to tips of antheridia 50–55 μ.

Female individual. Primary individual similar to the male, but stouter; terminated by a similar blackish projection. The fertile branch arising laterally from the subbasal cell, growing down in the form of a slender filament variable in length, usually of two or three cells, enlarging abruptly to form the single secondary receptacle. Secondary receptacle pale dirty brownish yellow, consisting of a vertical series of cells partly double above, simple below; the cells thick-walled, the long (transverse) axes directed obliquely upward and outward, about five to fifteen in number, their points of contact with the host surrounded by a slightly blackened irregular foot-like haustorial margin, and giving rise to single simple, or very rarely branched, haustoria which penetrate the wall of the spine at right angles to its surface; the cells all sterile with the exception of the proximal one from which arises the solitary, nearly erect perithecium. Perithecium relatively large, rather stout; the ascigerous portion large, slightly inflated, longer than the outwardly curved distal portion, which tapers to the bluntly pointed unmodified apex; the tip bent abruptly outward. Spores 30–35 × 3 μ. Perithecia 125–185 × 35–48 μ. Total length of primary individual 35–40 μ. Secondary receptacle 35–75 × 22 μ.

On spines of legs, antennae and anal appendages of *Periplaneta* sp., Zanzibar, Africa; Mus. Comp. Zoöl. On *Periplaneta* sp., Mauritius; Mus. Comp. Zoöl., No. 1357.

Herpomyces Periplanetae nov. sp.

Very variable according to the host and the position of growth.

Male individual consisting of four superposed cells, the two upper, in simpler individuals, producing one or two antheridia which are either sessile or borne on a single stalk-cell: in more highly developed individuals the two distal cells producing short branches which may bear several antheridia directly, or on secondary branchlets, some of which appear to be occasionally sterile; the total number of antheridia sometimes six or more. Greatest length of well-developed forms, to tips of antheridia, 90 μ, of small specimens 60 μ.

Female individual, hyaline or nearly so. Primary receptacle surmounted by two or more sterile cells, the uppermost often asymmetrical, ending in a terminal spinous process and bearing the minute characteristic black projection laterally: the subbasal cell sometimes several times divided, each resultant cell apparently giving rise to a single branch from which is developed the very variable secondary receptacle. Secondary receptacle *in simple individuals growing on spines of host;* consisting of a vertical series of from four to twenty or more obliquely superposed cells, alternating to form a double row, otherwise similar to that of *H. chaetophilus* and like it producing a single erect perithecium from one of its uppermost cells: *in individuals growing on the integument of host;* consisting of a variable number of cells; certain fertile ones disposed subhorizontally on either side of the primary receptacle, the fertile cells, of which there may be from one to six, together with the male individual when present, and the bases of the perithecia, protected by a shield or shell-like, usually very unequally bilobed cellular upgrowth, rounded or bluntly pointed above, the symmetrically curved successive cells which compose it enormously elongated transversely, their lumen scarcely wider than the intervening walls and forming a series of concentrically arranged arcs, the concavities directed downward. Perithecia one to six, commonly five in well-developed specimens, slightly divergent from the median line, long, pointed, tapering from the slightly, more or less asymmetrically inflated base; the distal portion not clearly differentiated, tapering more or less, curved, the slender upper portion bent abruptly inward toward the tip; the pointed apex bent inward, subtended externally by a terminal, slightly incurved, rather slender, bluntly pointed unicellular process, the cells of the cell row which it terminates distinctly larger than the other wall-cells; basal cells somewhat prominent. Spores 16 × 1.5 μ. Perithecia 145–220 × 30–36 μ, the process 14 μ. Secondary receptacle, including protective shield, in well-developed individuals 125 × 75 μ; in small specimens 35 × 50 μ; when vertically developed without shield 35–110 × 18 μ.

On *Periplaneta Americana* Sauss. (type form), Cambridge (Mr. Bullard): Bermuda; Mus. Comp. Zoöl. On *Periplaneta Australasiae* Sauss., Bermuda. On *Periplaneta* spp., Mexico, West Indies, Panama, Brazil, Africa, South Seas, China. All Mus. Comp. Zoöl. On *Stylopyga orientalis* Scudd., Boston, Mus. Comp. Zoöl.

Herpomyces arietinus nov. sp.

Male individual consisting of four superposed cells, the basal one relatively long, the distal ones bearing two to three antheridia. Length about 29 μ. Antheridia about 20 μ.

Female individual hyaline. Primary receptacle surmounted by two sterile cells, the upper terminated by an erect distally mucronate appendage; the subbasal cell giving rise to two branches (or to a branch which becomes immediately furcate?) each branchlet producing a secondary receptacle. Secondary receptacles two, symmetrically paired, each consisting of a horizontal series of about twelve or more vertically elongated, subfusiform, more or less curved cells, corresponding to and external to the fertile cell which bears the primary perithecium, the external margin free, other fertile cells (of which there appears to be but one in the type) completely hidden behind it. Ascigerous portion of the perithecium relatively long, hardly inflated, tapering slightly above, where it passes into the distal portion; which is about half as long, tapers very slightly, and is terminated by an incurved, tongue-like, slender, subcylindrical prolongation of the apex on the inner side, and by a relatively long straight erect slightly tapering subtending terminal unicellular process. Spores about 20 × 2 μ. Perithecia 100 × 22 μ, the terminal process 18 μ. Secondary receptacles, together, 55 × 18 μ.

On the antenna of a small brown wingless roach taken under stones and bark near the mouth of the Mammoth Cave, Kentucky; Mus. Comp. Zoöl., No. 1370.

Herpomyces Zanzibarinus nov. sp.

Male individual consisting of four cells, the distal one furnished with a sharp colorless terminal spine and a small blackish subterminal process, both of which become pushed to one side by the development of one, or sometimes two, sessile antheridia; the subterminal cell somewhat larger, sterile, or producing an antheridium laterally. Total length 28 × 6.5 μ. Antheridia 25 μ.

Female individual colorless. Primary receptacle surmounted by two sterile cells, the upper terminated by a sharp spinous process subtended by a blackish process, as in the male: the subbasal cell apparently divided at least once, the two (or more?) cells producing fertile branches from which, normally, two few-celled, paired, compact secondary receptacles are produced; each of which bears a single perithecium, the cells rather irregular, about five or six in number, subhorizontal in position, without characteristic modification. Ascigerous portion of the perithecium relatively stout, inflated on its inner side, almost twice as long as the distal portion, which tapers from it rather abruptly: distal portion curved inward abruptly at the pointed tip, which diverges from an erect blunt unicellular process which subtends it, at an angle of more than 90°. Perithecium, including base, to tip of terminal process, 125 × 28–32 μ; the process 10 μ long. The pair of fertile receptacles together about 30–35 × 45 μ broad (horizontally).

On the antenna of a large black wingless roach, Zanzibar, Africa; Mus. Comp. Zoöl., No. 1354.

Herpomyces forficularis nov. sp.

Male individual minute, consisting of four superposed cells, the three upper subequal, nearly round, constricted at the septa, the distal one terminated by a short bluntly pointed appendage resembling a minute foot, which is commonly turned to one side by the development laterally from the same cell of a single slender antheridium. Total length 18 × 5μ. The antheridium about 35 μ.

Female individual. Primary receptacle consisting of two superposed cells terminated by a single abruptly smaller sterile cell, which is slightly longer than broad and is terminated by a bluntly pointed appendage similar to that of the male, but larger; the subbasal cell much enlarged, somewhat inflated, the fertile branch apparently at once furcate so that two secondary receptacles are formed symmetrically placed on either side of and just below the primary receptacle. Secondary receptacles distinctly yellowish externally, consisting of a nearly horizontal series of about ten sterile cells, very narrow from the great elongation of their transverse axes which are vertical in position, and so arranged as to cover more or less completely the single fertile cell, which is subtriangular and gives rise to a solitary perithecium. Base of the perithecium nearly as broad as the secondary receptacle, forming a short stout neck: perithecium relatively large, the ascigerous portion somewhat longer than the distal part, very slightly inflated, nearly isodiametric, the base of the trichogyne persistent as a rather conspicuous hyaline projection between it and the distal part which is but slightly narrower, hardly tapering, the large lateral cells thick-walled, the rows similar on either side and terminating in large incurved tapering bluntly pointed brownish-yellow unicellular projections, which surmount the perithecium like a pair of mandibles, the inner somewhat shorter and straighter: the short, pointed apex included between their bases and bent slightly inward. Spores about 18 × 2 μ. Perithecia including base 200 × 36 μ; the terminal projections, longer, 35 μ. Secondary receptacles 35 × 20–35 μ. Primary receptacle including sterile terminal cell 20 × 7 μ. Total length to tip of perithecium 220–250 μ.

On antennae of a wingless roach, Mauritius (?); Mus. Comp. Zoöl., No. 1353.

Herpomyces Diplopterae nov. sp.

Male individual; four-celled, the two middle cells roundish-oblong, the distal longer and subcylindrical, terminated by the usual minute blackish projection; the subterminal and subbasal cells each producing one or two nearly sessile, or short stalked, antheridia, with well-differentiated slender necks. Total length to tips of antheridia about 50 μ.

Female individual. Primary receptacle similar to male, the subterminal and subbasal cells subcylindrical, longer than broad; the fertile branch producing two symmetrically placed secondary receptacles, as in *H. forficularis*, the sterile external cells yellowish, somewhat larger and more distinct, about twelve or more in number; the series extending externally and inferiorly to form a free buttress-like margin, much as in *H. Paranensis,* which almost wholly covers the single fertile cell. Perithecium yellowish, straight, nearly erect, the base bulging very slightly; but hardly broader than the ascigerous part, which is relatively large, long, subcylindrical, or slightly inflated; the distal part, relatively short, rather abruptly distinguished; the posterior cell row, which is external in relation to the host, more prominent, with larger thick-walled cells, the fifth from below prolonged to form a long, bluntly tipped, erect, horn-like, subterminal projection, distally curved inward above the short slightly incurved pointed subconical tip. Total length of perithecium (exclusive of base) to tip of process 150 μ; to tip 115 μ: ascigerous part 70–75 × 28–30 μ, distal part to tip 35–40 μ; the process, free part, 25–30 μ, whole cell 40–44 μ. Secondary receptacles both together 55–65 × 25 μ. Total length to tip of process 175–185 μ.

On *Diploptera dityscoides* Serv., Ascension Island, South Atlantic; Mus. Comp. Zoöl., No. 1371.

Herpomyces tricuspidatus nov. sp.

Male individual variably developed, the terminal cell rounded apiculate, but as a rule soon indistinguishable through proliferation, the simpler forms very similar to well-developed individuals of *H. Periplanetae,* the more complicated and most frequent type similar to the fertile branches of the male individual in *H. Ectobiae,* producing, through continued successive proliferation, a dense compact more or less appressed tuft of antheridia which appear to be associated with undifferentiated sterile branchlets. Antheridium long flask-shaped, hardly distinguished from the usually several celled branchlet which it terminates. Total length to tips of antheridia 75 μ.

Female individual. Primary receptacle small, surmounted by two rounded cells constricted at the septa, the distal one bearing a small sharp spine subtended by the usual minute blackish projection: the subbasal cell producing apparently a single fertile branch which divides at once, growing in opposite directions to form the somewhat irregular, and variably developed, secondary receptacles, which may creep extensively; the component cells, which are often very numerous, being vertically elongated and becoming arranged in two more or less complete rows; the inner mostly fertile, producing perithecia of which there may be twelve or rarely more; the outer becoming several times closely divided vertically, the cell-group which thus gives rise to the perithecial stalk, laterally connected with corresponding adjacent cell-groups throughout its lower third only, the upper two thirds forming a free, or nearly free, continuation of the perithecial stalk, forming no free protective margin above, while below they develop a continuous, irregularly lobed, spreading haustorial margin in close contact with the host. Perithecia raised on a well-developed stalk, consisting of two unequal and asymmetrical cells placed side by side; that on the side toward which the tip of the perithecium is bent (anterior) attenuated below and extending higher than the posterior, which becomes narrower upward from its broad base; the stalk becoming gradually and slightly broader from below upward, and directly continued by the base of the perithecium from which it is not distinguished. Ascigerous portion of the perithecium distinguished from the base by a very slight prominence, and about as long as the stalk and base combined; becoming distally slightly broader, the two lower tiers of wall-cells separated by a slight elevation; the third wall-cell of the anterior row small, and forming a prominent elevation followed by a depression which subtends a large, erect, tapering, bluntly pointed, distally incurved spinous process formed by an outgrowth of the lowest cell in the anterior row of wall-cells of the distal portion (fourth anterior wall-cell) which extends upward higher than the tip of the perithecium, its upper two thirds forming a free spine; the lower cell of one of the corresponding lateral rows (fourth lateral wall-cells) producing a similar process, shorter, slightly sharper, curved inward distally toward the apical pore, this process always external in relation to the host and thus developed on the right or left side according as the perithecium is formed from the receptacle at the left or at the right of the original insertion toward which the anterior sides of all the perithecia are turned: the rest of the terminal portion above the bases of these outgrowths short, abruptly tapering, its outer margin vertical, slightly prominent and not distinguished from the posterior margin of the ascigerous portion, which is directly continued by it; its inner margin running abruptly inward and upward from the base of the anterior process to the small blunt tip, which is curved abruptly inward and is subtended by a nearly erect, short, sharp spinous process; the whole nearly symmetrical with the anterior process which is very slightly longer. Spores about 12 μ long. Perithecia, ascigerous portion 45–50 × 25 μ, terminal part to tip 28 μ, anterior process 30–34 μ, apical process 8 μ, stalk, including basal cells, 45–58 × 16–18 μ. Total horizontal extent of larger individuals including both receptacles 220 μ. Fertile cells below perithecial stalks 30–40 × 13–15 μ. Total height to tip of perithecial process 175–200 μ.

On *Blabera* sp. and *Epilampra* (?) sp., Panama; Mus. Comp. Zoöl., No. 1364. On *Epilampra* sp., No. 1360, St. Kitts, W. I. (type), No. 1366, Hayti. On a wingless form labelled "China?" All Mus. Comp. Zoöl., and in all cases on the antennae.

Herpomyces Paranensis nov. sp.

Male individual similar to the simpler forms of *H. tricuspidatus*, but the terminal cell distally modified to form a long slender flexuous tapering unicellular prolongation extending above the tips of the antheridia. Total length to tip of terminal prolongation 250 μ; the prolongation 185 μ. Antheridia about 60–70 μ.

Female individual. Primary receptacle very small, the distal cells rounded, the uppermost prolonged as in the male. Secondary receptacles developed on either side of the primary, the cell series apparently turning inward from either side so that the perithecia are more or less clustered (younger ones appearing behind the two primary ones), and protected by a shield-like structure external to the base of the first fertile cell, developed like a buttress, the outer and upper margins of which are free, consisting of sterile cells which are greatly elongated vertically and very narrow, similar and successively fewer-celled buttresses being formed behind the primary one in connection with each of the remaining perithecia, of which there may be from four to six. Perithecia very similar to those of the preceding species, but with the following differences: the greatly elongated fertile cell of the receptacle extends nearly to the base of the perithecium, the posterior stalk-cell extending downward beside it nearly to its base, covered by the protective shield except at its distal end, which is connected by a narrow isthmus with an abrupt short broad terminal enlargement; the anterior stalk-cell small, short, subtriangular in outline; the base of the perithecium abruptly somewhat broader, its cells protruding more or less distinctly; the ascigerous region thus somewhat clearly distinguished, especially posteriorly, relatively distinctly larger than in the preceding species, somewhat inflated; the conformation of the distal portion similar in general, but the third wall cell of the anterior row is not modified to form a prominence at the base of the anterior spiniferous cell, the upper half of which forms a free spinous process slightly incurved distally and equalled or even exceeded by the lateral spinous process; the free tip of the perithecium about twice as long relatively, slightly incurved, the erect incurved spinous process, which subtends it externally, more than twice as long as that of the preceding species and sublateral; the tip between the base of this spine and the inner angle of the anterior spine relatively shorter and broader. Spores about 15 × 1.6 μ. Perithecium: ascigerous portion 58 × 30 μ; terminal portion to tip, 40 μ; anterior process, free portion, 22–25 μ, whole cell, 50–54 μ; subterminal process, free part, 17 μ; total length, 123 μ, including basal cells. Width of two outer buttresses together 85–100 μ; height of shield-like upgrowth 36–50 μ; length from insertion to base of perithecium 60–80 μ; total length from insertion to tip of perithecial spine 180–218 μ.

On antenna of a wingless roach (? *Blabera*), Para, Brazil; Mus. Comp. Zoöl., No. 1362.

Herpomyces Ectobiae nov. sp.

Male individual consisting of four superposed cells terminated by the characteristic blackish projection, the distal cells producing a dense appressed tuft of coherent antheridial branchlets and antheridia; the subbasal cell usually giving rise to a fertile branch, simple or furcate, which produces a secondary male receptacle consisting of an irregularly double straggling series of cells, some of which are sterile, while others bear short-stalked, unilateral, dense antheridial tufts similar to the primary one (which may sometimes be lacking ?).

Female individual colorless. Primary receptacle as in the male, terminated by two short cells, the upper of which bears distally the characteristic blackish minute foot-like projection, the subbasal cell producing a simple or furcate fertile branch. The fertile receptacles, like those of the male, often creeping extensively, consisting of an irregularly double series of obliquely seriate cells, sterile or fertile without definite sequence, the whole plant producing sometimes twelve or even more perithecia developed as a rule in irregularly acropetal succession. Perithecia subsessile, inflated below, attenuated above, the extremity bent or sometimes slightly recurved, the apex unmodified. Spores 20 × 2.5 μ. Perithecia, including base, 80–90 × 20 μ. Total length of secondary receptacle, longer, 200–225 × 15 μ. Primary individual 22 μ.

On *Ectobia Germanica* Scudd., Cambridge, Mr. Bullard. On *Ectobia* sp., Zanzibar; Mus. Comp. Zoöl., No. 1357; St. Kitts, West Indies; Mus. Comp. Zoöl., No. 1361. Although aberrant from the fact that the male develops secondary receptacles, this species corresponds exactly to the generic type in all other respects.

Corethromyces longicaulis nov. sp.

Receptacle abnormally developed, very small, the basal and subbasal cells arising almost side by side immediately above the foot; the basal cell long and narrow, strongly curved so as to become concave externally, its wall very thick, the cavity becoming almost obliterated; the distal half nearly horizontal, slightly bulging and becoming wholly deep black brown, except along its upper (in position) margin which is transparent yellowish and closely applied to the lower surface of the basal cell of the appendage, beyond which it hardly projects externally and which thus appears to arise from it; the subbasal cell larger, nearly hyaline externally, convex, bulging below, with a more or less distinct constriction below its slightly enlarged upper portion, which gives rise distally to the broad stalk-cell of the perithecium and sublaterally to that of the appendage. The appendage much reduced, nearly hyaline, consisting of three superposed cells; the basal (stalk-cell) squarish or rounded, the lower half or less of its inner margin connected with the subbasal cell of the receptacle. its subbasal cell smaller, bearing usually a single short antheridial branchlet; the upper cell still smaller, often hardly distinguishable, bearing one or two short antheridial branchlets and a short sterile terminal branch. Antheridia terminal, one to three in a series. Stalk-cell of the perithecium relatively very large, often curved, usually as large as the other parts of the plant combined, brown, slightly constricted at its insertion, nearly cylindrical, slightly and gradually broader toward and below the basal cells, which are relatively small and barely separate the cavity of the perithecium from that of the stalk-cell; body of the perithecium concolorous with the stalk-cell, slightly inflated, tapering distally almost symmetrically to the blunt tip, which is somewhat asymmetrical from the slightly greater prominence of one of the lip-cells; the series of wall-cells strongly spiral, completing as a rule somewhat more than one whole turn. Spores $30 \times 3\,\mu$. Perithecia $65 \times 20\,\mu$, the stalk-cell $90–110 \times 12\,\mu$. Receptacle $25\,\mu$. Appendage $30–40\,\mu$. Total length to tip of perithecium, average $200\,\mu$.

On *Stilicus angularis* Lec., at the base of the head on the upper side; Arlington, Mass., June. Sometimes associated on the same host with *C. Stilici* Thaxt.

ACOMPSOMYCES Thaxter.

The past season has yielded two additional species of this well-marked genus, the material of which is sufficiently abundant to determine with greater accuracy than was formerly possible its distinctive characters.

The appendage is very uniform in type, clearly distinguished above its basal cell: the terminal cell bearing distally a single antheridium, which is furnished with a spinous process; the subterminal cell is sterile in all cases; the cell next below producing, normally, from one to three antheridia laterally and somewhat irregularly; the antheridia being of characteristic form with large appressed venters and stout necks curved outward. Although in the single type the cavity of the perithecium appears to be, as was formerly stated, continuous with that of the stalk-cell, as in genera like *Dimorphomyces*, this is certainly not the case in the new forms described below, the basal perithecial cells of which are clearly defined. As it is highly improbable that *A. Corticariae* differs from other species in this respect, it may be assumed that the basal cells in the unique type are obscured by the abrupt curvature of the type specimen in this region.

Acompsomyces Atomariae nov. sp.

Colorless or very faintly brownish. Receptacle short, the distal cell squarish; the basal cell twice as large, narrow below, bulging beneath the base of the antheridial appendage from which it thus appears to arise terminally. Basal cell of the antheridial appendage rather long and narrow, not distinguished from the receptacle, to the distal cell of which its lower half is closely applied, while its upper half is in equally close contact with the stalk-cell of the perithecium; the rest of the appendage free, compact, slightly inflated, with evenly curved outline, faintly tinged with brown, consisting of three cells: the lower subtriangular in outline with the largest angle outward, bearing distally three closely appressed antheridia neither of which arises from its outer side; the cell next above somewhat larger, sterile, subtriangular with the largest angle external; the terminal cell smaller, separated by a horizontal septum from the terminal antheridium, the neck of which is curved inward, the spinous process conspicuous and external. Stalk-cell of the perithecium well developed, rather slender, about as long as the receptacle, the basal cells well distinguished: body of the perithecium narrower below, its inner margin nearly straight with slight constrictions at the septa, the outer bulging distinctly and more or less symmetrically; the tip distinctly but not abruptly distinguished, short, stout, slightly but rather abruptly expanded below the flat-conical apex, from the middle of which project abruptly the small short appressed prolongations of the lip-cells, forming a terminal apiculus. All the cells thick-walled. Spores very slender, 44 × 3 μ. Perithecia 36–46 × 25–30 μ, the stalk-cell 25–30 × 10 μ.

Receptacle 25 μ. Free appendage to tip of spinous process 36 × 12 μ. Total length to tip of perithecium 125–150 μ.

On elytra of *Atomaria ephippiata* Zimm., Kittery Point, Maine, and Intervale, N. H.

Acompsomyces pauperculus nov. sp.

Hyaline or nearly so. Receptacle short, somewhat bent; the distal cell very small, irregular, sharply pointed below, externally separated from the basal cell by an oblique septum; the basal cell three or four times as large, narrow below, expanded above its distal point, forming a right angle, the septum on one side applied to the base of the stalk-cell of the appendage. The latter subtriangular, its lower half in contact on the inner side with the two cells of the receptacle, its upper with the stalk-cell of the perithecium; the rest of the appendage free, relatively large, hardly inflated, its lower cell about as large as the two upper combined and bearing commonly one, sometimes three, antheridia as in *A. Atomariae;* the two cells above it nearly equal or the upper often smaller and bearing its antheridium subterminally, so that the spinous process of the latter appears to terminate the appendage, the wall distinguishing this antheridium being commonly invisible from its obliquity. Stalk-cell of the perithecium about as large as that of the appendage and similarly shaped, except that its position is reversed, separated distally from the basal cells of the perithecium by a very oblique septum: the body of the perithecium short, stout, bent asymmetrically, and considerably inflated; the inner margin straighter, the short squarish tip rather abruptly distinguished, the apex subtended by four distinct symmetrical prominences, which form a crown surrounding the four appressed prolongations of the lip-cells, which appear as a blunt, conical protuberance within it: all the cells rather thin-walled. Spores 45 × 4 μ. Perithecia 70 × 30–35 μ, the stalk-cell 12–18 × 9. Receptacle 20–30 μ. Free appendage 32–36 μ. Total length to tip of perithecium 110 μ.

On elytra, prothorax, and legs of *Atomaria* sp., Kittery Point, Maine, June.

ACALLOMYCES nov. gen.

Receptacle consisting of two superposed cells, the lower sometimes apparently obliterated or indistinguishable from the foot, the upper bearing a single perithecium and an antheridial appendage. Appendage consisting of six superposed cells, the basal cell closely associated with the receptacle and the stalk-cell of the perithecium; the terminal-cell bearing a spine-like process and becoming converted into a simple antheridium, the subterminal-cell cutting off a cell laterally which becomes an antheridium, the remaining cells sterile. Perithecium stalked, normal in structure.

A genus of very simple structure, closely allied to *Acompsomyces*, from which it differs chiefly from the characters of its antheridial appendage.

Acallomyces Homalotae nov. sp.

Hyaline, becoming faintly tinged with straw color. Receptacle very small, the lower half becoming tinged with smoky-brown; the basal cell hardly distinguished from the foot and commonly obliterated by a thickening of the walls in this region, so that the receptacle appears to be one-celled. Basal cell of the antheridial appendage separated from the distal cell of the receptacle by a somewhat oblique septum, and nearly similar to that of the stalk-cell of the perithecium, to which it is closely united on its inner side; the subbasal cell very small, and becoming scarcely distinguishable in mature individuals; the two cells above it similar, rather distinctly differentiated, the pair forming a slight symmetrical enlargement; the subterminal cell above larger and longer than these two combined, the base of the antheridium extending its whole length; both this and the terminal antheridium above it relatively large, the necks very long, straight, or but slightly bent, and somewhat divergent. Stalk-cell of the perithecium somewhat broader than long, the basal cells small, the perithecium proper relatively large and somewhat inflated above the base, distally tapering gradually to the tip; the lip-cells forming four corresponding projections, the anterior larger and broader, the posterior narrow and bluntly pointed, subtended by a more or less well-defined hump, the two lateral usually shorter, blunt, slightly divergent, sometimes not clearly differentiated, varying in shape and position. Spores 35 × 3.5 μ. Perithecia 75–95 × 25–32 μ, the stalk-cell 10–12 μ. Appendage, above basal cell to tip of terminal antheridium, 36 μ. Antheridia 21–25 μ. Receptacle 20–25 μ. Total length to tip of perithecium 125–150 μ.

On the superior surface of the abdomen of *Homalota* sp.; Intervale, N. H. Apparently very rare.

Stichomyces Stilicolus nov. sp.

Hyaline, becoming tinged with yellowish, the perithecia becoming pale amber-brown. Subbasal cell of the receptacle about twice as long as the basal cell; the stalk-cell of the perithecium and that of the appendage symmetrically divergent, or the former lateral in position. The appendage consisting of a free, relatively long, distally somewhat enlarged basal cell, and two to four much smaller distal cells; the lower of which give rise from their upper angles to short branches bearing free antheridia, usually in pairs; the distal cells producing sterile branches which are more or less elongate, hyaline, and sparingly branched. Stalk-cell of the perithecium very large and stout, inflated or distally enlarged below the rather inconspicuous basal cells; body of the perithecium relatively small, becoming amber-brown, curved strongly inward, tapering slightly to the blunt unmodified tip. Spores 35 × 3 μ. Perithecia 60–90 × 18–28 μ, the stalk 90–100 × 20 μ. Receptacle 30–40 × 18 μ. Appendages 150–200 μ.

On the thorax prothorax and adjacent legs of *Stilicus angularis* Lec., on the right side; Arlington, Mass., June.

Although very different from the type of *Stichomyces* this species seems referable to no other genus, although it might possibly be included in *Rhizomyces*.

Rhachomyces anomalus nov. sp.

Receptacle small, tapering to the large sharply pointed foot, the basal cell relatively large, the subbasal bearing a pair of stout, bristle-like, simple, black-brown, septate, hyaline-tipped appendages distally and posteriorly; the cell above it bearing a similar pair arising close to the base of the short stalk-cell of the perithecium; the terminal portion of the receptacle consisting of one or two small appendiculate cells, anterior to the stalk-cell: the five or six appendages relatively large, appressed, together more or less completely surrounding the base of the perithecium, and sometimes extending nearly to its tip. Perithecium tinged with purplish brown or nearly hyaline, the stalk and basal cells small and concealed, the body relatively long, very slightly inflated, tapering to the blunt tip, which is bent slightly toward a large, curved, horn-like, hyaline outgrowth which, arising just below the tip, extends upward beyond the apex over which it is slightly recurved (or more rarely curved away from the tip), ending in a bluntly pointed extremity. Spores 46 × 4 μ. Perithecia 75–90 × 20–25 μ, the outgrowth 35–45 × 14 μ. The longer appendages 75 μ. Receptacle 35–45 μ. Total length to tip of perithecium 110–125 μ.

On *Conosoma pubescens* Payk. Growing appressed on the bristles which cover the abdomen near its tip., Waverly, Mass. An anomalous form placed only provisionally in the present genus.

Ceraiomyces Selinae nov. sp.

Receptacle pale dirty brownish, becoming gradually somewhat broader distally, the foot blackish brown, not clearly distinguished, the subbasal cell longer than the basal. Stalk-cell of the perithecium slightly longer than broad, the basal cells forming a hyaline neck bent above the stalk-cell and not distinguished except in color from the body of the perithecium, which is nearly straight, symmetrical, dull purplish brown, tapering gradually to the slightly asymmetrical apex; the tip hardly distinguished by a slight external elevation. Stalk-cell of the appendage more or less strongly curved, or recurved so that the appendage bends downward; the insertion-cell slightly broader than long, blackish purple below, separated by a slightly oblique septum from the much larger cell above it, from which four or five branches arise distally; the branches simple, or sparingly branched, hyaline or reddish brown, sterile or bearing antheridia. Perithecia 125–150 × 30–35 μ, the stalk-cell 22–25 μ. Receptacle 200–325 × 25–30 μ. Appendages 185–400 μ. Total length to tip of perithecium 219–490 μ.

On the prothorax of *Selina Westermanni* Mostch., India Orient. (labelled "Tranquil"), Berlin Museum, No. 976.

ECTEINOMYCES nov. gen.

Receptacle consisting of a single series of superposed cells, becoming variable in number as a result of intercalary division: bearing distally a single perithecium of normal type and an antheridial appendage. The appendage consisting of a series of superposed cells, several of which above its basal or stalk-cell, bear simple antheridia, which are separated distally by oblique septa on one or both sides, much as in the genus Compsomyces.

Ecteinomyces Trichopterophilus nov. sp.

Wholly hyaline with the exception of the typically blackened foot. Receptacle varying from a very long and slender to a comparatively short and stout form, the number of superposed cells varying in number from five to fifteen, or rarely more; the cells often hardly larger distally; the series as a whole often irregular, the successive cells irregularly unequal in length. Appendage similar to the receptacle, consisting of five or six superposed cells; the basal cell united to the stalk-cell of the perithecium; the subbasal cell sterile; two or three of the cells above it bearing one to two antheridia each, which may be replaced by sterile branches similar to those which always arise from the terminal cells of the series, both laterally and terminally. Stalk-cell of the perithecium short, not distinguished from the basal cells; the body of the perithecium oblong or oval, tapering abruptly distally to form the slender well-distinguished tip, which tapers slightly to the blunt unmodified apex. Spores 25 × 3 μ. Perithecia 55–65 × 22–28 μ. Appendage including branches 75–100 μ. Receptacle 25–140 × 7–12 μ. Total length to tip of perithecium 100–220 μ.

On the elytra and abdomen of *Trichopteryx Haldemani* Lec., Intervale, N. H., August, 1901.

Laboulbenia acanthophora nov. sp.

Pale smoky brown. Perithecium relatively very large and elongate, the lower wall-cells forming a well-developed stalk, slightly narrower and paler than the main body, which is very long, straight, but slightly inflated, tapering slightly at the relatively short, stout, more deeply suffused tip; one of the lip-cells forming a median, terminal, erect, slightly curved and tapering, blunt-tipped, dark brown projection, the lower half broader, the whole more than twice as long as a similar second outgrowth which, arising near its base, curves outward on the right side (the perithecium being considered anterior). Basal cell of receptacle subhyaline, more than twice as long as broad, the subbasal subisodiametric, darker brown than the other cells; cells III and VI subequal, lying perpendicularly side by side, both several times longer than broad; cell IV about as long as cell III; cell V small, obliquely separated. Insertion-cell black, well defined. Outer appendage (broken) simple (?) its basal cells blackened; the branches curved outward, externally blackened below, and giving rise above to several successive branchlets. Spores 75 × 4.5 μ. Perithecium (main body) 220 × 50 μ; stalk 60 × 30 μ; terminal appendage 60 μ. Receptacle 220 × 65 μ. Appendages (broken) 150 μ. Total length to tip of perithecium 580 μ.

On the elytra of *Pericallus* sp.; Sharp Collection, No. 1202; East Indies.

Laboulbenia Bledii nov. sp.

Color uniformly pale dirty yellowish. Perithecium more than two thirds free, rather slender, tapering to the tip, which is more or less well distinguished; the lip-cells rather coarse and prominent, with a subterminal blackish shade on the inner side. Appendages divergent and free through the enlargement of cell V: the outer appendage simple, short, four- to five-celled; the basal cell of the inner appendage somewhat smaller than that of the outer, bearing a branch on either side similar to the outer appendage, the three straight, closely applied in a more or less compact group. Receptacle rather coarsely punctate, the basal cell slender below and suffused above the foot, rather short, and separated from the usually very long subbasal cell by a more or less distinct constriction; cells III, IV and VI subequal, more than half the upper margin of cell V free. Perithecia 100–125 × 35–40 μ. Receptacle 220–275 × 45 μ. Appendages about 90 μ. Total length to tip of perithecium 290–360 μ.

On elytra and abdomen of *Bledius jacobinus* Lec., California; Sharp Collection, No. 1174. A short stout form which is perhaps a variety of this species occurs on *Bledius basalis* Lec. in Florida (Henshaw Collection).

Laboulbenia Borneensis nov. sp.

General color pale dull straw yellow, transparent with brownish suffusions. Perithecium about two thirds free, rather stout and relatively small, externally convex, straight; a prominent hump, nearly horizontal above and distally broadly edged with contrasting black, protruding externally just below the narrow prominent abruptly differentiated tip, which is bent strongly inward and is wholly deep black; except the coarse, contrasting, hyaline, outwardly oblique lip-cells, which, on the inner side, are edged with black to the pore, the external black suffusion of the tip continuous with that of the hump. Receptacle relatively large, long and slender, the basal cell rather slender, hyaline below, distally concolorous with the subbasal cell which is much longer, distinctly stained with smoky brown, and more or less distinctly ridged or wrinkled; cell III twice as long as cell IV; cell V small, narrow, triangular; cell VI more than four fifths as long as cell V; cell VII roundish or squarish, and forming a rather abrupt external prominence below the base of the perithecium. Appendages appressed against the inner margin of the perithecium, the tip of which they hardly exceed: the insertion-cell thick, black, well defined, not deeply constricted; the outer appendage apparently simple, the basal cell slightly longer than broad, tinged with brownish; the inner appendage consisting of a basal cell almost similar to that of the outer, bearing two branches (possibly only one) which are in turn several times branched. Spores about 45 × 4 μ. Perithecia 100–125 × 35–50 μ (distally including hump × 48 μ). Receptacle 250–290 μ. Appendages (broken) 90 μ. Total length to tip of perithecium 350–380 μ.

On *Thyreopterus* (?), sp., S. Borneo; Dr. Sharp's Collection, No. 1201.

Laboulbenia cauliculata nov. sp.

Perithecium short and stout, straight or slightly curved, inflated more prominently on its inner side; dull olivaceous brown, translucent, or becoming opaque; the basal wall-cells forming a well-defined pale olivaceous or hyaline narrow stalk; the tip black, broad, not distinguished, except by its color; the rather coarse, but not prominent, lip-edges hyaline and outwardly oblique. Receptacle relatively short and small, the basal cell usually curved, narrow and subhyaline below, becoming opaque and punctate distally; the subbasal cell having only a small portion of its posterior margin free, cell III being subtriangular and overlapping it; cell IV nearly as broad as, and somewhat larger than cell III, separated from cell V, which may equal it in size, by a vertical or but slightly oblique septum; cells II–VI mostly translucent yellowish brown, often becoming opaque. Insertion-cell rather broad: basal cells of the appendages hardly distinguishable, the outer giving rise abruptly to usually two erect, or slightly divergent, stiff, bristle-like, curved branches, which are blackish brown, externally opaque, producing short hyaline branchlets along their hyaline inner margins: the basal cell of the inner appendage producing a branch on either side consisting of from one to two cells from which arise several branchlets similar to those of the outer appendages. Spores 58 × 5 μ. Perithecia 125–165 × 45–60 μ, including stalk (18–22 μ). Receptacle 90–100 × 35–40 μ. Appendages 125–185 μ. Total length to tip of perithecium 220–290 μ.

On *Colpocaccus Lanaiensis* var., No. 1226; *C. marginatus* Sharp, Kaui, No. 1228; on *Atelothrus depressus* Sharp, Lanai, No. 1231; *A. constrictus* Sharp, Molokai, No. 1234; *Mesothriseus Hawaiiensis* Sharp, Hawaii, No. 1238; on other numbers of the Perkins Collection of Hawaiian Carabidae as follows: Kaui, Nos. 1220, 1246–1248; Tahura, No. 1236; Molokai, No. 1250; Oahu, Nos. 1241, 1244; Maui, Nos. 1214, 1242, 1253, 1255.

Var. **prolixa** nov. var. Perithecium straight or slightly curved, large and greatly elongated, the body opaque; the neck usually elongate, hyaline or translucent. Receptacle as in the type, but usually translucent brownish yellow. Branches of the appendages usually more numerous and more slender, the two main branches of the inner often consisting of from three to four cells. Spores 70 × 5 μ. Perithecia 365–540 × 55 μ, including the stalk (90–220 × 36 μ). Appendages 220 μ. Total length to tip of perithecium 450–670 μ.

On *Mesothriseus tricolor* Sharp, Molokai, No. 1239; *M. collaris* Sharp, No. 1240, Molokai; on No. 1235, Maui, and No. 1260, Oahu, in Perkins Collection.

Var. **spectabilis** nov. var. Perithecium large, long, tapering more or less symmetrically above and below, strongly curved (or often recurved) outward throughout its length, including the clearly distinguished hyaline neck; dark olive brown, becoming nearly opaque, the black tip slightly distinguished, the lip-edges hyaline. Receptacle as in the type. Appendages as in the type, but one or both of the primary branches of the inner appendage hyaline, contrasting, many celled, and variably elongated through continued terminal proliferation, black branchlets arising one from each successive cell and alternating on opposite sides of the primary branch, often very long, curved upward and inward so as to cross one another, when two primary branches are present, with such regularity as often to form a lattice-work pattern; the series often complicated by the production of copious slender hyaline branchlets from the upper side of the secondary branches. Perithecia, main body, 165–200 × 40–48. Total length of appendages 290–360 μ.

On Hawaiian Carabidae from Perkins Collection, No. 1261, "*Discolus caliginosus* Blkm.," Honolulu, Oahu. No. 1179, *Metromenus mutabilis* Blkm., Oahu. On Nos. 1254 and 1256, Molokai. On No. 1259 (type), Oahu.

Laboulbenia Columbiana nov. sp.

Uniform amber yellow. Perithecium straight, free, converging abruptly from the base toward the appendages at an angle of about 45°, slightly and nearly symmetrically inflated, the tip relatively broad, with darker subterminal suffusions; the lip-cells somewhat spreading, the posterior and two lateral ones forming distinct ear-like lateral horizontal prolongations. Receptacle normal, tapering to the pointed foot, the basal and subbasal cells relatively large, the latter largest, cells IV and VI subequal; cell V separated obliquely from cell III, and somewhat longer than cell IV, which is separated by a horizontal septum. Insertion-cell thick, somewhat translucent, purplish. Outer appendage simple, short; the basal cell faintly purplish, more or less abruptly convex externally, separated by a constriction and dark septum from the subbasal cell, which is also distinguished above by a dark septum, basal cell of the inner appendage smaller than that of the outer, bearing a simple short erect branch on either side. Spores 50 × 4 μ. Perithecia 20–25 × 90–110 μ. Receptacle 150 × 30 μ. Appendages 40 μ. Total length to tip of perithecium 200–250 μ.

On bristle-like hairs on the elytra of *Anchonoderus concinnus* Reiche, Columbia; Berlin Museum, No. 1023. A single specimen 680 μ long, with a free perithecium curved outward, with only lateral lip-like projections and with appendages 300 μ in length which are otherwise exactly like those of the type, was found on the same host; but in the absence of further material it is impossible to determine whether or not it should be considered a mere variety.

Laboulbenia concinna nov. sp.

Perithecium opaque, nearly symmetrical, rather long and slender, straight, the lower half slightly inflated; tapering very gradually distally to the broad, truncate often symmetrical tip, which is barely differentiated above an inconspicuous elevation; the flat lip-edges slightly translucent brown: the basal cells forming a well distinguished short hyaline stalk, the curvature of which bends the perithecium away from the appendages at an angle of nearly 45°. Appendages forming a dense rather short slightly spreading tuft, the axis of which is coincident with that of the receptacle, copiously branched, the branchlets unilaterally disposed, rather closely septate; the septa mostly dark brown, except the very numerous paler slightly tapering extremities, which hardly reach the middle of the perithecium. Receptacle subclavate, the basal cell rather large, hyaline, narrower below; the cells above yellowish brown, inconspicuously punctate; cells III and IV bulging symmetrically and prominently below the well-defined insertion-cells; cells IV and V separated by a nearly vertical septum. Perithecia, exclusive of stalk, 15–180 × 33–45 μ, stalk 25 × 30 μ. Receptacle 150–185 × 55–65 μ. Appendages (longest) 125 μ. Total length to tip of perithecium 325–375 μ.

On upper surface of abdomen, at tip, of *Casnonia* sp., Buitenzorg, Java. On a single specimen among numerous hosts kindly communicated by Prof. H. M. Richards.

Laboulbenia corniculata nov. sp.

Hyaline becoming pale yellowish. Perithecium becoming tinged with brown, usually bent inward at the base at an angle to the axis of the perithecium, sometimes horizontal (the appendages and their insertion undergoing a corresponding change in position), relatively stout and short, somewhat irregular in form, distally slightly inflated, with a more or less well-marked external elevation just above the base; the tip rather abruptly distinguished, relatively narrow, the apex broadly hyaline, with coarse irregularly prominent lips, the lower half black, subtended externally by a well-developed, horn-like, blunt-tipped, hyaline outgrowth, which is larger than the whole tip and slightly curved outward. Receptacle colorless or pale yellowish, the basal and subbasal cells of about the same diameter throughout, subequal, rather long and slender in contrast to the small, compact distal portion; cells III and IV subequal, or cell IV larger, the septum between them nearly horizontal; cell V triangular, relatively small and, like cell IV, slightly suffused above with brown, below the very broad, thin, clearly defined, black insertion-cell; the upper hyaline angle of which protrudes between the basal cells of the appendages. Appendages similar in type to those of *L. Galeritae*, hyaline, the outer consisting of about five to seven very obliquely superposed cells, bearing the branches distally and externally; the basal part of each branch consisting usually of two short brownish inflated or squarish cells, distinguished by constrictions and dark septa, the distal part about equal to the basal in length, or somewhat longer, often unicellular, hyaline, stained reddish in the types, very thick-walled; abruptly broader, as a rule, than the basal part; the tip bent slightly, tapering or even slightly inflated. The inner appendage similar to the outer, the basal part of the branches usually three-celled, the lowest, and sometimes that next above it, consisting of a single cell which bears terminally a single antheridium, the long neck of which is rather abruptly bent near the middle. Spores 90–110 × 7 μ. Perithecia 175–240 × 60–80 μ. The horn-like projection about 40 × 22 μ. Receptacle 325–475 μ. Greatest total length of appendages 450–600 μ.

On inferior surface of *Galerita carbonaria* Mannerh., Brazil; Berlin Museum, No. 960.

Laboulbenia Craspidophori nov. sp.

Perithecium relatively large, almost wholly free, erect, usually bent slightly inward toward the base and outward toward the tip, becoming almost opaque dark brown; the tip rather small, hardly distinguished, bent slightly outward; the lip edges pale, outwardly oblique. Receptacle usually rather slender, the basal and subbasal cells relatively large and long, faintly suffused with brown, the upper longer, separated from cell III by a narrow horizontal septum and from cell VI by a very oblique or almost vertical one; the distal portion of the receptacle narrow, but rather abruptly distinguished, especially cells III and IV, more or less deeply suffused, becoming concolorous with the perithecium. Outer appendage long, slender, simple, or rarely distally furcate, curved outward; the basal cell of the inner appendage relatively very small, bearing a single, erect, short, one-celled branch; which, after bearing several antheridia distally, soon disappears. Spores 75 × 5 μ. Perithecia 180–210 × 48 μ. Receptacle 150–325 μ. Appendages longer, 300–375 μ. Total length to tip of perithecium 280–500 μ.

On *Craspidophorus tenuipunctatus* Laf., East Indies; British Museum, No. 592. On *Panagaeus* (*Eudema*) *Symei* Murr., Old Calabar, Africa; British Museum, No. 594. On *Microsomus* (*Eudema*) *vicinus* Murr., Gaboon, French Congo, Africa; Berlin Museum, No. 920. On *Craspidophorus* (?) sp., U. S. National Museum, Liberia, Africa, No. 8.

Laboulbenia curvata nov. sp.

Perithecium large and long, strongly curved inward throughout, rich deep red-brown, the inner half usually opaque, the outer more or less translucent or becoming opaque; the basal wall-cell forming a well-defined, short, hyaline stalk; the tip short and broad, suffused about the terminal nearly median pore. Receptacle short and stout, translucent, more or less deeply and unevenly suffused with dirty brown; the basal cell paler, or nearly hyaline below, often as long as the rest of the receptacle above it, obscurely punctate; the subbasal cell short, a very small part only of its anterior margin free; cells III and IV subequal, or cell IV larger, separated by a very oblique septum, a small portion only of the outer margin of cell IV free. Insertion-cell slightly oblique, brown or black, the suffusion involving the basal and even the subbasal cells of the appendages. Appendages similar to those of *L. perplexa*, consisting of from six to eight obliquely superposed cells, the branchlets usually much shorter than the perithecium, the two lowest cells of the branches of the inner appendage bearing distally a pair of deep brown divergent antheridia, the long slender necks curved rather abruptly distally. Spores 70–75 × 4 μ. Perithecia 325–400 × 50–70 μ, including the stalk (40–55 μ). Receptacle 185–220 × 75–90 μ. Total length to tip of perithecium 500–600 μ.

At base of anterior legs of *Galerita carbonaria* Mannerh., Brazil; Berlin Museum, No. 960: and of *Galerita* sp., Hope Coll., No. 259.

Laboulbenia dentifera nov. sp.

Perithecium relatively large, as long or longer than the receptacle, straight, erect, almost wholly free, rather dark dull brown; the outer margin distally converging rather abruptly in an almost straight line to the apex; one (the right) of the lateral lip-cells prolonged obliquely inward and upward to form a large tooth-like projection. The receptacle relatively short and stout, the basal cell longer, hyaline and contrasting, except distally, where it is involved in the general uniform dark dirty olive-brown suffusion of the rest of the receptacle, the cells of which are short and broad, punctate, hardly distinguishable. The insertion-cell thick, black, rather narrow, the basal cell of the outer appendage short and stout, bearing distally an inner and an outer branch, the basal cell of the outer bearing two branches, the black contrasting constricted base, only, of the outer persisting; the appendages otherwise hyaline, stout, tapering slightly; the basal cell of the inner appendage very small, roundish, bearing a branch on either side with single antheridia near the base. Perithecium 125 × 32 μ, the tooth-like appendage 20 μ. Receptacle 115 × 55 μ. Appendage 220 μ. Total length to tip of perithecium 240 μ.

A single specimen on the margin of the elytra of *Notiobia disposita* Bates; British Museum No. 678; Chontales, Nicaragua.

Laboulbenia Disenochi nov. sp.

Perithecium from two thirds to three quarters free, transparent, yellowish, becoming more or less irregularly and for the most part rather faintly suffused with blackish brown; the tip relatively large, black, more or less well distinguished, the black suffusion not abruptly limited and extending some distance downward externally; the lips outwardly oblique, hyaline about the pore. Receptacle rather short and stout, normal, pale yellowish, often becoming tinged with brown distally where it is rather faintly striate. Insertion-cell well defined, slightly oblique. Outer appendage consisting of an erect series of usually four hyaline cells, each of which bears externally (the upper terminally) a stiff simple branch of variable length, curved outward and upward, deeply blackened externally, the notched often broadly hyaline upper (inner) margin contrasting: the basal cell of the inner appendage bearing a branch on either side, mostly two- to three-celled, and resembling the outer appendage, except for the presence of basal antheridial branchlets on which the brown antheridia are borne terminally, usually in pairs. Spores 65 × 6 μ.

Perithecia 150–165 × 58–62 μ, sometimes smaller. Receptacle 185–220 μ. Total length 290–360 μ.

On *Disenochus fractus* Sharp, Kaui, No. 1222; *D. agonoides* Sharp, Haleakala, Hawaii, No. 1229; *D. aterrimus* Sharp, Kaui, No. 1218; *D. sulcipennis* Sharp, Kaui, No. 1219; *Brosconegneus optatus* Sharp, Oahu, No. 1215.

Laboulbenia Dryptae nov. sp.

Perithecium usually nearly straight, often nearly symmetrical, the region corresponding to the basal wall-cells usually distinctly differentiated, hyaline or transparent, brownish, becoming darker in older individuals, concolorous with the basal and other cells below; but in general contrasting strongly, but not abruptly, with the rich dark brown of the rest of the perithecium above it, which may become opaque; the tip often bent slightly outward, not abruptly distinguished; the inner lip-cells more prominent and deeply suffused, except the edges. Receptacle rather short, punctate, becoming more or less, and rather irregularly, stained with brown, often blackish along the posterior margin up to the insertion-cell, which is thick, well defined, contrasting. The appendages rather slender; the outer nearly straight in normal individuals, simple, divergent, rigid, tapering; the basal cell more than twice as broad, subhyaline, with brownish suffusions next its deep black contrasting outer wall, the opacity sometimes involving the subbasal cell, which is similar and similarly blackened externally, as is the cell next above: the inner appendage simple, or more often once branched above the subbasal cell, divergent like the outer, brownish yellow, the antheridia borne singly from the lower cells. Perithecia 110–150 × 35–40 μ. Receptacle 135–180 μ. Longer appendages 220 μ. Total length to tip of perithecium 220–250 μ.

On *Drypta ruficollis* Dej., Natal, Africa; British Museum, No. 506.

Laboulbenia dubia nov. sp.

General form closely resembling that of *L. rigida*, the shape of the perithecium similar, except that the tip is smaller, more abruptly distinguished; the lips outwardly oblique, the blackish inner lip-cell more prominently rounded and subtended by a slight, usually distinct rounded elevation, which gives the tip a characteristic outline; the body dark translucent brown, wholly free. The receptacle very thick-walled, pale dirty yellowish brown, deeper distally. The basal cells of the appendages subequal, the outer bearing two branches radially which are simple, or the inner once branched above its subbasal cell; the antheridia borne singly at the lower septa or on short branchlets: all the branches of both appendages relatively stout, erect, rather closely septate, about six- to eight-celled, curved slightly outward, tapering to the blunt extremities, which but slightly exceed the apex of the perithecium. Spores 55 × 5 μ. Perithecia 165–185 × 55–65 μ. Receptacle 220–325 × 75 μ. Longer appendages 220 μ. Total length to tip of perithecium 440–500 μ.

On the abdomen of *Philonthus politus* Linn., Alverstoke, England; British Museum, No. 363. A form which is doubtfully separated from the almost endless variations of the *L. flagellata* type and which, though sufficiently well marked in the type material, may prove a mere variety. No other closely allied form, however, is known to occur on species of *Philonthus*.

Laboulbenia Euchilae nov. sp.

Slender, nearly uniform pale dirty yellowish throughout. Perithecium relatively small with a faint brownish tinge, about four fifths free, erect, straight; the tip fairly well distinguished, the inner lip-cells large, prominent, rounded, the lip-edges outwardly oblique. Receptacle slender and long, punctate; the subbasal cell much longer than the basal, nearly isodiametric, or swollen above the basal cell and broader than the distal part of the receptacle, which is separated from its distal end by a prominent constriction; cells III and VI subequal, or cell III larger; cell V very small; cell IV longer than broad, becoming externally tinged with brownish. Insertion-cell relatively thick, free, narrow. Appendages becoming wholly dark brown, in contrast to the rest of the plant; the outer simple or once furcate above the third cell; the basal cell more than twice as long as broad; the basal cell of the inner appendage less than one half as large as that of the outer, bearing a usually simple branch on either side, which bears single antheridia near its base. Spores 60 × 4.5 μ. Perithecia 100 × 30 μ. Receptacle 220–250 × 35–40 μ. Appendages 150 μ. Total length to tip of perithecium 300–350 μ.

At the base of the anterior legs of *Euchila flavilabris* Dej., Brazil; Berlin Museum, No. 938.

Laboulbenia Eudaliae nov. sp.

Perithecium transparent brown, nearly straight externally, the inner margin somewhat convex; the inner lip-cells deeply suffused, contrasting, coarse, erect, prominent, the lip edges hyaline, outwardly oblique. Receptacle normal, rather short and stout, somewhat prominently rounded below the perithecium; cell III smaller than cell VI; cells IV and V broad, equal in length, prominent below the broad insertion-cell. Outer appendage consisting of from three to four mostly roundish or flattened superposed cells, constricted at the blackened septa, which become oblique by the proliferation of each cell from its inner side to form a single, usually simple, erect branch; the branches eventually stout, rather closely septate, thick-walled; the basal cell of the inner appendage half as large as that of the outer, producing one or more simple branches similar to those of the outer appendage, and short antheridial branchlets, which bear the brown, slightly curved antheridia in a rather dense group. Spores 72 × 5 μ. Perithecia 150–155 × 55 μ. Receptacle 170–220 μ. Appendages 180–225 μ. Total length to tip of perithecium 250–350 μ.

On *Eudalia latipennis* MacLeay, Port Denison, Australia. On elytra and legs; Berlin Museum, No. 952.

Laboulbenia exigua nov. sp.

Perithecium relatively very small, the upper third or less free, reddish brown, deeper than the receptacle, erect, straight, nearly symmetrical; the tip clearly and rather abruptly distinguished; the lip-cells with darker shades, distinctly spreading, especially externally, the edges hyaline, nearly horizontal. Basal and subbasal cells of the receptacle subequal, small. The insertion-cell broad, well marked. The basal cells of the appendage clearly distinguished, subequal, about as broad as long; the outer bearing a single, simple erect branch, the basal cell of which is abruptly narrower, squarish, deeper brown, the septa dark, the rest of the branch simple, obscurely septate, distally hyaline, tinged with pale brown below; the inner appendage like the outer, shorter, paler, the brownish short basal portion of the single simple branch smaller and without septa. Spores about 40 × 4 μ. Perithecia 86 × 26 μ. Receptacle 220 μ. Appendages 184–150 μ. Total length to tip of perithecium 250–290 μ.

On the inferior thorax of *Chlaenius biguttatus* Motsch., Japan; Berlin Museum, No. 923.

Laboulbenia flaccida nov. sp.

Perithecium free except at its base, brown, straight, slightly inflated below, tapering gradually to the broad blunt apex; the tip scarcely if at all distinguished, marked by darker shades; the lips coarse, not prominent. Receptacle yellowish or nearly hyaline, becoming slightly tinged with brownish and faintly punctate above the basal cell, which, as well as the subbasal, is relatively large, the two subequal, usually more or less abruptly distinguished from the broader compact portion, the cells of which are relatively small, the upper half or less of cell IV free, projecting externally to the insertion cell, which is rather small and thick; deep, slightly reddish, brown, the deep suffusion continuous with a corresponding external coloration of the basal cell of the outer appendage and the three to four lower cells of the outer branch, which arises from it and which curves more or less strongly outward; the distal portion curved upward, hyaline, tapering, flaccid, each of the suffused cells giving rise distally, from the inner side, to a more or less erect, simple (or the lower sometimes furcate) hyaline branchlet, the lower cells of which are rather long and slender, inflated; the distal portion tapering, thin-walled, usually becoming flaccid; the inner branch of the appendage furcate above its erect basal cell, the branchlets divergent and similar to those of the outer branch, or short and bearing antheridia usually in pairs; the inner appendage consisting of a basal cell about as large as the outer, bearing a hyaline branch on either side which is usually furcate above its basal cell, the branchlets similar to those of the outer appendage. Spores about 40 × 4 μ. Perithecia 80–90 × 25–30 μ. Receptacle 90–125 μ. Appendages 150–200 μ. Total length to tip of perithecium 175–220 μ.

On legs of *Casnonia subdistincta* Chaud.; British Museum, Biologia Collection, No. 704.

Laboulbenia Tachyis nov. sp.

Form slender. Perithecium tinged with brown, relatively small, narrow, erect, very slightly curved outward distally, about three quarters free, tapering slightly to the relatively broad, hardly differentiated tip; the lip-cells coarse, outwardly oblique, with hyaline edges. Receptacle slender, the basal and subbasal cells hyaline, elongate, nearly equal in length; the distal portion tinged with brown. Basal cell of the outer appendage forming a characteristically modified free papillate prominence which occupies the outer third of the insertion-cell, projecting slightly beyond it, its inner half giving rise to a single rather short branch which may bear one or two branchlets: the basal cell of the inner appendage giving rise to a small erect branch on either side. Perithecia 60 × 16 μ. Receptacle 110 μ. Appendages 45 μ. Total length to tip of perithecium 155 μ.

On *Tachys incurvus* Say, Cambridge. On *Tachys* sp., Cocoanut Grove, Florida, December.

Laboulbenia Formicarum nov. sp.

Basal cells of the perithecium small, compact, not distinguished from the body, which is suffused by darker brown shades and tapers from near the broad base, with but slight inflation, to the well-distinguished tip; which is more or less distinctly curved outward, with subterminal blackish suffusions on both sides; the rather blunt, coarse-lipped apex outwardly oblique. Receptacle abnormal in form, very short and stout, the basal cell small hyaline, narrow below, abruptly broader distally below the subbasal cell, which is broader than it is long and bulges prominently externally, giving the plant a humpbacked habit; cell III small, broader than it is long; cell IV small, squarish, separated from cell V, which is slightly smaller, by a nearly vertical septum. Appendages normal, insertion-cell blackened, but not deeply, the outer appendage simple, the three lower cells inflated, the subbasal cylindrical and distinguished by dark septa. Spores 30 × 2.5 μ. Perithecia 50–60 × 16–18 μ. Receptacle 30–35 × 18 μ. Longest appendages 90 μ. Total length to tip of perithecium 70–80 μ.

On all parts of *Lasius Americanus* M. and of *Formica neogagates* M., Cambridge, Mass. I am much indebted to Mr. Theo. Pergande for the identification of these hosts. *Rickia Wasmanni* Cavara, a genus with compound antheridia belonging among the Peyritschiellae, is the only other form that has been found on true ants.

Laboulbenia fusiformis nov. sp.

Perithecium relatively long and slender, becoming nearly opaque and marked by scattered blackish points more conspicuous near the base, sometimes nearly obsolete, erect, or slightly divergent above the stalk, straight, subfusiform; the basal wall-cells forming a short hyaline stalk; the tip often rather abruptly differentiated, darker, often curved outward; the lips suffused, or the edges hyaline. Receptacle very long and slender; the basal cell short, nearly hyaline; the rest pale dull browish, indistinctly transversely striate-punctate; cell II very long, isodiametric throughout, except where its distal end is obliquely overlapped for a short distance by cell VI, separated by a short horizontal septum from cell III, which is much elongated and is separated from cell IV by an oblique septum, above which the receptacle shows a slight but abrupt contraction in diameter; cell IV more than twice as long as cell V.

Insertion-cell black and thick, the blackening involving the greater portion of the basal cell of the outer appendage. Appendages very similar to those of *L. perplexa,* the basal cells of the branches somewhat darker, the distal part somewhat shorter. Spores 90 × 7 μ. Perithecia 450–580 × 60–75 μ including stalk (40 μ). Receptacle 500 μ to over 1 mm. × 70–75 μ. Appendages 275–375 μ. Total length to tip of perithecium 1.65 mm.

On the inferior surface of the prothorax of *Galerita carbonaria* Mannerh., Brazil; Berlin Museum, No. 960.

Laboulbenia Hawaiiensis nov. sp.

Perithecium variously suffused with dark olive brown, sometimes transparent brownish yellow, becoming nearly opaque, about one quarter, or only the tip, free from the receptacle, rather short and stout; the tip black, often bent outward, tapering rather abruptly; the lip-edges hyaline, rather prominent and outwardly oblique. Receptacle pale reddish amber or yellowish, becoming variously suffused with dark olive brown, especially distally; the basal cell more commonly narrow and hyaline below; abruptly broader and suffused with dark olive brown below the base of cell II, which is often abruptly broader through a slight inflation in this region, and similarly suffused, the suffusions often faint; cells III to V often deeply suffused with olive brown, faintly striate; cells III and IV subequal; cell V extending more or less prominently upward along the inner margin of the perithecium which is further bordered by cells III and IV; the insertion-cell carried obliquely outward by these modifications. Basal cell of the outer appendage usually hyaline, bearing normally two branches; the basal cell of the inner hyaline, that of the outer often small and blackened, bearing two branchlets; the outer shorter, more slender, opaque (usually broken); the basal cell of the inner appendage similar to that of the outer, often protruding somewhat inward, bearing two branches like the outer, or less often laterally, that next the perithecium bearing one or more antheridial branchlets; the antheridia terminal in groups of two or more, sometimes densely clustered; the other bearing similar antheridial branchlets or more often one or more long sterile branchlets like the outer appendage; all the sterile branches usually elongated in a sweeping curve toward the perithecium, commonly rich brown or nearly opaque, sometimes hyaline, sometimes multiplied by branching close to the base. Spores 65 × 5 μ. Perithecia 90–125 × 40–55 μ. Receptacle 200–335 μ. Appendages 290–725 μ. Total length to tip of perithecium 230–360 μ. Specimens on "*Cyclothorax*" and "*Bembidium*" smaller.

On *Atelothrus erro* Blk., Maui, No. 1230; *A. gracilis* Sharp, Lanai, No. 1232; *Disenochus sulcipennis* Sharp, Kaui, No. 1219; *Mauna frigida* Blk., Maui, No. 1221; *Colpodiscus lucipetens*, Maui, No. 1217; *Colpocaccus Hawaiiensis* Sharp, Hawaii, No. 1224; *C. Lanaiensis* Sharp, Lanai, No. 1225; *C. posticatus* Sharp, Kaui, No. 1227; *Mesothriscus muscicola* Blk., Hawaii, No. 1237; *M. tricolor* Sharp, Molokai, No. 1239; *M. alternans* Sharp, Kaui, Nos. 1220 and 1243. On the following numbers in the Perkins Collection: Kaui, Nos. 1248, 1249, 1251, 1257, 1270; Maui, Nos. 1214, 1219, 1226, 1263, 1264, 1266, 1269, 1270, 1271; Molokai, Nos. 1250, 1267, 1272, 1274; Hawaii, Nos. 1265, 1268.

Laboulbenia Helluodis nov. sp.

Perithecium becoming rich brown, free, usually somewhat broader distally, slightly inflated; the distal wall-cells forming a clearly defined, somewhat constricted, short neck; the tip rather abruptly distinguished, bent very slightly outward, opaque except the broadly hyaline or subhyaline, flat-topped lips. Basal cell of receptacle hyaline or subhyaline, inflated, broader than, and contrasting with, the opaque subbasal cell, which is of about equal length and forms a constricted region; the distal portion of the receptacle small, rather abruptly broader, subsymmetrically triangular, the external margins even; cells III, IV and VI subequal, more or less deeply suffused with brown; cells III and IV more deeply suffused externally, about half the upper margin of cell IV free and forming a distinct prominence external to the jet black insertion-cell; cell V small and roundish. Appendages very similar to those of *L. Planetis;* the outer consisting of a divergent main axis of three cells, broadly and deeply blackened externally and at the septa; the two lower bearing a single, usually simple, divergent branch distally from the inner side; the upper bearing two such branches terminally, the outer deeply suffused with blackish brown, especially toward its base, and directly continuous with the suffused portion of the main axis; the basal cell of the inner appendage half as large as that of the outer, giving rise to a rather short simple branch on either side, or often itself simple, bearing one or two solitary antheridia near its base. Spores about 85 × 5 μ. Perithecia 150–165 × 40–55 μ including the stalk (20 μ). Receptacle 185–220 × 60 μ. Longest appendages 290 μ. Total length to tip of perithecium 360 μ.

On *Helluodes Nebrioides* Nietn., Ceylon; Berlin Museum, No. 1050.

Laboulbenia Helluomorphae nov. sp.

Perithecium straight or slightly divergent, hyaline to yellowish, becoming somewhat tinged with smoky brown; the basal wall-cells forming a scarcely apparent short stalk; the blackish tip abruptly distinguished; the coarse translucent lip-edges outwardly oblique. Receptacle normal hyaline to straw-yellow, distally obscurely punctate and slightly suffused with faint brownish shades; cells III, IV and VI subequal; cell V rather large, its inner margin more than one half free, nearly vertical, the thick slightly oblique insertion-cell thus free above the base of the perithecium. Outer appendage of the *L. Galeritae* type; the basal cell large, more or less suffused; the two or three cells above it obliquely superposed, hyaline; all the cells bearing distally and externally single simple branches, the basal part consisting of two cells, becoming clear brown, constricted at the dark septa; the distal part hyaline, or more faintly brownish, distinguished by a deep brown suffusion at the base: the inner appendage consisting of a very small hyaline basal cell, giving rise on either side to a very short branch, consisting of from one to two cells; the basal one bearing an antheridial branchlet, consisting of a single cell terminated by a solitary abruptly brownish antheridium. Spores 70 × 7 μ. Perithecia 130–140 × 35–40 μ. Receptacle 220–180 μ. Appendages 185–300 μ. Total length to tip of perithecium 290–325 μ.

On *Helluomorpha melanaria* Reiche, Ega, Amazon; British Museum, No. 527. On *Pleuracanthus brevicollis* Dej., Surinam; Berlin Museum, No. 942. On the elytra.

Laboulbenia humilis nov. sp.

Rather slender, nearly uniform dirty olivaceous. Perithecia relatively large, straight, erect, the outer margin nearly straight to the lips, or slightly convex; the tip not well distinguished, with darker shades below the rather coarse, pale, outwardly oblique lips. Basal cell of the receptacle relatively large, somewhat paler; the cells above it darker, and transversely, rather coarsely, striate-punctate; cell III separated from cell II by short horizontal septum; cell VI separated from cells II and III by oblique septa; the anterior margin of the receptacle slightly convex, the posterior strongly divergent above cell II. Insertion-cell relatively narrow and thick, the outer appendage short, simple, tapering, four to five-celled, slightly divergent above the basal cell, which is rather long, irregular and paler; the subbasal cell separated by a more or less distinctly oblique septum, where the appendage is slightly geniculate; the inner appendage erect, simple, or the small basal cell producing two pale, short, few-celled, simple branches. Spores about 50 × 5 μ. Perithecia 100–125 × 30 μ. Receptacle about 185 μ. Appendages 60–75 μ. Total length to tip of perithecium 250–275 μ.

On the elytra of *Chlaenius monogrammus* Laf., Hong Kong; British Museum, No. 606. On *C. cyaniceps* Bates, Hong Kong; Berlin Museum, No. 925.

Laboulbenia incerta nov. sp.

Perithecium erect or slightly divergent, rather pale or rarely darker translucent dirty brown, considerably, almost symmetrically, inflated, slightly broader below; the basal wall-cells forming a very short, often almost obsolete, hyaline stalk; the tip rather abruptly distinguished, usually bent slightly outward, short, stout, wholly suffused, darker below; the apex evenly rounded as a rule; the outer lip-cells curved outward to the external pore. Receptacle normal, hyaline, or faintly yellowish; the distal portion short and broad; the basal cell relatively short; cells III and IV subequal. The insertion-cell slightly oblique, black, contrasting, the opacity usually involving a part or the whole of the basal cells of the outer and inner appendages. Appendages similar to those of *L. perplexa*, more compact, with shorter branches; outer appendage consisting of about six to eight obliquely superposed cells, the branches divergent, curved upward; the two cells of the basal part stout, clear brown, constricted at the dark septa, roundish to long-oblong; the distal part rather stout, unicellular, tapering to a blunt apex and seldom reaching beyond the tip of the perithecium; the two branches of the inner appendage similar to the outer, except that the two lowest branches consist of a single cell, its basal septum alone dark and constricted, bearing distally a single brown antheridium. Spores 95–100 × 8 μ. Perithecia 185–250 × 60 μ. Receptacle 185–290 × 70–80 μ. Appendage 200–250 μ. Total length to tip of perithecium 360–500 μ.

On the superior and inferior surface of *Galerita carbonaria* Mannerh., Brazil; Berlin Museum, No. 960.

Laboulbenia insignis nov. sp.

Perithecium free except the base, straight, nearly symmetrical, brown, translucent or transparent, paler below; the wall-cells spiral, describing about half a turn from the base to the tip; which is abruptly distinguished, nearly symmetrical, abruptly opaque below the broadly subhyaline, faintly brownish lips, which are not prominent and form a somewhat angular-truncate or very slightly oblique apex. Distal and basal portions of the receptacle very thick-walled, punctate, abruptly distinguished; the basal part hyaline or faintly reddish brown, parallel with the perithecium; the subbasal cell larger and longer, more or less prominently and gradually constricted toward the middle; the straight anterior margin of the distal portion abruptly divergent and formed by cell VI and the secondary stalk-cell, both of which become deeply suffused with black brown, except the base of cell VI which is concolorous with the subbasal cell, from which it arises laterally and subterminally: cell III and IV subequal and separated by a slight constriction, faintly brownish or subhyaline, their margins slightly convergent toward the thick, jet black, constricted, slightly oblique insertion-cell; the inner margin of which is free from the base of the perithecium. Basal cell of the outer appendage squarish or slightly longer than broad, bearing above its outer upper angle a single opaque contrasting short branch (broken in the types but evidently bearing several branchlets); the basal cell of the inner appendage somewhat smaller, bearing a branch on either side; each branch thrice closely branched, their short basal cells, which are subhyaline or faintly reddish brown, each successively bearing two or three divergent branchlets; the series ending in branchlets of the fourth or fifth order, which are deep brown, slender, stiff (the extremities broken in the types), divergent, thirty or more in all. Spores 185 × 6.5 μ. Perithecia 290–310 × 80–87 μ. Receptacle 365–540 × 150 μ. Appendages (broken) 2207 μ or more. Total length to tip of perithecium 600–650 μ.

On inferior thorax of *Thyreopterus brevicollis* Kl., Madagascar; Berlin Museum, No. 934.

Laboulbenia Japonica nov. sp.

Short and stout, unevenly suffused with smoky or faintly olive brown. Perithecium relatively very large and long, more or less distinctly curved toward the appendages; the base subhyaline, the body evenly dark, slightly olivaceous brown, scarcely inflated, tapering very slightly to the stout, evenly rounded, opaque, hardly differentiated tip; the longitudinal series of wall-cells slightly spiral, describing about one quarter of a turn or somewhat more. Receptacle relatively small, short and stout, the basal and subbasal cells hyaline, contrasting, the latter somewhat larger, separated by an oblique partition from cell III, which is small, subtriangular and deeply suffused; cell IV larger, suffused, as is cell V, which is relatively large and long-oval; cell VI deeply suffused, extending down beside and to the base of cell II; the cells above it also suffused and more or less indistinguishable. Insertion-cell large and unmodified, bearing a group of cells as in *L. orientalis*, which give rise to three crest-like series of branches; one outer and two inner and lateral; the outer series antero-posterior, the inner ones obliquely lateral; the branches closely set, about four or five in each series, and two to four times successively branched in the plane of the series; the basal cells sometimes bearing more than two branchlets, the cells subequal, slightly inflated, the septa, except those of the distal cells, dark blackish; the whole mass curved outward, and slightly downward, forming a short dense, rather compact tuft. Spores 80 × 5 μ. Perithecia 235 × 70 μ. Receptacle 185 × 70 μ. Appendages 100–150 μ. Total length to tip of perithecium, average, 420 μ.

On anterior legs of *Brachinus* sp., Japan; Sharp Collection, No. 1188.

Laboulbenia Latonae nov. sp.

Perithecium and receptacle much as in the larger and more typical forms of *L.* "*elongata*"; the perithecium usually deep brown, nearly opaque; the receptacle often enormously developed through the elongation of cell II. The outer appendage consisting of a large, broad, subhyaline basal cell, which gives rise to two or more, often three, or rarely more, branches in a crest-like radial series, their basal cells small squarish subhyaline; producing, as a rule, a pair of greatly elongated simple deep rich brown flexuous tapering branchlets; the basal cell of the inner appendage giving rise in general to a branch on either side, one of which resembles the branches of the outer appendage, and gives rise to long slender brown branchlets; the other commonly short, and bearing one or two antheridial branchlets; the small straight antheridia borne in compact groups of two to four members. Spores 70 × 5 μ. Perithecia 175–220 × 60 μ. Longest appendages 1150 μ, the average 750 μ. Total length to tip of perithecium 325–1125 μ.

On all parts of *Latona Spinolae* Guér., Bogota; Berlin Museum, No. 834.

Laboulbenia media nov. sp.

Perithecium usually narrow, straight, erect, sometimes slightly inflated, the basal wall-cells forming a relatively long, well-distinguished, hyaline or subhyaline stalk; the basal cells vertically elongated; the ascigerous portion marked by distinct transverse blackish striations throughout, and becoming opaque or nearly so; the tip erect, the apex slightly asymmetrical, subtended by a darker shade on the inner side.

Receptacle medium to short, the basal cell hyaline, becoming dirty brownish yellow; cells III and IV deep brownish, the suffusion more or less confined to these cells or involving also cell II and other adjacent cells, the more deeply suffused parts coarsely punctate; cell III extending upward beside cell IV and distally forming a more or less well-defined external prominence, sometimes as distinct as that formed by the basal cell of the outer appendage; cells V and VI wholly or partly hyaline, becoming dirty brownish yellow. Insertion-cell small, almost wholly external to cell V, hardly distinguishable, translucent and only finally involved in the opacity of the adjacent cells. Appendages resembling those of *L. Galeritae*, the basal cell of the outer appendage large, subtriangular, forming distally and externally a rounded prominence, from the upper nearly horizontal surface of which arises the first of the oblique series of branches; the remaining cells of the appendage hyaline, successively smaller, from about six to ten in number, forming the oblique series characteristic of this type of appendage; each bearing externally a single branch; the branches relatively short, the basal cell more elongated than the subbasal, both purplish brown, slightly constricted at the dark septa; the rest of the branch shorter or not much longer than the basal part and consisting of from one to three cells, hyaline or brown below, blunt-tipped; the basal cell of the inner appendage giving rise to the characteristic series of superposed ramiferous cells on either side which are closely united to the outer appendage, the four or five lower cells of each series producing, as a rule, unicellular branchlets terminated by a pair of brown long-necked antheridia; the sterile branchlets of the upper and distal cells similar to those of the outer appendage. Spores 45 × 4.5 μ. Perithecia 180–290 × 40–48 μ, the stalk 55–125 × 25–35 μ. Receptacle 165–220 μ. Appendages, greatest total length, 165–185 μ. Total length to tip of perithecium 350–725 μ.

On *Galerita* sp. (tips of elytra), Venezuela; Paris Museum, No. 75. On tip of abdomen, inferior, of *Galerita Lecontei* Dej.; British Museum, No. 521; Costa Rica.

Laboulbenia Megalonychi nov. sp.

Hyaline, becoming very faintly tinged with yellowish brown. Perithecium about one half or more free, slightly divergent, short, stout, tapering slightly from the broad basal half to the tip; which is, as a rule, bent abruptly inward, rarely outward, or erect, rather small and well-distinguished by its long external contrasting broadly blackened margin, and by a shorter broadly blackened area below the small, prominently rounded inner lips; the lip-edges horizontal, or nearly so. Receptacle rather long and slender; cells I and II of nearly equal diameter, the latter large; while between it and cell III and VI the receptacle may be abruptly and strongly twisted; cells III and VI subequal, the former higher; cell IV slightly smaller; cell V relatively large, the inner half or less of its inner margin free between the perithecium and the broad, thick, black, contrasting insertion-cell. Outer appendage consisting of three superposed, distinctly brown, successively smaller cells, each nearly twice as long as broad; each of the two lower producing distally from its inner side a simple hyaline abruptly erect branch, the terminal one bearing two such branches distally. Basal cell of the inner appendage half as large as that of the outer, bearing on each side a branch which bears one or more branchlets and, bending across the outer appendage, is often characteristically recurved beyond it. Spores 58 × 5 μ. Perithecia 100–125 × 40–50 μ. Receptacle 275–360 μ. Appendages 185 μ. Total length to tip of perithecium 300–435 μ.

On the right inferior prothorax of *Megalonychus patrobioides* Chincoxo, E. Africa; Berlin Museum, No. 1037. On *M. Angolensis* Harold, No. 1039.

Laboulbenia notata nov. sp.

Perithecium straight, erect, hardly more than the tip free, almost or quite opaque distally, paler below; covered with scattered wart-like protuberances; the tip abruptly distinguished, opaque, separated from the body of the perithecium by a subhyaline zone; the apex broadly hyaline, truncate, somewhat angular. Receptacle elongate throughout; the basal cell relatively small, dark brown, somewhat contrasting, the rest pale brownish yellow to dark brown; distally more or less conspicuously marked by scattered brown warts; cell VI very long and slender, overlapping cell II for some distance, the cells above it also unusually elongate; cell III very long and slender, separated from cell II by a narrow horizontal septum; cell V relatively small. Insertion-cell somewhat oblique, thick, black, not abruptly narrower than the cells below it. Outer appendage simple, short, the basal cell several times longer than broad, the remainder more slender, brown, contrasting with the basal cell. Inner appendage consisting of a basal cell more than half as long as that of the outer, bearing distally on either side a short simple branch similar to the outer appendage but paler. Perithecia 271 × 55 μ.

Receptacle 1100 × 75 μ. Appendages about 185 μ. Total length to tip of perithecium 1150 μ.

A single mature specimen on *Thyreopterus armatus* Cast., Madagascar; Berlin Museum, No. 933.

Laboulbenia obliquata nov. sp.

Color pale amber yellow. Perithecium almost wholly free, distally bent abruptly outward or almost recurved; the base slightly inflated; tapering somewhat, distally, to the apex; the tip hardly distinguished, irregularly bent or twisted; the asymmetrical lip-cells forming irregular projections, the two inner subtended by unequal dark patches. Subbasal cell of the receptacle somewhat longer than the basal, nearly as broad as the distal portion of the receptacle; cells III and VI subequal; cell IV subtriangular; cell V broad and short, carrying the thick contrasting black insertion-cell out free from the perithecium. Outer appendage simple (or distally branched?), slightly divergent; the basal cell somewhat longer than broad, its basal third or more obliquely involved by the opacity of the insertion-cell: the basal cell of the inner appendage similar to that of the outer, without suffusion, bearing a short branch on either side, each of which may bear several branchlets. Spores 45 × 3.5 μ. Perithecia 165–170 × 40 μ. Receptacle 185–200 × 45 μ. Total length to tip of perithecium 325–360 μ.

On elytra of *Coptodera gagatina* Dej., Brazil; Berlin Museum, No. 978.

Laboulbenia Oedichiri nov. sp.

Tinged with smoky brown. Perithecia falcate, strongly bent toward the appendages, pale smoky brown, tapering symmetrically, or nearly so, to the base and apex; the basal cells forming a short narrow hyaline stalk; the tip not differentiated, suffused with deep blackish brown, except the pale or hyaline blunt apex, the outer lips most prominent. Receptacle more or less distinctly punctate, relatively long and of nearly equal diameter throughout, often slightly geniculate through an enlargement of the basal cell, which is relatively very large just below the subbasal cell, which is smaller and separated by a more or less oblique septum; cell III relatively small, distally more prominent on the inner side; cells IV and V very small, nearly equal; cells III and VI subequal. The insertion-cell small, rather thick, wholly free and separated from the stalk of the perithecium by about half the width of cell V. The basal cell of the outer appendage larger, sometimes inflated, bearing distally, as a rule, one terminal and two lateral branches which are usually simple, distally hyaline, somewhat constricted at the septa, hardly reaching to the tip of the perithecium; basal cell of the inner appendage very small, circular in outline, bearing apparently three branches like the outer. Spores 55 × 5.5 μ. Perithecia 125 × 32 μ. Receptacle 150 × 30 μ. Appendages about 90 μ. Total length to tip of perithecium about 250 μ.

On the elytra and upper surface of the abdomen of *Oedichirus* nov. sp.; Sharp Collection, No. 1154; Rio de Janeiro, Brazil.

Laboulbenia pallida nov. sp.

Hyaline, becoming faintly suffused with yellowish brown. Perithecium one half or less free, concolorous or slightly darker, stout, erect; the tip small, usually well distinguished, bent outward; the lip-cells outwardly oblique, the inner much more prominent, rounded distally, wholly suffused, except the edges, with dark distinctly reddish brown. Receptacle normal, or often abnormally septate, more commonly as a result of one or two distal divisions of cell V. Insertion-cell broad, reddish brown, transparent but contrasting, irregular in form, often oblique in position, being carried out free from the base of the perithecium from which it is separated by the partly free upper margin of cell V, sometimes once divided vertically; an external distinct similar accessory insertion-cell, bearing a single appendage and standing in direct relation to one of the subdivisions of cell V is rarely present. The appendages hyaline, in general normal, the outer basal cell twice or more than twice as large as the inner, bearing usually two branches which may be once branched, one of them sometimes an antheridial branch; the basal cell of the inner appendage producing usually a branch on either side, which may bear only short antheridial branchlets or longer simple sterile ones. Antheridia relatively large, stout, single or in pairs. Spores 75 × 6 μ. Perithecia 125–150 × 60–70 μ. Receptacle 220–300 μ. Longer appendages 150 μ. Total length to tip of perithecium 290–380 μ.

On elytron of *Clivina fasciata* Putz., St. Geronima, Guatemala; British Museum, No. 674. On *C. dilutipennis* Putz., Mexico; British Museum, No. 675.

Laboulbenia perplexa nov. sp.

Perithecium very large and long, nearly twice as long as the receptacle, dull translucent olive brown, or becoming blackish brown, straight, usually erect, nearly isodiametric, or often subclavate, tapering slightly below to a short rather narrow hyaline contrasting stalk formed by the basal wall-cells; the subbasal wall-cells slightly spiral, making from one quarter to one half a turn; the tip short, blunt, blackish, generally not well distinguished, sometimes bent rather abrutly inward. Receptacle short, the basal cell nearly hyaline, somewhat elongate, the rest of the receptacle hardly exceeding it in length, dirty olive brown, becoming more deeply suffused with brown or blackish brown and somewhat mottled; cell II small, separated from cells III and VI by very oblique septa, its lower third or fourth, only, free; cells II, III, VI and often IV not differing greatly in size, their inner angles often converging to a common point at about the centre of the distal portion of the receptacle; cell II triangular or subtriangular, extending upward to a point just below the insertion-cell, and downward nearly to the base of cell II; cell V but slightly smaller than cell IV. Insertion-cell slightly oblique, well defined, rather thick, broad, black. Appendages corresponding in type to those of *L. Galeritae,* hyaline or the lower cells becoming suffused more or less, but not deeply, with brown; the outer and inner free above the base; the outer consisting of from six to sometimes ten or more obliquely superposed cells each of which bears externally a simple branch consisting of a two-celled basal part, the cells slightly longer than broad, constricted at the dark septa, clear brown; the distal part long, becoming slender, flexuous and hyaline toward its extremity, which may reach, or even exceed, the tip of the perithecium. The basal cell of the inner appendage producing a free branch on either side similar to the outer appendage, and bearing similar branchlets; except that the lowest, and usually the next above it, consist of single cells bearing terminally single brownish, slightly curved antheridia. Spores 90 × 7 μ. Perithecia 290–360 × 50–60 μ, including the stalk (40 μ). Receptacle 220 × 60–70 μ. Appendages including branches 200–360 μ. Total length to tip of perithecium, average, 500 μ.

On the elytra and prothorax of *Galerita carbonaria* Mannerh., Brazil; Berlin Museum, No. 960. Allied to *L. incerta* and *L. fusiformis.*

Laboulbenia Planetis nov. sp.

Perithecium large and stout, clear transparent brown, except the subhyaline base, which is separated by a more or less clean-cut line of demarcation; the wall-cells with a slightly spiral twist, the tip abruptly distinguished, symmetrical or nearly so, black, except the evenly rounded subhyaline symmetrical lip-edges. Receptacle rather slender, subhyaline with pale brownish shades; cell II several times as long as the basal cell; cell IV bulging rather prominently externally and more deeply suffused with brown. The insertion-cell deep black, broad, thick. The appendages erect, their branches curved toward the perithecium: outer appendage normally consisting of a main axis of three superposed cells, externally broadly blackened, the blackening involving the septa and adjacent external portions of the cells; the distal cell bearing three branches; an outer and an inner simple, and a median one, once branched, the outer deeply suffused about its base, which forms a direct continuation of the suffused external area of the main axis; the remaining cells each producing a simple branch distally on its inner side; all the branches long, slender, distally thin-walled and hyaline, curved toward the perithecium, the lower somewhat constricted, and more or less deeply suffused at the lower septa: basal cell of the inner appendage half as large as that of the outer, bearing a branch on either side, from the basal cells of which arise often two branchlets, similar to those of the outer appendage. Spores 65 × 5 μ. Perithecia 155–165 × 55–60 μ. Receptacle 290 μ. Appendages 360–435 μ. Total length to tip of perithecium 400–425 μ.

On the thorax of *Planetes bimaculatus* MacLeay, Java; Berlin Museum, No. 1047.

Laboulbenia Platyprosopi nov. sp.

Uniform transparent pale reddish amber, the cell-walls very thick. Perithecium straight, wholly or nearly free, erect, the margins slightly convex; of nearly equal diameter throughout to the rather abruptly differentiated short erect blackish tip; the hyaline lip-edges outwardly oblique: the base sometimes abruptly broader than the ascigerous portion. Cells I–VI of the receptacle not differing very greatly in length, except cell V, which is relatively large; cells III and VI paired; cell IV prominent externally below the thin contrasting distinctly reddish insertion-cell. The appendages quite hyaline, the basal cell of the outer several times larger than that of the inner, and bearing two to four branches in an antero-posterior series; the basal cells of which usually bear each a pair of branchlets in the same plane: the small basal cell of the inner appendage producing a branch on either side bearing branchlets similar to those of the outer appendage; the antheridia crowded on special branchlets, curved, hyaline, small and closely appressed. Spores about 55 × 5 μ. Perithecia 150–185 × 36–45 μ. Receptacle 150–220

× 60–70 μ. Appendages, longest, 360 μ. Total length to tip of perithecium, 275–350 μ.

On the elytra and abdomen of *Platyprosopus Beduinus* Nordm.; Berlin Museum, No. 810; Nubia.

Laboulbenia producta nov. sp.

Perithecium faintly tinged with pale dirty brown, rather long and slender, nearly erect, free except the rather narrow subhyaline base; the lips somewhat prominent, outwardly oblique, the inner subtended by a dark suffusion. Receptacle tapering to a slender pointed base, the basal cell hyaline, becoming faintly tinged with dirty brownish, contrasting with the opaque subbasal cell which is less than twice as long; cells III and VI subequal, nearly opaque, reddish brown; cell IV nearly opaque, continued upward to form a free blunt well-defined prominence which projects beside and slightly beyond the basal cell of the appendage; cell V relatively very large, extending to the base of cell IV, wholly pale dirty brown like the perithecium, contrasting. Insertion-cell relatively large, slightly oblique inward, resting below on cell V and laterally and obliquely on cell IV. Basal cells of the appendages nearly equal, subisodiametric, the outer bearing an outer and an inner simple hyaline branch distally, the basal cells of which, especially the inner one, are more or less inflated, roundish, with dark septa, and tinged with dirty brown; the basal cell of the inner appendage bearing a branch on either side similar to those of the outer, the branches hardly extending to the tip of the perithecium. Spores 55 × 4 μ. Perithecium 90–100 × 25 μ. Receptacle 110 μ to insertion-cell, the projection above 10–12 μ. Total length to tip of perithecium 220 μ.

Growing appressed on the bristle-like hairs on the elytra of *Anchonoderus concinnus* Dej., Columbia; Berlin Museum, No. 1023.

LABOULBENIA PROLIFERANS, Thaxter.

This species, which is very widely distributed throughout the warmer regions of the eastern hemisphere, presents very numerous variations in form, size, color, degree of proliferation and character of appendages, among which the following seem sufficiently constant to warrant a varietal name:

Var. LIBERIANA Thaxter. Cell V constantly once proliferous, the accessory appendage simple, distinguished by a thick, jet black base. On *Eudema* sp., Liberia. On *E. arcuatocolle* Murr., Old Calabar, Africa; British Museum, No. 591. On *Eudema* sp., Congo, Africa; Paris, Nos. 84 and 152.

Var. **atrata** nov. var. Cell V two to five times proliferous, the accessory appendages distinguished by thick black bases, simple or once to twice divergently branched. On *Chlaenius Dohrini* Bert., Tropical E. Africa; Berlin Museum, No. 226. On *Euchlaenius trochantericus* Kolbe, Njam Njam, Semnio, Africa; Berlin Museum, No. 921. On *Craspedophorus nucata* Harold, "Regne Lugna"? Africa; Berlin Museum, No. 908. On *C. Westermanni* Laf., Togo, Africa; Berlin Museum, No. 909. On *Rhopalomelus angusticollis* Boh., Port Natal, Africa; British Museum, No. 630. On *Rhopalomelus* sp., Africa; Hope Collection, No. 311.

Var. **cincta** nov. var. Like var. *atrata*, the perithecium borne on a black base, differentiated in a manner similar to that of the typical insertion-cells of the genus. On *Eudema sinuaticolle* Laf., Lagos, Guinea; British Museum, No. 590. On *E. grossum* Hope, Old Calabar, Africa; British Museum, No. 587. On *Episcosomus* (*Eudema*) *grossus* Hope, Africa; Paris Museum, No. 81. On *Craspedophorus adaequatus* Kolbe, Bismarkdorf, Togo, Africa; Berlin Museum, No. 919, and Kamerun, Africa; Berlin Museum, No. 912. On *C. Congoanus* Kolbe, Kimpoko, Congo, Africa; Berlin Museum, No. 907. On *C. Preusii* Kolbe; Barumba Station, Kamerun, Africa; Berlin Museum, No. 910. On *C.* (*Eudema*) *Strachani* Hope, Togo, Africa; Berlin Museum, No. 913.

Var. **divaricata** nov. var. Cell V once proliferous or sometimes normal without proliferation, the primary appendages copiously several times branched, the branches or branchlets commonly more or less marked by basal blackish suffusions, divaricate, slender, tapering, flexuous. On *Chlaenius* sp., Sandakan, Borneo; British Museum, No. 664. On *Chlaenius* ? sp., Victoria, Australia; Hope Collection, No. 339. On *Notonomus viridescens* Chaud. ? New Zealand; Paris Museum, No. 159. On *Bembus gigas* Bates, Japan; Paris Museum, No. 133. On *Rhembus laevis* Lesne, Java; Paris Museum, No. 109.

Laboulbenia sphyriopsis nov. sp.

Perithecium free, projecting at right angles to the axis of the receptacle or recurved, opaque, blackish brown, slightly asymmetrically inflated below, the tip not distinguished, very broad, truncate or bluntly rounded, concolorous. Receptacle relatively slender, hyaline, cells I and II relatively very large, the rest small; cell I shorter, often tinged with brown;

cells II and III edged with blackish brown externally; cell IV wholly blackish brown; cell V hyaline or partly suffused, the insertion-cell thick. Basal cell of the outer appendage more or less suffused, giving rise to an outer and an inner branch; the basal cell of the former giving rise to an outer, shorter, and an inner branchlet; the inner branch simple: basal cell of the inner appendage bearing a simple branch, distally furcate above its basal cell; the branchlets both fertile, short, with a single terminal antheridium; or one of them sterile and similar to the outer appendage, short, stout, nearly straight, stiff, opaque, hyaline-tipped: the antheridia relatively large, solitary; the broad neck bent abruptly at right angles. Perithecia 90–110 × 40–48 μ. Receptacle 220–250 μ. Appendages 150–185 μ.

On *Metronemus caliginosus* Blk., Oahu, Honolulu; No. 1261 and 1262. Perkins Collection, Hawaiian Carabidae. Right inferior surface of prothorax.

Laboulbenia Stomonaxi nov. sp.

Perithecium hyaline, becoming tinged with brown, less than one half free; short, the outer margin nearly straight, bending rather abruptly inward at the tip; the inner margin converging to the apex from its point of union with cell V; the tip rather abruptly distinguished, broad; the apex nearly flat, the lips hardly prominent. Receptacle yellowish or hyaline, with faint brownish shades; the basal cell usually bent, the subbasal with deeper brown suffusions; cell III more or less deeply suffused with brown, its thick outer wall opaque or nearly so; cells III and IV subequal, several times longer than broad; cell V long-triangular. Insertion-cell thick, black, contrasting, wholly free, separated from the perithecium by nearly the whole upper margin of cell V. Outer appendage curved strongly outward, hyaline on its inner side, deeply suffused on the outer concave side, the suffusion continuous with the insertion-cell; consisting of a small number of cells, the basal relatively long, the rest producing suberect hyaline branchlets distally on the upper side: basal cell of the inner appendage smaller, the subbasal cell usually bearing two branchlets (the appendages broken in the types). Spores 50 × 5 μ. Perithecia 65 × 25 μ. Receptacle 100–110 μ. Appendages 50–75 μ approximately. Total length to tip of perithecium 130–140.

On the margin of the left elytron of *Stomonaxus* sp., Java; Paris Museum, No. 93.

LABOULBENIA TEXANA Thaxter.

An examination of further material from tropical or subtropical species of *Brachinus* seems to indicate that several forms, two of which have already been described as species, are better associated as varieties of this remarkable type. The simplest of these is that I have formerly described as *L. tibialis*, while *L. rostellata* is perhaps the most peculiar. The varieties may be enumerated as follows:

Var. TIBIALIS. On *Brachinus* sp. from Eustis, Florida.

Var. ROSTELLATA. On the same host and from the same locality.

Var. **incurvata** nov. var. Corresponding closely to the type in the form and coloration of the receptacle. The outer appendage nearly straight, divergent, hardly incurved, becoming more or less brownish, relatively somewhat longer and more tapering than in the type, the subbasal cell producing from its left side a short well-developed five- to six-celled branch, recurved, and bearing branchlets of the characteristic type from its convex side; the second cell producing on its concave side a two-celled branchlet, the lower cell of which bears one and the distal two of the characteristic branchlets. The inner appendage closely resembling that of the type, hyaline, incurved, its third cell producing a well-developed similar branchlet from its concave side. Perithecium as in the type, but the tip quite different, narrow, bent abruptly inward at right angles, the lips compressed. On *Brachinus geniculatus* Dej., Montevideo, Uruguay; Berlin Museum, No. 992.

Var. **pendula** nov. var. Longer and more slender than the type; cell √ not greatly broadened, so that the perithecium and appendage diverge but slightly. The outer appendage nearly erect, brown, relatively very long, tapering; one or sometimes more than one of the three lower cells producing externally or laterally well-developed branches of the characteristic type, which are long, slender and more or less pendulous. The inner appendage recurved, more or less pendulous, relatively slender and similar to a well-developed branch which arises on its inner side from the third cell above its base. Perithecium nearly erect, the tip well distinguished, bent slightly inward; the lips somewhat spreading. On *Brachinus geniculatus* Dej., Montevideo, Uruguay; Berlin Museum, No. 992. On the inferior surface of the prothorax. A form somewhat similar also occurs on *B. lateralis* Dej. from Oaxaca, Mexico (Brit. Mus., No. 721); a single specimen only having been obtained it is impossible to say how constant its differences may prove.

COREOMYCES nov. gen.

Mature individual consisting of a single series of superposed cells terminated by a single perithecium. Receptacle attached by a more or less rhizoid-like foot and consisting of three superposed cells, the upper of which becomes divided distally by successive transverse septa; these divisions resulting in a series of superposed cells, from each of which arises, on one side, a single branched appendage; the members of the series thus resulting being superposed in a single vertical row. Perithecium consisting of an undifferentiated stalk-cell immediately above the appendiculate cells, which is followed directly by the ascigerous cavity, the septa which separate the basal and wall-cells being obliterated in mature individuals.

The antheridia of this anomalous genus have not been definitely distinguished, but appear to be similar to those of Ceratomyces, to which it otherwise does not appear to be nearly related. The development of the perithecium, in so far as it is shown by the material available, seems distinctly unlike that of any other genus.

Coreomyces Corisae nov. sp.

Curved or straight, pale dull brownish, nearly transparent. Cells of the receptacle distinguished by more or less distinct constrictions, the three lower somewhat variable in length, the basal smaller, usually tapering somewhat to the foot; the subbasal larger than either of the others, more or less inflated, often as broad as the perithecium; the upper cell somewhat narrower, followed by the two to four, usually flattened, appendiculate cells, which are successively longer, though usually slightly narrow from below upward, the uppermost becoming about half as long as broad. The appendages long slender hyaline, sparingly branched, slightly divergent; usually extending upward to, or considerably beyond, the tip of the perithecium. Stalk-cell of the perithecium not differentiated from the cells below it, large, as broad as the body of the perithecium above it, cylindrical or slightly inflated, about as long as broad or slightly longer: body of the perithecium usually straight, somewhat darker dull brown especially where it tapers rather gradually to the small tip which becomes distinctly symmetrically four-papillate, and is usually more or less conspicuously bent to one side. Spores 85 × 6 μ. Perithecia 100–110 × 25–35 μ, the stalk-cell 30–33 × 32–35 μ. Receptacle, exclusive of the appendiculate cells, 75–100 μ. Appendages 200–250 μ. Total length to tip of perithecium 275–290 μ.

On inferior surface of abdomen of *Corisa Kennicottii* Uhler, Arlington, Mass. On *Corisa* sp., Iowa. This interesting form, which is the only member of the group occurring on an Hemipterous insect, was discovered by Mr. Charles Bullard, who has also found on a different host a somewhat larger form that may prove to be distinct.

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CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORY OF HARVARD UNIVERSITY.—LI.

ON *CAULOGLOSSUM TRANSVERSARIUM* Fries (Bosc).

By John R. Johnston.

With a Plate.

CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORY OF HARVARD UNIVERSITY. — LI.

ON CAULOGLOSSUM TRANSVERSARIUM Fries (Bosc).

By John R. Johnston.

Received May 15. Presented May 14, by Roland Thaxter.

The remarkable fungus which forms the subject of the present paper was described under the name *Lycoperdon transversarium* as early as 1811, by M. Bosc (3). It has since been reported only occasionally, and the numerous descriptions of it which exist are mostly copies of previous ones, no further information having, up to the present time, been available in regard to its structure or development than is contained in the early description of Bosc.

Through the kindness of Professor Thaxter, I have had the use for study of alcoholic and of dried material of this form, which was collected by him in Florida during the month of October, and which included both mature and young specimens.

General Structure.

The mature specimens examined are club-shaped, stipitate, and olive-brown; from three to seven centimeters in height. They vary from narrowly to broadly clavate, that is, from 1.5 cm. to 2.5 cm. in diameter, in specimens of the same length. Fig. 2 represents a narrowly clavate form with fibrillose base; in fig. 3 is illustrated an individual with a much greater diameter, the base being expanded for attachment to the substratum. The stipe is distinguished usually as the narrowed base of the fruiting body and by its whitish color. The peridium is simple; it ruptures irregularly and indefinitely, and covers the entire gleba. It consists of loosely interwoven hyphae, which become spread apart at various points, exposing the chambers of the gleba beneath (figs. 3 and 11); and in some mature specimens is even more or less evanescent, the exposure of the glebal chambers giving a honeycombed appearance to the entire surface. An area of this nature, in which the characteristically anastomosing glebal folds are thus exposed, and in which the chambers are in free communication with the air, is shown, considerably enlarged, in fig. 11. The peridium, which thus encloses the gleba often more or less incompletely, is further continuous with the columella where it emerges at the apex (fig. 10) and also below with the stipe.

In the interior of a typical specimen, the columella, which is a direct continuation of the stipe, runs to the apex and is externally visible, either as a more or less orbicular depression or a slight elevation. Though this is usually at the apex of the body, it is sometimes a little at one side (fig. 5). The firm sub-gelatinous axis consists of branching hyphae closely interwoven, and running more or less parallel. From the columella proceed the tramal hyphae toward the peridium. In a section of a mature specimen the tramal plates appear to branch and anastomose so as to form closed chambers, while from the trama arise many hyphae which bear basidia in groups (figs. 14, 15). The basidia, growing at right angles to the trama, form, as usual, the hymenial layer which lines the chambers (z′, fig. 7). The gleba (z) is thus formed between the peridium (x) and the columella (y), and consists of a series of chambers which appear to be more or less continuous with one another, and are lined with typical 4-spored basidia. The spores are brown, smooth, ovate to elliptical, borne on well-developed sterigmata, and without any distinguishing peculiarities. Usually each basidium (fig. 15) has at least a basal septum, and in some cases there are also indications of one more or less median septum.

DEVELOPMENT.

Alcoholic material was used in studying the young stages. About a dozen specimens, ranging from 3 mm. to 8 mm. in height, were imbedded in paraffin, and cut into sections 10 μ thick. For staining, Haematoxylin, Safranin, and Bismark Brown, were employed, the Haematoxylin giving better results than the two others. Dried material soaked in potassium hydrate and picked apart under a dissecting microscope, showed the groups of basidia, their form and septation, very clearly.

The youngest specimens examined, which were merely pointed upgrowths from the running mycelium, showed no indication of differentiation. In a specimen 3 mm. long, for example, there is no differentiation of gleba, but simply a loose interweaving of superficial hyphae which is the first indication of a peridial layer. There is also in these specimens, at their apex, a sheaf-like spreading of the parallel hyphae, which is very marked in the older ones (figs. 8 and 9).

The youngest stages examined in which any differentiation of importance was observed is represented in fig. 8, in which a distinct layer (x) of interlacing hyphae covers the more compact columella (y). Between the two is a deeply staining region (z) running lengthwise with short folds. This layer may be seen to consist of the tips of hyphal branches arranged in a columnar fashion, closely packed, and arising from a looser indefinite region below, which is made up of branching hyphae proceeding directly from the columella. This compact, deeply staining layer, which is the initial gleba, has no connection with the peridium lying next to it, but becomes gradually merged at both ends into the hyphae of the columella. The peridium is thus pushed away from the axis of the fruiting body by the growing gleba; and its further growth, which is relatively inconsiderable, is independent of the structures which it surrounds. At the base and the apex of the fruiting body, its connections are a little more evident. At the apex the peridium is indistinctly differentiated from the columella, and though the hyphae of both anastomose in this region, this differentiation becomes more marked with further growth; not to the extent, however, of final separation. At the distal end of the stipe the hyphae of the peridium and of the axis are closely intermingled, anastomosing freely, as do also the hyphae of the peridium with those at the apex of the columella; the distinction between the axil and the peripheral layer in each case lies in the fact that the hyphae of the axis constitute a firm coherent mass, while the hyphae of the peridium form a thin flocculent layer. These conditions are represented in fig. 13, which is an enlarged view of a portion of fig. 8 at the apex of the fruiting body. The fact that the initial gleba (z) is not connected with the peridium (x) and arises from the columella (y) is clearly shown in this figure. Here, too, the columnar layer extends gradually into the columella near the apex, at the right in the figure which here includes the ill-defined region of separation between the peridium and the columella. At this point there are thus clearly distinguished in this stage three regions: the peridium, the gleba, and the columella; the gleba consisting as yet merely of folds from the surface of the columella which result from the more rapid multiplication of the elements of the columnar layer in one part than in another.

In a somewhat older stage (fig. 6), these three regions are seen to have been still more clearly differentiated from one another. Distinct groups of hyphae run out from the columella into the folds the surface of which, as previously mentioned, consists of a layer of columnar elements. As the fold pushes farther out and its surface increases, new columnar cells are constantly coming to the level of the rest in the formation of the hymenial layer. Secondary folds arise from the primary ones by a similar process of development which ceases only at the maturity of the fruiting body. A repetition of this folding, which causes the trama to appear branched in section (fig. 7), results in a general enlargement of the gleba while the peridium is pushed farther away from the columella, the gross form of the plant becoming thus changed from almost columnar to broadly clavate. This excessive growth of the gleba causes it to equal or to exceed the columella in thickness within the very thin peridium, a condition presenting a distinct contrast to that of a younger stage, in which the peridium as well as the columella is much thicker than the gleba.

Little has been said as yet in regard to the growth of the peridium and its conditions at maturity. That it originates from the part of the mycelium which was not differentiated into central axis and gleba has been already mentioned, and also that its development after this differentiation is independent of the development of axis or gleba. With the increase in the size of the gleba is associated a corresponding increase in the extent of the peridium, which does not, however, become any thicker. In mature specimens it is exceedingly thin in comparison with the columella, so that at the apex the continuity of the two regions is very inconspicuous (fig. 5); and even in so young a stage as that represented in fig. 7, it is only the peripheral hyphae of the columella and of the peridium which anastomose at the apex. The distinction between the two regions about the stipe also is clear in this figure. From the fact that in some mature specimens (fig. 2) the relatively exceedingly thin and delicate peridium is continuous over the gleba, one must assume that its growth has kept pace to a certain extent with that of the elements within it. In a majority of specimens, however, (fig. 3) the chambers are more or less exposed in places, as if excessive growth within had stretched the peridium and finally caused it to become pulled apart over certain areas. This condition is probably further accentuated by the irregular wearing of the delicate weft which constitutes the peridium in the older stages; and in some specimens the honeycombed surface of the gleba may be almost entirely exposed.

This partially exposed condition of the hymenial region is no doubt connected, in a measure, with the dissemination of the spores, many of which must, in such cases, be discharged directly into the air. Observations made by Dr. Thaxter in the field on abundant material in all conditions of development show, however, that with the exception of this limited dispersal from the more or less exposed glebal chambers, no means of dissemination appears to exist except such as may be accomplished by the ultimate decay of the mature plant or its possible ingestion by animals. The history of the gleba in this respect is thus wholly similar to that which characterizes the Hymenogastraceae in general, a powdery condition occurring only when the glebal mass has dried after decomposition. It may be mentioned that in the locality examined by Dr. Thaxter it was noticed that this general decay was often hastened by the attack of the parasitic *Sepedonium chrysospermum.*

To sum up the development of the fruiting body: the upgrowth from the mycelium is differentiated into a cortical and a medullary layer. The cortical layer becomes the peridium, whose further growth is independent of the medullary layer, from which it differs in the arrangement of its hyphae, in that they form a loose irregular network instead of running more or less parallel to form a firm subgelatinous mass. The medullary layer becomes the stipe and the columella, with which the hyphae of the peridium are continuous. The surface of the columella becomes differentiated into folds which form the gleba. The surface of these folds consists of columnar elements which constitute the basidia of the mature specimen.

HISTORY.

This species was first described as *Lycoperdon transversarium* by M. Bosc (3) (1811), as follows:

"VESSELOUP TRANSVERSAIRE. V. sessile en forme de massue irrégulière, traversée par un axe conique, de contexture fibreuse qui part des racines et se termine au sommet.

"On rencontre cette espèce dans les bois sabloneux de la basse Caroline, mais elle n'est nulle part commune. Sa forme la rapproche de la Vesseloup pistillaire et son axe de la Vesseloup axate que j'ai décrite et figurée dans les mémoirs de la société d'histoire naturelle de Paris. Sa masse est composée, comme dans les autres vesseloups, d'un reseau très serré entre les mailles duquel sont nichées les semences, mais j'ignore si elle s'ouvre au sommet ou sur les côtés pour les repandre. Je crois que ces caractères suffisent pour en faire un genre nouveau dont feroit partie ma vesseloup axate."

This description, as well as the drawing which accompanies it, answers very well for the plant under consideration, although the latter is merely an outline of the specimen, indicating its club-shaped form.

In 1829 Elias Fries (10) transferred the *Lycoperdon transversarium* of Bosc to the genus *Cauloglossum*, Grev., giving the generic characters of *Cauloglossum*, and referring in his Syst. Orb. Veg., I. p. 136, to *Schweinitzia* Grev., as a synonym. An explanation of this is found in a paper by Greville (13) entitled "Observations on a New Genus of Plants belonging to the Natural Order Gastromyci," published in 1823, in which he describes the genus *Schweinitzia* as follows: "gen. char., *peridium stipitatum ad basin dehiscens. Stipes percurrens.*" Under this he includes two species, *S. pistillaris*, the type, synonymous with *Scleroderma pistillare* Pers. (*Lycoperdon pistillare* L.); and *S. carcinomalis* synonymous with *Scleroderma carcinomale* Pers. (*Lycoperdon carcinomale* L.). These two species, from the characters of the genus and their synonymy, will be readily recognized as forms which are correctly placed at present in the genus *Podaxon*. A few months later Greville published the following note in the Scottish Cryptogamic Flora (14): "In the 8th volume of the Edinburgh Philosophical Journal, I proposed to name a most curious new genus of *Gastromyci*, SCHWEINITZIA, after the above mentioned excellent botanist. I have since ascertained that a genus of phenogamous plants has been recently dedicated to him by Mr. Elliot. I wish, therefore, to substitute the name *Cauloglossum:* the character is *Peridium stipitatum ad basin dehiscens. Stipes percurrens.* The species are *C. pistillare* and *C. carcinomale*, both imperfectly described by Persoon (24) in his Synopsis Fungorum, under *Scleroderma*." It is thus evident that the name *Cauloglossum* was given to a genus which included only the two species just mentioned, both of which are at present correctly placed in *Podaxon*. Since the genus *Cauloglossum* is thus a synonym of *Podaxon*, having been originally proposed for the reception of forms generically quite distinct from *C. transversarium*, as has been shown in the preceding account, its use in the present instance would lead to confusion, and it has therefore seemed desirable to examine in some detail the further literature and synonymy relating to it.

In Fries's Systema Mycologia (10), in which, as above mentioned, *Lycoperdon transversarium* Bosc, is first referred to Cauloglossum, Greville's description of that genus is reproduced without modification, and the following synonyms are given: *Cauloglossum transversarium* Fries (Bosc): *Penis caninus virginianus*, Pluk. phyt. t. 184. f. 6.? (Cfr. *Spadoniam*): *Lycoperdon transversarium*, Bosc and Nees: *Cionium Carolinense* Sprengel. (*Herculea* Fries., inscr.).

The record in the reference to Plukenet (25) (1720) reads thus: fig. 6: "*Fungus phalloides, parvus, s. Penis caninus Virginianus;* D. Banister." The plate shows a figure of an erect specimen of almost the same diameter throughout, but with rounded and scaly ends, and is altogether too meagre a drawing to admit of any definite determination. It is probably the phalloid, *Cynophallus caninus.*

The third citation, Nees (23), refers merely to a copy of Bosc's figure under the name *Lycoperdon transversarium* Bosc.

Cionium Carolinense, which is further referred to by Fries as a synonym, is described by Sprengel (28) as follows: "*Cionium carolinense*, sessile oblongum irregulare glabrum, columella elongata. *Carolina* (Lycoperdon transversarium Bosc)." The characters of *Cionium*, however, as given by Link (17) who instituted the genus, do not justify Sprengel in making this reference, as is manifest from the following description: "*Cionium. Sporangium subglobosum, aut difforme. Peridium simplex, membranaceum, rumpens, squamulatim fatiscens. Flocci intus versus basin adnati. Columella intra peridium. Sporidia coacervata.*

"Sporangium prima aetate molle. . . . Sporidia majuscula, globosa, plerumque atra. Hujus loci Didymium complanatum, farinaceum."

Finally the only reference to the name *Herculea* is that of Fries (9), where he simply states that he had, in preparing his manuscript, called the genus containing the species "*elatum*" and "*transversarium*" *Herculea*, but that Greville had forestalled him by the publication of the name *Cauloglossum*, which should stand.

In the Genera Plantarum, 1836, Endlicher enumerates among the Gastromycetes *Cauloglossum transversarium* Fries.; while Fries again in 1845, writing on the vegetation of Scandinavia, mentions *Cauloglossum* Fr. under the Podaxinei, a group of the Lycoperdaceae. He places the name in brackets, intending doubtless to indicate that it is not found in Scandinavia.

In the Vegetable Kingdom, 1853, John Lindley includes *Cauloglossum* Fr. under the Podaxinei.

Another and fuller record by Fries (12) is to be found in the Nov. Symb. Mycol. 1857: "*Cauloglossum transversarium.* S. M. III, p. 61. In Carolina, Curtis, et in Mexico. Secundum haec specimina juvenilia a *Secotio* hoc genus tantum differt peridio clavato et stipite contiguo, observante jam Berkeleyo. Habitu et facie adeo exacte convenit cum *C. elato*, exoleto modo lecto, ut de genericu identitate non dubitem."

In Grevillea (1), 1873, is the following record:

"Cauloglossum transversarium Fr. Car. Inf. No. 267. In wet, sandy places. Dull brownish yellow." M. J. Berkeley.

In Grevillea (4), 1878, a further record reads thus:

"Cauloglossum transversale Fr. — Rav. Fung. Amer. No. 13. In moist woods. Gainesville, Florida." M. C. Cooke.

In the Curtis Herbarium at Harvard University the species is represented by the following specimens: "*Cauloglossum transversarium.* ad. terram arenosam ad margines sylvarum Nov. 1846: Wilmington, N. C." "*Secotium transversarium* B. & C., Tuskegee, Ala., aestate, 1853, Beaumont (31)." "*Cauloglossum transversarium;* Pinetis arenosis. N. Cesaraea, C. F. Austin (166)." "*Cauloglossum transversarium* Fr., Santee Canal, S. C., Ravenel."

In the Ellis Herbarium at the New York Botanical Gardens is a specimen of *Cauloglossum transversarium* collected near Wilmington, North Carolina, in November, 1880, by Dr. Thos. F. Woods. In a letter * Dr. Woods says, "It grows along the moist margins of a mill-pond near Wilmington in a loamy soil under thin undergrowth."

C. L. Shear (27) in an article published in the Asa Gray Bulletin, 1899, practically repeats the early description of Bosc, having obtained no new data or specimens upon which to work. He has, however, taken the generic description from that of Zobel (29) in Corda, which was a modification of Greville's description made to suit *C. aegyptiacum*, a form entirely different from *C. transversarium*, so that as a result Mr. Shear really attributes to *C. transversarium* some characters which it distinctly does not possess, as "basidia (?) stellate-glomerate" and "base (of stipe) clothed with the rudiment of the volva, which is incomplete and coriaceous."

The last record concerning it which I have at hand is that of Dr. Thaxter in 1897 at Eustis, Florida. Dr. Thaxter's notes read as follows: "abundant material was found growing out of the bases of living or dead trees or upon rotten stumps or fallen logs of *Pinus palustris* Mill., or among rubbish on the ground close by. When young the gleba is dirty gamboge yellow; when exposed by injury becoming dirty olive brown, eventually dark. Externally dirty brownish or buff yellow. Stem nearly white."

An examination of all the records obtainable thus indicates that the plant under consideration is identical with the *L. transversarium* of Bosc, and it is further evident that its reference by Fries to *Cauloglossum* was incorrect, since the last mentioned genus including its synonyms *Herculea* Fries and *Schweinitzia* Grev. is a synonym of *Podaxon*. *Cionium* Lk., the only other generic name that has been applied to it, represents a plant of an entirely different character, undoubtedly referable to the Myxomycetes. In view of these facts it seems necessary to propose a new generic name for this form.

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* For the quotation from this letter and a portion of one of Dr. Woods's specimens I am indebted to Prof. F. S. Earle of the N. Y. Bot. Gard.

In order to determine whether species other than *C. transversarium* should be similarly treated, it seems desirable in this connection to examine the published data which are available concerning *Cauloglossum elatum* Fr., *C. aegyptiacum* (Mont.) Zobel, and *C. novo-zelandicum* (Henn.) Fischer, which appear to be the only forms, with the exception of the two species of *Podaxon* and the *Penis caninus virginianus* Plukenet already mentioned, that have ever been referred to *Cauloglossum*.

Cauloglossum elatum, a form occurring in East India, is described by Fries (10) as possibly synonymous with *Clavaria pistillaris* Lour. (19). In general form and in the presence of an axil columella it resembles *C. transversarium*. In the minor characters, its white color, small size (only half an inch in height), it seems to differ distinctly, while the fact that the peridium falls away, only scaly remnants being left at the base and apex, separate it clearly from *C. transversarium*, and suggests that it belongs rather in the Podaxaceae.

Cauloglossum aegyptiacum (Mont.) Zobel, which was originally described under *Podaxon* by Montagne (22), is figured and described in Corda (29). Zobel, however, modified Greville's original generic description of *Cauloglossum* by adding the following points: "basidiis (?) stellato glomeratis; . . . stipes firmus, erectus, basi rudimentis volvae vestitus. Volva incompleta, cum stipite connata, coriacea." A species to which such characters apply cannot be included in the same genus with *C. transversarium*; and it seems probable that Montagne was right in placing it in *Podaxon*. It is a form that has been reported but once, by M. Bové, from the lands between Suez and Gaza.

The last form that has been placed under *Cauloglossum* is *Clavogaster novo-zelandicum* described by P. Hennings (15) and referred to *Cauloglossum* by Fischer. *Clavogaster* was a new genus proposed by Hennings for a New Zealand form, the characters of which he described as follows: "Perithecium subcoriaceum, persistens e stratis binis discoloribus efformatum, clavatum, stipitatum. Capillitium subfasciatum in cellulas favosas, polyedras, sporis levibus, ellipsoideis, coloratis, pedicellatis. *Hippoperdo* peraffinis."

In view of the fact that the presence of a capillitium and the absence of a columella seem sufficient to exclude such a form from *Cauloglossum* as heretofore understood, it is not clear why Professor Fischer should have changed the name employed by Hennings, who, moreover, could hardly have omitted all reference to a columella were one present in the species.

Whether the name *Clavogaster* should stand, or should be considered as synonymous with some other genus is, however, of no importance in the present connection, since the type species seems widely distinct, generically, from the form under consideration.

It is evident, then, that, in so far as can be determined by published data concerning them, none of the forms which are above referred to can be properly associated with *Cauloglossum transversarium*, which at present must be considered to stand as the only representative of a monotypic genus. As I have before shown that *Cauloglossum* is a synonym of *Podaxon*, it seems desirable to substitute a new name for the genus which may be distinguished by the following characters:

RHOPALOGASTER nov. gen. Fruiting body clavate, stipitate, traversed by a firm sub-gelatinous axil columella continuous with the stipe. Stipe firm, erect with naked base. Peridium simple, continuous with the stipe below and with the columella at the apex, more or less evanescent-indehiscent. Gleba persistent. Tramal plates extending from the columella toward the peridium. Basidia clavate, in groups, 4-spored, spores simple, borne on well-developed sterigmata.

RHOPALOGASTER TRANSVERSARIUM (Bosc). *Lycoperdon transversarium* Bosc in Gesell. Naturf. Freunde, Berl. Mag., Vol. V, p. 87, pl. VI, f. 9. *Cauloglossum transversarium* (Bosc), Fr. in Syst. Myc. III, p. 61. *Cauloglossum transversale* Fr., Cooke in Grevillea, No. 40, p. 133 (1878). *Secotium transversarium*, B. & C. in herb. Curtis.

Narrowly to broadly club-shaped, 3–7 cm. high; the distal end of the columella appearing at or near the apex, either as a shallow orbicular depression, or a slight protuberance. Peridium dirty brownish or buff yellow. When young the gleba is dirty gamboge yellow; when exposed by injury becoming dirty olive brown, eventually dark. Stem nearly white when fresh. Spores ovate-elliptical, 3.6–4.3 $\mu \times$ 5.8–7.2 μ, yellow brown, borne on long slender sterigmata. The plants grow out of the bases of living or dead trees or upon rotten wood, stumps or fallen logs, or among rubbish on the ground close by in wet pine lands. September–November. S. Carolina (Bosc, 1811, Curtis, 1857?); Santee Canal, S. C. (Ravenel); Wilmington, N. C. (Woods, November, 1880, Curtis, November, 1846); Carolina (Berkeley, 1873); Gainesville

Fla. (Ravenel, 1878); Eustis, Fla. (Thaxter, October, 1897); Tuskegee, Ala. (Beaumont, aestate, 1853).

AFFINITIES.

In comparing *Rhopalogaster* with other genera, certain general resemblances are evident in several directions. Its gross characters have led, as has been already mentioned, to its close association with *Podaxon* in the group of Podaxaceae, which in turn have been placed by different authorities in different Gastromycetous orders. Fries, for example, as well as a majority of the earlier writers, regarded *Cauloglossum transversarium* as closely related to the species of *Podaxon* and *Secotium*, while Fischer (7), in his recent revision of the Gastromycetes (1898) has removed *Podaxon* to a new group, the Plectobasidiineae, leaving *Cauloglossum* with *Secotium* in the Hymenogastrineae.

That *Rhopalogaster* should be separated from the Podaxaceae, where, however, the other species of *Cauloglossum*, namely, *C. elatum*, *C. aegyptiacum*, and *C. novo-zelandicum*, should probably be referred, is clearly evident when one examines the structure characteristic of this group. The stipe of *Podaxon* is hollow and usually bulbous at the base; the peridium is scaly, and opens either by the falling off of the scales or simply by longitudinal cracking; capillitium threads are present, either rudimentary or well developed; the spores differ in form; and the gleba becomes a powdery mass at maturity. As all of these characters are distinctly absent in *Rhopalogaster*, and as *Podaxon* is the only one of the Plectobasidiineae which is at all similar to the type under consideration, *Rhopalogaster* manifestly cannot be included with the Podaxaceae. The only alternative group which remains to be considered is that of the Hymenogastrineae under which *Secotium* is now placed by Fischer.

According to Fischer's revision, the Hymenogastrineae include three sub-groups, the Secotiaceae including *Cauloglossum*, the Hysterangiaceae, and the Hymenogastraceae. The Secotiaceae according to Fischer's definition are characterized as follows: "Die Gleba ist von einer axilen, sterilen Columella durchsetzt, welche sich nach unten in einer Stiel fortsetzt. Die Tramaplatten gehen von der Peridie ab und sind deutlich gegen den unteren Teil der Columella gerichtet," and also "Bei der Reife löst sich meist die Peridie an ihrem unteren Rande von Stiele ab und kann sich mehr oder weniger deutlich ausbreiten nach Art des Hutes der Agaricaceen." As has been shown on a preceding page, the tramal plates in *Rhopalogaster*, on the contrary, extend from the columella toward the peridium, and the persistent peridium does not break away in pieces. It is certain, then, that *Rhopalogaster* cannot be placed in the Secotiaceae.

The two remaining groups, the Hysterangiaceae and the Hymenogastraceae, are described by Fischer as possessing a "Gleba ohne oder nur mit kurzer, nicht bis zum Scheitel reichender Columella." This characterization, if adhered to strictly, would exclude *Rhopalogaster* from both groups, but since it seems more than probable that it is most nearly related to forms included in them, it will be necessary to examine into the structure of the different genera in order to determine, if possible, which are most nearly related to it.

In the Hymenogastraceae the resemblances to *Rhopalogaster* are to be found in the hymenial structure. In this group, especially *Hymenogaster* and *Octaviana*, the basidia are clavate, septate, and are arranged surrounding irregular chambers. The direction of the tramal plates is away from the peridium, however, and the columella is but rudimentary or entirely lacking. The genera *Rhizopogon*, *Sclerogaster*, and *Lycogalopsis* seem rather to exhibit affinities with the next great group, the Lycoperdineae.

In the Hysterangiaceae the absence of a complete columella is the most noticeable point of difference. *Gautiera* resembles *Rhopalogaster* in its irregularly chambered gleba, and in its thin peridium, which is at maturity lacking to such an extent as to expose the glebal chambers just as is shown in fig. 11 of the accompanying plate.

Gymnoglossum, a form reported by G. Massee (20) from New South Wales, is distinguished from *Rhopalogaster* mainly by the absence of a peridium at every stage of development, and by the fact that the columella does not reach more than half-way to the apex of the peridium. The points of similarity are the "irregular cavities lined with large clavate tetrasporous basidia, sterigmata elongated, slender, spores colored; stem distinct, elongated, solid, passing up into the gleba." *Protoglossum*, Mass., exhibits similar affinities to *Rhopalogaster*.

The genus *Hysterangium* shows a columella reaching almost to the apex of a well-developed peridium. A longitudinal section of an immature specimen of *Hysterangium clathroides* represented in a drawing by Rehsteiner (18) is strikingly similar to some of my sections of *Rhopalogaster*. It shows the thin flocculent peridium enclosing the stipe and gleba, and from the sterile part which is a continuation of the stipe there arise folds to form the gleba. The only essential difference in gross structure which is here apparent is due to the fact that in *H. clathroides* the columella falls short of the apex of the peridium. The basidia, however, which seem to be twisted in the drawing of Tulasne, perhaps owing to shrinkage in the specimen, bear but two spores on short sterigmata.

Protubera, Möller, and *Phallogaster*, Morgan, like *Hysterangium*, resemble *Rhopalogaster* through the presence of an incomplete columella; but although the early development of *Phallogaster* is as yet unknown, both genera appear to possess characters which should associate them much more closely with the Phalloideae than with the Hysterangiaceae.

In summing up this comparison with the three divisions of the Hymenogastrineae it is evident that *Rhopalogaster* is similar to the Secotiaceae only through the presence of a central columella running to the apex of the peridium. The differences lie in the nature of the peridium, in the origin and development of the tramal plates, and in general habit.

The Hysterangiaceae show resemblance in glebal structure, and in the origin and development of tramal plates. The differences lie rather in the basidia and spores and in the possession of only an incomplete columella.

Affinities with the Hymenogastraceae lie in the similarity of glebal characters; the differences are apparent in the lack of an axil columella, in the origin and development of the tramal plates, and in general habit.

It will thus be seen that although *Rhopalogaster* resembles the Secotiaceae and the Hymenogastraceae in certain respects, the origin of its gleba excludes it absolutely from both these families; and that, on the other hand, except for the presence of a complete columella, it may be very closely associated with forms belonging to, or at least that have been placed among, the Hysterangiaceae.

Gymnoglossum, for example, although its columella is incomplete and the peridium is said to be entirely lacking in all stages of development, seems very similar in general structure and habit as well as in the possession of tetrasporous basidia, bearing brown ovoid spores on well-developed sterigmata. *Gautiera* also offers points of resemblance in possessing an evanescent peridium, and distinct, though rudimentary, columella. The genus *Hysterangium*, to which reference has previously been made, although dissimilar in respect to its basidia and spores, possesses an almost complete, though much less clearly limited, columella; from which the glebal structures develop almost exactly as in Rhopalogaster.

Although, even excluding *Protubera* and *Phallogaster*, the Hysterangiaceae of Fischer include a somewhat miscellaneous assortment of forms, none of which present even general points of structure to which our plant can be said strictly to conform, it seems desirable, in view of the resemblances noticed, to include it in this family. It may be suggested, however, that the discovery of additional forms, or a fuller knowledge concerning some of those already recorded, may possibly necessitate the formation of a fourth order for its reception among the Hymenogastrineae.

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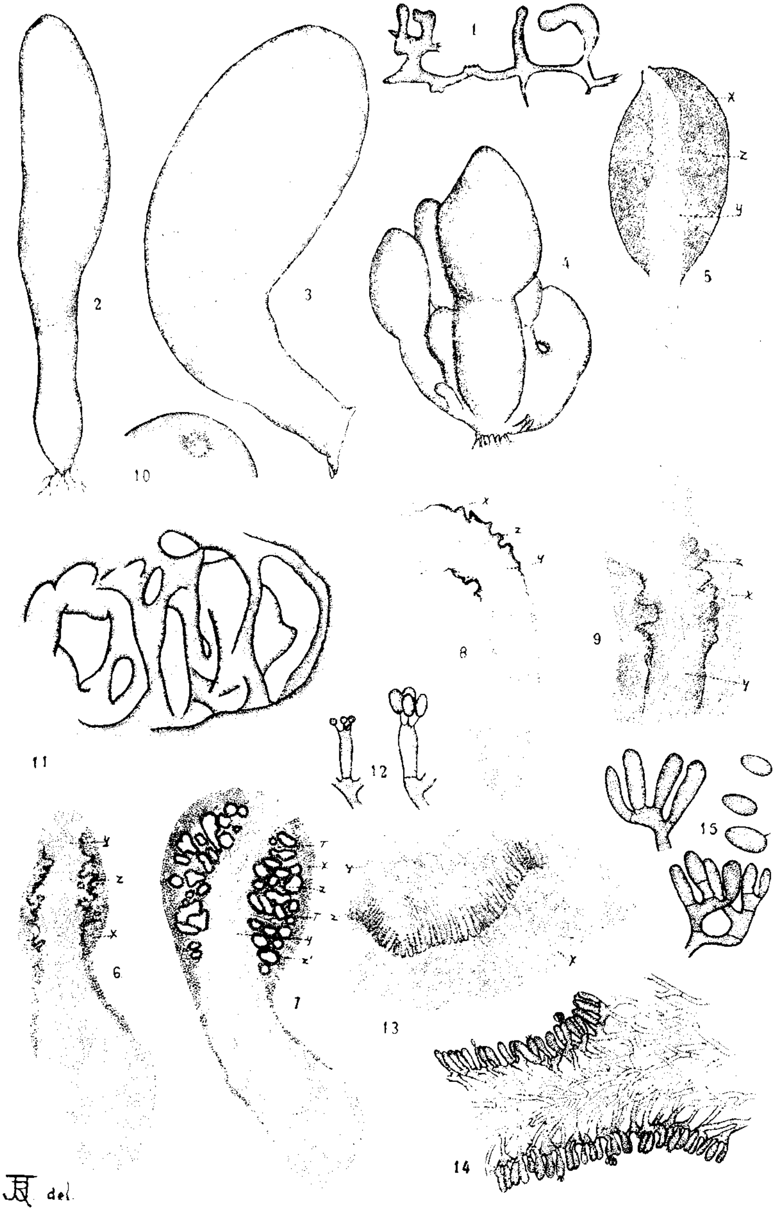
EXPLANATION OF PLATE.

Figs. 1–5 inclusive are about three-fourths natural size as reduced. The remainder of the figures were drawn with a camera lucida with the magnifications indicated, and reduced one-third in the reproduction.

Fig. 1.	Several young fruiting bodies on a common mycelium.
Fig. 2, 3, 4.	Specimens illustrating different habits. Fig. 3 shows chambers exposed in a few places.
Fig. 5.	Median longitudinal section of mature specimen. x = perid., y = col., z = gleba.
Fig. 6, 7, 8. 9.	Sections of various stages. Fig. 8, × 16; Fig. 9, × 16; Fig. 6, × 12; Fig. 7, × 10.
Fig. 10.	The end of columella seen from outside.
Fig. 11.	Chambered structure seen from outside where peridium has opened, × 19.
Fig. 12.	A young and a mature basidium bearing spores.
Fig. 13.	Portion of Fig. 8. 625 diam. x, y, z, as above.
Fig. 14.	Portion of chamber wall, showing trama and basidia.
Fig. 15.	Groups of basidia and spores. Oc. 4., $\frac{1}{12}$ oil im.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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

NEW SERIES. — NO. XXIV.

(PAPERS FROM THE HOPKINS-STANFORD EXPEDITION TO THE GALAPAGOS ISLANDS.)

FLORA OF THE GALAPAGOS ISLANDS.

BY B. L. ROBINSON.

(With the collaboration of specialists.)

CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY, NEW SERIES, NO. XXIV.

By B. L. Robinson.

Presented January 8, 1902. Received May 20, 1902.

FLORA OF THE GALAPAGOS ISLANDS.

The peculiar character of the vegetation on the Galapagos Islands was brought to scientific attention in 1847 by Sir Joseph Dalton Hooker. His flora of the archipelago, based chiefly upon the collections of Charles Darwin, showed clearly that these islands, although small in extent, volcanic in character, and only five hundred to six hundred miles from Ecuador, possess a vegetation striking in individuality and very unlike that of the adjacent mainland. Darwin, who visited the Galapagos on the memorable voyage of the "Beagle," landed on four of the islands (Chatham, Charles, James, and Albemarle). He was able to spend only a few days upon each, and naturally much of his attention was devoted to the general topography, the geology, and especially the peculiar fauna; moreover the season was one of drought; yet he was able to collect 201 different plants. These, supplemented by smaller collections (chiefly those of Douglas & Scouler, Macrae, and Edmonston), enabled Hooker to include in his enumeration 239 species, of which no less than 107 were described as new to science, thus indicating an extraordinary endemic element of nearly forty-five per cent of the whole flora of the islands, a proportion which became still more remarkable when the imperfectly represented thallophytes, and certain phanerogams, believed to be artificially introduced weeds, were omitted from consideration.

In 1852 Prof. N. J. Andersson, well known for his critical studies in *Salix*, visited the Galapagos Islands on a voyage of the Swedish frigate "Eugenie." He landed upon five of the islands, four of them being the same as those visited by Darwin, and the fifth (Indefatigable) until then entirely unknown botanically. The sailing schedule of his vessel permitted him to remain but a few hours upon some of the islands, and upon none more than two or three days. Nevertheless, he was able to secure an excellent collection of plants, including 338 numbers, many of which were represented by several specimens. These plants, critically determined by Andersson himself and distributed to the leading herbaria of various countries, have long furnished the chief reference specimens for the interesting vegetation they represent. In 1853 Andersson published a detailed flora of the islands, and in 1857 he republished the same work in revised form and well illustrated.

Since the appearance of these classical papers by Hooker and Andersson, there has been no general revision of the Galapageian flora. The islands have been visited, however, and plants collected upon them, — in 1868–1869 by Dr. A. Habel, in 1871 by the Hassler Expedition under the direction of Professor Louis Agassiz, in 1875 by Dr. Theodor Wolf, in 1884 by Lieutenants Chierchia and Marcacci, in 1888 by Professor Leslie A. Lee, in 1891 by Mr. Alexander Agassiz, also by the late Dr. Georg Baur and his assistant, Mr. C. F. Adams, and finally by Messrs. Robert E. Snodgrass and Edmund Heller of the Hopkins-Stanford Expedition. Not only have these collectors secured much additional material from the five larger islands, visited by Darwin and Andersson, but many specimens are now at hand, chiefly through the efforts of Dr. Baur and Messrs. Snodgrass and Heller, to illustrate the florulae of no less than twelve of the smaller islands, of which nearly all include new and peculiar species or forms. Furthermore, since the appearance of Andersson's works there have been many scattered notes, in monographic treatments of families and genera, throwing new light upon the identity, affinities, and nomenclature of Galapageian plants. It has therefore seemed desirable during the study of the rich botanical collections secured by the Hopkins-Stanford Expedition and referred by the Zoölogical Department of Stanford University to the Gray Herbarium for examination, to undertake a general recension of the flora of the Galapagos Islands, and to bring together its now more extensive bibliography, synonymy, and records of distribution. This has appeared the more worth while because some of the species regarded as new in the earlier treatments of the flora have dropped into synonymy and others formerly supposed peculiar to the islands are now known to occur in other regions; so that without a comprehensive revision it would be well nigh impossible to draw any statistical summary or show (1) in how far the vegetation of the archipelago is really peculiar, (2) to what other floras it is most nearly related, and (3) the complicated affinities existing between the florulae of the different islands. Finally, to these incentives there has been added a wish to derive, if possible, new light upon the origin of the islands themselves.

I am greatly indebted to my colleague, Professor W. G. Farlow, for the treatment of the difficult orders of *Fungi*, *Lichenes*, *Algae*, and *Musci*, to Dr. A. W. Evans of Yale University for the treatment of the *Hepaticae*, and to Professor K. Schumann of the Royal Botanical Museum at Berlin for the elaboration of the *Cactaceae* of the Hopkins-Stanford Expedition. Further expert assistance has been kindly and very promptly given by Mr. Casimir de Candolle of Geneva (*Peperomia*), Mr. C. B. Clarke of Kew (*Kyllinga*), Professor A. Cogniaux of Verviers (*Miconia*), Professor F. Lamson-Scribner of the United States Department of Agriculture (*Chloris*), Dr. Gustav Lindau of Berlin (*Justicia*), and Dr. Hans Hallier of Hamburg (*Ipomoea*). I am likewise indebted to Sir W. T. Thiselton-Dyer and Mr. W. Botting Hemsley for some comparisons at the Royal Gardens at Kew and for a list of the plants collected upon the Galapagos Islands by Dr. A. Habel. Mr. F. V. Coville and Dr. J. N. Rose of the United States National Museum have generously furnished me with duplicates of many of the plants secured on the Galapagos and Cocos Islands by Messrs. A. Agassiz and Lee, and also sent several unicates for examination. Professor William Trelease has obligingly forwarded the Galapageian *Cactaceae* from the Engelmann Herbarium for comparison. Sir J. D. Hooker has kindly given me interesting data concerning the history of the botanical exploration on the islands, Mr. J. Henry Blake, artist on the "Hassler," has furnished information regarding that expedition, and Miss Mary A. Day, librarian of the Gray Herbarium, has rendered efficient assistance in bibliography and tabulation as well as in an exhaustive search for information relating to the early expeditions to the islands. The plates have been drawn by Mr. F. Schuyler Mathews.

The bryophytes and thallophytes, as yet known to occur on the Galapagos Islands, are so few that they cannot be supposed in any adequate sense representative of the great groups to which they belong. It has, therefore, seemed best to exclude them from the tabular statistics. It may be remarked, however, that their inclusion would not have significantly altered the numerical relations presented.

Of the following plants, said to have been collected on the Galapagos Islands by Dr. Habel, the identifications (which cannot now be controlled by a reëxamination of the specimens) have seemed too doubtful to include in the catalogue: *Boerhaavia diffusa*, L., *Neptunia triquetra*, Benth., *Rhynchosia Senna*, Gillies, *Stylosanthes humilis*, HBK., *Tribulus terrestris*, L. var., *Acalypha parvula*, vars., *Waltheria ovata*, Cav., and *Evolvulus alsinoides*, L. var.

From Table I, reviewing the exploring expeditions which have done botanical collecting upon the Galapagos Islands, it will be seen that plants have been secured under no less than 2300 numbers and that about two-thirds of these have been examined during the preparation of this paper. With the exception of two cacti in the Engelmann Herbarium I have been unable to locate the phanerogams said to have been collected by Dr. Thomas Hill on the Hassler Expedition, and I should be greatly obliged for any information regarding their disposition.

Although great pains have been taken to summarize in this paper every bit of available information regarding the flora of these islands, which have now been visited by more than a score of collectors, it must be admitted that our knowledge of the subject is still very incomplete. With all due appreciation of the valuable and self-sacrificing services which zoölogists and geologists have rendered in collecting plants in the Galapagos, it should be remembered, when interpreting the general results, that during the last half century the islands have been visited by only one trained botanical systematist and by him very hurriedly. It cannot be doubted that much remains to be done in exploring and recording the Galapageian flora, and it is to be earnestly hoped that future collectors will endeavor to secure *specimens* of *each* plant from *every* island, since these insular races are in many cases so similar that mere lists, based upon hurried field determinations, are almost valueless.

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—— (6) In Hook. Icones Plantarum, xxviii. 1901, t. 2715–2719.

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Portulaceae from the Galapagos Islands. Lond. Jour. Bot. v. 1846, pp. 108–109.

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FUNGI.

By W. G. FARLOW.

Favolus, Fries.

F. ciliaris, Mont. Ann. Sci. Nat. Bot. ser. 2, xx. 364, t. 15, f. 2 (1843). — Narborough Isl.: southern part, alt. 615 m., Apr. 1899, *Snodgrass* & *Heller* (a single specimen). Further distrib. Madagascar and Japan.

FOMES, Fries.

F. LUCIDUS, Fries, Nov. Symb. 61 (1855). *Boletus lucidus*, Leysser, Fl. Hal. 300 (1783). — ALBEMARLE ISL.: Tagus Cove, *Snodgrass & Heller.* Further distrib. general.

SCHIZOPHYLLUM, Fries.

S. ALNEUM, Schroeter, Pilz. Schlesien, i. 553 (1889). *S. commune*, Hook. f. (3), 164; Anderss. (1), 123, & (2), 35. *Agaricus alneus*, L. Sp. 1176 (1753). — CHARLES ISL.: *Darwin* acc. to Hook. f., l. c. Further distrib. general.

Besides the *Favolus* and *Fomes* above mentioned, the collection of Messrs. Snodgrass & Heller contains a small dried *Polyporus* and a dried agaric, also alcoholic material of two *Agaricini* and a small tremelline fungus; none of these being in condition to determine.

LICHENES.

BY W. G. FARLOW.

Judged by the collections of previous expeditions as well as those of Messrs. Snodgrass & Heller, the lichen flora of the Galapagos Islands must be striking in appearance and abundant. Naturally the collections include principally the larger and more showy species of foliose and fruticulose habit, which are in most cases identical with species found along the Pacific Coast of America from California to Chili. Hooker (3), 164, records three lichens collected by Darwin on James Island; Andersson secured nine species from James and Chatham Islands, and Baur six from Charles, Albemarle, Hood, and Chatham Islands. The lichens collected by Hill on the Hassler Expedition were determined by Tuckerman and included three new species, *Lecanora glaucovirens*, *Rinodina mamillana*, and *Pertusaria albinea*, the second species having been found previously in Hawaii. Willey in his Synopsis of the Genus Arthonia, 1890, described an *Arthonia nivea* from the Galapagos Islands (collected by Hill), and reported *A. platyspeilea* Nyl., as well as a third unnamed species. The last was sent to Nylander, but I am unable to ascertain whether it was ever named by him. Apparently a considerable number of lichens were collected by the Hassler Expedition, but no list of them was ever made so far as I am aware, and as the specimens are scattered through the Tuckerman Herbarium no enumeration could be made without a long search. In the following catalogue reference is made to such of these specimens as have come under my notice.

ARTHONIA, Ach.

A GREGARIA, Koerb. Syst. 291 (1855); Willey, Synop. of gen. Arthonia, 7. *Sphaeria gregaria*, Weig. Obs. Bot. 43, t. 2, f. 10 (1772). — DUNCAN ISL.: on twigs, May, 1899, *Snodgrass* & *Heller*. Cosmopolitan.

A. NIVEA, Willey, Synop. of gen. Arthonia, 5 (1890). — GALAPAGOS IDS.: Hassler Exped. [*Hill*]. Endemic.

A. PLATYSPEILEA, Nyl. Pr. N. Gr. 99 (1863); Willey, Synop. of gen. Arthonia, 16. — GALAPAGOS IDS.: Hassler Exped. [*Hill*] acc. to Willey, l. c. GARDNER ISL.: May, 1899, on twigs with *Pyrenula cerina*, *Snodgrass* & *Heller*. Also found in Florida.

A. sp. Willey, Synop. of gen. Arthonia, 41 (1890). — GALAPAGOS IDS.: Hassler Exped. [*Hill*] acc. to Willey, l. c.

BUELLIA, De Not.

B. **straminea**, Tuck. in herb. — ALBEMARLE ISL.: north of Pt. Christopher, 21 Jan. 1899, *Snodgrass* & *Heller*.

This small specimen of a crustaceous *Buellia* on a volcanic rock is identical with a large specimen in herb. Tuckerman, Galapagos Ids., Hassler Exped. [*Hill*] which is labelled *B. straminea*, Tuck., which, however, appears never to have been described. Why it was never published may perhaps be due to the fact that there is another specimen in herb. Tuckerman from the Galapagos Ids., Hassler Exped. [*Hill*], marked *B. flavo-areolata*, Nyl. and that Tuckerman was in doubt whether *B. straminea* was distinct from that species. There is, however, no note of Tuckerman stating his opinion. The Snodgrass & Heller lichen agrees perfectly with the specimen of *B. straminea*. In both the asci are obovate, 32–42 μ by 14–17 μ; spores fuscous, 2-celled, 11 μ by 8–9 μ; paraphyses hyaline, 2 μ in diam., branching only at the apex where they end in globose tips, 5 μ in diam., the upper half of which is dark colored. Iodine turns the hymenium blue. The specimen of *B. flavo-areolata* in herb. Tuckerman differs only in having spores rather larger and narrower, those of *B. straminea* being nearly globose, in which respect it corresponds better with the description of *Lecidea flavo-areolata*, Nyl., Ann. Sci. Nat. Bot. ser. 4, iii. 166 (1855). It seems to me probable that the two species are really the same, but in the absence of material to show the variations of the spores, it can only be said that our lichen is certainly *B. straminea* which differs from *B. flavo-areolata* in the shorter and comparatively broader spores.

B. sp. — BINDLOE ISL.: on rocks, 20 June, 1889, *Snodgrass* & *Heller*.

A single specimen agreeing with some of the specimens referred to *B. lactea* in herb. Tuckerman, but it is not certain how far these agree with the views of other lichenologists.

CHIODECTON, Ach.

C. SANGUINEUM, Wainio, Lichens du Brésil, ii. 143 (1890). *Byssus sanguinea*, Sw. Prodr. Fl. Ind. 148 (1783). *Hypochnus rubrocinctus*, Ehrh. Hor. Phys. Berol. 84 (1820). — ABINGDON ISL.: alt. 250 m., 25 June, 1899, *Snodgrass* & *Heller*. Widely distributed in the warmer parts of the world.

CLADONIA (Hill) Wainio emend.

C. PYCNOCLADA, Nyl. Jour. Linn. Soc. ix. 244 (1866). *Cenomyce pycnoclada*, Gaudichaud, Ann. Sci. Nat. Bot. v. 95 (1825). *Cladonia rangiferina*, var. auct. var. — ALBEMARLE ISL.: mountain east of Iguana Cove, alt. 925 m., Feb. 1899, *Snodgrass* & *Heller*; mountain east of Tagus Cove, alt. 770 m., Feb. 1899, *Snodgrass* & *Heller*. Further distrib. S. Am., Africa, Asia, Australia.

LECANORA, Ach.

L. GLAUCOVIRENS, Tuck. Proc. Am. Acad. xii. 172 (1874). — GALAPAGOS IDS.: Hassler Exp. [*Hill*]. Endemic.

PARMELIA, Ach.

P. PERLATA, Kremph. Flora, lii. 222 (1869); Anderss. (1), 124, & (2), 36. — ABINGDON ISL.: *Snodgrass* & *Heller*. ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*. CHARLES ISL.: *Andersson* (specimen not seen by the writer). The specimens of Messrs. Snodgrass & Heller are small and sterile but appear to belong to this common species.

P. sp. (*P. physodi*, Fries affinis) Andersson (1), 124, & (2), 36. — CHARLES ISL.: *Andersson*. Sterile and indeterminate, not seen by the writer.

PERTUSARIA, DC.

P. ALBINEA, Tuck. Proc. Am. Acad. xii. 177 (1877). — GALAPAGOS IDS.: on bark, Hassler Exp. [*Hill*]. Endemic.

PHYSCIA (DC.) Th. Fr.

P. LEUCOMELA (L.) Michx. Fl. Bor.-Am. ii. 306 (1803). *Lichen leucomelas*, L. Sp. ed. 3, 89 (1764). *Parmelia leucomela*, auct. var.

P. leucomela, var. *filiformis*, Anderss. (1), 124, & (2), 36. *Borrera leucomelas*, var. *filiformis*, Hook. f. (3), 164. — JAMES ISL.: *Darwin*, acc. to Hook. f., l. c. who states that "the same slender variety is common on the Peruvian Coast."

PSEUDOCYPHELLARIA, Wainio.

P. AURATA, Wainio, Lichens du Brésil, i. 183 (1890). *Sticta aurata*, Ach. Meth. Lich. 277 (1803); Hook. f. (3), 164; Anderss. (1), 124, & (2), 36. — ALBEMARLE ISL.: mountain east of Iguana Cove, alt. 925 m., Feb. 1899, *Snodgrass* & *Heller*. JAMES ISL.: *Darwin*. NARBOROUGH ISL.: southern part, alt. 615 m., March, 1899, *Snodgrass* & *Heller*. Further distrib. general.

PYRENULA, Fée.

P. CERINA, J. Muell, Flora, lxvii. 667 (1884). *Verrucaria cerina*, Eschw. Bras. 133 (1833). *Pyrenula aurantiaca*, Fée, Suppl. 82, t. 37, f. 1 (1837). — NARBOROUGH ISL.: southern part. alt. 615 m., 5 April, 1899, *Snodgrass* & *Heller*. On branches of the palo santo (*Bursera graveolens*) with spermagonia and spores. The latter are obtusely elliptical, 28 μ by 14 μ, at first hyaline and multiguttulate but becoming dark and 4-celled. Further distrib. Bermuda, S. Am.

RAMALINA, Ach.

R. COMPLANATA, Ach. Lich. Univ. 599 (1810). *Lichen complanatus*, Sw. Fl. Ind. Occ. iii. 1911 (1806). — GALAPAGOS IDS.: Hassler Exp. [*Hill*]. CHARLES ISL.: Cuevas Bay, *Baur*. GARDNER ISL.: May, 1899, *Snodgrass* & *Heller*. HOOD ISL.: May, 1899, *Snodgrass* & *Heller*. TOWER ISL.: June, 1899, *Snodgrass* & *Heller*. Further distrib. warmer parts of America, East Indies, Australia.

R. INDICA, Fr. Kongl. Vet. Ak. Handl. 43 (1820); Anderss. (1), 123, & (2), 35. — CHARLES ISL.: on trunks of trees in the upper region, *Andersson*. Not seen by the writer. Further distrib. India.

R. USNEOIDES, Fr. Lich. Eur. 468 (1831). *Parmelia usneoides*, Ach. Meth. Lich. 270 (1803). — BINDLOE ISL.: 20 June, 1899, *Snodgrass* & *Heller*, a small form. A characteristic species of the warmer regions of America. A specimen in herb. Tuckerman collected on the Galapagos Ids. by the Hassler Exp. [*Hill*], and marked *Alectoria sarmentosa* appears to belong rather to this species.

RINODINA, Mass.

R. MAMILLANA, Tuck. Proc. Am. Acad. vii. 226 (1866), & xii. 174. — GALAPAGOS IDS.: Hassler Exp. [*Hill*]. Further distrib. Oahu, Hawaiian Ids.

ROCCELLA, DC.

The most striking feature of the maritime lichen flora of the Galapagos Islands as well as of the west coast of America from California southward is the abundance of species of *Roccella* of which very fine specimens were collected by Messrs. Snodgrass & Heller. In most herbaria the *Roccellae* of this region are somewhat vaguely placed under the species *R. fuciformis* and *R. tinctoria*, the complanate forms being referred to the former and the more or less terete forms to the latter. The excellent Monographia Roccelleorum of Darbishire, Stuttgart, 1898, affords the means of a more satisfactory determination. The separation of *Dendrographa* and *Roccellaria* from *Roccella* on the structure of the cortex is well founded. All the specimens of this group collected by Messrs. Snodgrass & Heller belong to the genus *Roccella* proper.

R. PERUENSIS, Darb. Bull. Herb. Boiss. v. 763 (1897), & Monogr. Rocc. 47, t. 18, 19, f. 83–88 (1898). *R. Montagnei*, var. *peruensis*. Kremph. Verh. Zoöl. Bot. Gesell. Wien, xxvi. 443 (1876). *R. fuciformis*, C. E. Cummings in Rob. & Greenm. (1), 149, not Ach. — ALBEMARLE ISL.: *Baur*. BARRINGTON ISL.: *Snodgrass & Heller*. CHARLES ISL.: *Baur*. CHATHAM ISL.: *Snodgrass & Heller*. DUNCAN ISL.: *Snodgrass & Heller*. GARDNER ISL.: *Snodgrass & Heller*. HOOD ISL.: *Baur; Snodgrass & Heller*. TOWER ISL.: *Snodgrass & Heller*. A considerable number of specimens were collected, none, however, with apothecia. The yellow basal filaments described by Darbishire are present in this species, which grows on shrubs. The specimens are larger and broader than the figures of this species in Darbishire and some resemble more closely his figure of *R. portentosa*, t. 8, f. 32. It may be questioned whether that figure really belongs to *R. portentosa*. In all specimens having this habit there were present the yellow base and lignicolous habitat which indicate *R. peruensis*.

R. PORTENTOSA, Darb. Ber. Deutsch. Bot. Gesell. xv. 4 (1897), & Monogr. Rocc. 29, t. 7–11, f. 27–41 (1898). *R. tinctoria*, var. *portentosa*, Mont. in Gay, Pl. Chili, viii. 841 (1852). — BARRINGTON ISL.: May, 1899, *Snodgrass & Heller*. GARDNER ISL.: May, 1899, *Snodgrass &*

Heller. James Isl.: *Hill* of the Hassler Exp. (in hb. Tuckerman as *R. tinctoria*). Seymour Isl.: south, May, 1899, *Snodgrass* & *Heller*. Further distrib. Peru (a specimen in hb. Farlow coll. by Gaudichaud and distrib. as *R. tinctoria*, f. *dichotoma*).

R. intricata, C. E. Cummings in Rob. & Greenm. (1), 149, coll. by Baur upon Charles Island, has the cortex of *Roccella* proper and therefore cannot be placed in *Roccellaria* where *R. intricata* properly belongs but is a narrow form of *R. peruensis*. The same may be said of some, but perhaps not all, of the *R. phycopsis*, Hassler Exp. [*Hill*] from the Galapagos Islands in herb. Tuckerman.

Sticta, Schreb.

S. Weigelii, Wainio, Lichens du Brésil, i. 189 (1890). *S. damaecornis*, var. *Weigelii*, Ach. Lich. Meth. 446 (1810). — Albemarle Isl.: Iguana Cove, 30 Dec. 1898, *Snodgrass* & *Heller*. This is the *S. quercizans*, common throughout the warmer parts of America.

Teloschistes, Norm.

T. flavicans, Norm. Gen. Lich., 17, 1852. *T. chrysopthalmus*, var. *flavicans*, Auct.; C. E. Cummings in Rob. & Greenm. (1), 149. *Lichen flavicans*, Sw. Prodr. 147 (1788). *Evernia flavicans*, var. *crocea*, Anderss. (1), 123, & (2), 35. — Albemarle Isl.: *Snodgrass* & *Heller*. Charles Isl.: *Andersson*; *Snodgrass* & *Heller*. Chatham Isl.: *Baur*, acc. to C. E. Cummings, l. c. Duncan Isl.: *Snodgrass* & *Heller*. Further distrib. general.

Usnea, Dill.

U. arthroclada, Fée, Ess. Crypt. Écorc. xcvii. & ci. 3, f. 4, 5 (1824). — Narborough Isl.: southern part, alt. 615 m., 5 April, 1899, on *palo santo* trees (*Bursera graveolens*) with *Ramalina usneoides*, *Snodgrass* & *Heller*. Further distrib. Brazil.

This has a habit resembling *U. longissima* but stouter. With potash the medulla turns reddish brown, and this fact with the hard and polished cortex often breaking and forming articulations indicates *U. arthroclada*.

U. ceratina, Ach. Lich. Univ. 619 (1810). *U. barbata*, Ach. f. *ceratina* Schaer, Spec. 505 (1840). — Narborough Isl.: *Snodgrass* & *Heller*. Cosmopolitan.

U. plicata, Ach. Syn. Meth. 305 (1814); Hook. f. (3), 164; Anderss.

(1), 124, & (2), 36 — CHARLES ISL.: *Andersson*. JAMES ISL.: *Darwin*. Further distrib. general.

ALGAE.

BY W. G. FARLOW.

CYANOPHYCEAE.

SCYTONEMA, Agardh.

S. MYOCHROUS, Ag. Disp. Alg. Suec. 38 (1812). *Conferva*, Dillw. Brit. Conferv. t. 19 (1802). — ALBEMARLE ISL.: Mangrove Swamp, Turtle Pt., *Snodgrass* & *Heller*. Further distrib. general.

CHLOROPHYCEAE.

BRYOPSIS, Lamx.

B. RAMULOSA, Mont. Cuba, 16, t. 3, f. 2 (1838). *B. pennatula*, J. G. Agardh, Oefver. Kongl. Vet. Akad. Förh. iv. 6 (1847). — WENMAN ISL.: Dec. 1898, *Snodgrass* & *Heller*. Further Distrib. W. Ind., Fla.

CAULERPA, Lamx.

C. RACEMOSA, Weber van Bosse, Ann. Jard. Bot. Buitenzorg, xv. 357 (1898). *Fucus racemosus*, Forsk. Flor. Aegypt. 191 (1775). *Caulerpa clavifera*, Ag. Spec. Alg. 437 (1823). — ALBEMARLE ISL.: Jan. 11, 1899, *Snodgrass* & *Heller*. Further distrib. tropical seas.

ENTEROMORPHA, Link.

E. CLATHRATA, Kütz. Phyc. Germ., 247 (1845). *Conferva clathrata*, Roth. Cat., III, 175 (1806). — ALBEMARLE ISL.: Turtle Pt. in mangrove swamp, *Snodgrass* & *Heller*. Further distrib. cosmopolitan.

OEDOGONIUM, Link.

O. sp. — ALBEMARLE ISL.: Iguana Cove with *Najas*, *Snodgrass* & *Heller*. An entangled mass of a sterile *Oedogonium*.

RHIZOCLONIUM, Kütz.

R. HOOKERI, Kütz. Tab. Phyc., III, Pl. 67, f. 2 (1853). — ALBEMARLE ISL.: Tagus Cove, Jan. 11, 1899, *Snodgrass* & *Heller*. Further distrib. Nicobar Ids., Chiloe, etc.

Entangled masses with cells 120–140 μ in breadth and of the same length or a little longer. Cell-walls rather thick. The rhizoidal branches seen were very scanty, but that is often the case in other species of the genus. This seems to be the same as the *R. Hookeri* of Grunow, Novara, 37, and his remarks on the Nicobar specimens apply equally to ours. Hohenacker, 477, Chiloe, has cells rather longer and thinner. In spite of the occasional rhizoid processes, it seems to me possible that this may be an old, prostrate form of *Chaetomorpha antennina* (Bory) corresponding to the similar form known in *C. aerea*.

ULVA (L.) Wittr.

U. FASCIATA, Delil. Egypt, 153, Pl. 158, f. 5 (1813). — ALBEMARLE ISL.: Tagus Cove, Feb. 1899, *Snodgrass* & *Heller*. Further distrib. warmer parts of Atlantic and Pacific Oceans, Ceylon.

*PHAEOPHYCEAE.**

CARPOMITRA, Kütz.

C. CABRERAE, Kütz. Phyc. Gen. 343 (1843). *Fucus cabrera*, Clemente, Cat. 313; Turner, Hist. Fuc. Pl. 140 (1811); Piccone (1), 40. — CHATHAM ISL.: *Marcacci*, acc. to Piccone, l. c. Further distrib. southern Eu., Africa.

FUCODIUM, J. Ag.

F. GALAPAGENSE, Picc. & Grun. in Piccone (1), 40, t. 1, f. 1, t. 2, f. 3, & (2), 22. — CHARLES and CHATHAM IDS.: *Marcacci*, acc. to Piccone, ll. cc. Endemic.

GLOSSOPHORA, J. Ag.

G. galapagensis, nov. sp.

Frons usque 18 cm. longit., .6 cm. latit. linearis-elongata, saepe dichotoma, axillis apicibusque obtusis, margine distincte spinosa; undique praecipue in parte inferiore phylliculis vestita quae basem versus deorsum ad instar radicorum prolongantur. Sori oosporiferi minuti irregulariter supra frondem sparsi. — ALBEMARLE ISL.: Turtle Pt., March, 1899, *Snodgrass* & *Heller*.

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* *DIATOMEAE.* Many species of this order from the Galapagos Islands are enumerated by Ehrenberg (2), and the following are recorded by Cleve in his paper On some new and little known Diatoms, Kongl. Svenska Vetensk. Akad. Handl. xviii. no. 5, pp. 26, Pl. I–VI, 1881. *Mastogloia panduriformis*, Cl.; *M. submarginata*, Cl. & Grun.; *Navicula Eugeniae*, Cl.; *N. Hennedyi*, var. *undulata*, Cl.; *N. Hennedyi*, var. *minuta*, Cl.; *N. Platessa*. Cl.; *N. Anderssonii*, Cl.; *N. gallapagensis*, Cl.; *N. jugata*, Cl.; *Surirella degenerans*, Cl.; *S. formosa*, Cl.; *Campylodiscus peramplus*, Cl.; *Plagiogramma spinosum*, Cl.; *Rutilaria recens*, Cl.; *Melosira tuberculosa*, Cl.; *Auliscus insignis*, Cl.; *Biddulphia gallapagensis*, Cl.; *Triceratium Tripos*, Cl.; *T. Anderssonii*, Cl.; *T. Cleve*, Cl.; *T. gallapagense*, Cl.; *T. Margaritiferum*, Cl.

In all the specimens there are leaflets on both surfaces of the fronds except near the tips. They arise from the cortical cells, and have no connection with the fructification which is found on the frond itself. Near the base the leaflets grow downward, become somewhat thicker and twisted and bear on their lower surface the proper rhizoids so that the fronds are attached by what is generally called a fibrous base. In spite of the leaflets which would lead one to refer this alga to the genus *Glossophora*, it seems to me to be less closely related to *G. Kunthii*, J. Ag. of Peru, the type of the genus, than to *Dictyota crenulata*, J. Ag. of the Pacific Coast of Mexico. As originally described from Mexico that species has no leaflets and the base is described as stupose. In 1884 Grunow described a variety from the Canary Islands with leaflets, but its relation to that species is uncertain. The Galapagos species differs so much from *D. crenulata* as originally described, and from the var. *canariensis* that it must be kept distinct. If material hereafter collected should show that the two species are really forms of a single species, a new description will be needed. The following notes on the microscopic structure of the Galapagos alga will be of interest.

In the upper part the frond is about 100 μ thick, becoming 200 μ thick below. Above, there is a single medullary layer of large cells 70 μ × 70 μ × 168 μ and a single layer of colored cortical cells 14 μ × 14 μ in section. Below, the cortical cells become rather deeper, 22 μ × 14 μ in section, and the medullary cells thicker-walled and comparatively narrower, 112 μ × 56 μ in section, and between the medullary and cortical cells is a single layer of flattened colorless cells, 28 μ × 28 μ × 90 μ. The tangential walls of the medullary cells are pitted to an extent seldom seen in algae of this order. The oogonia are 80 μ or more in diameter and found in small numbers in scattered sori on the frond. The spines are sharper and more solid than in most of the ciliate species, 24–32 mm. long, the tips deeper colored than the base.

PADINA, Adams.

P. DURVILLAEI, Bory, Coquille, 147, t. 21, f. 1 (1829). — ALBEMARLE ISL.: Tagus Cove, and Turtle Pt., Feb. & Mar. 1899, *Snodgrass* & *Heller*. CHATHAM ISL.: north, Aug. 1891, *Baur* as *Zonaria lobata*. Further distrib. W. Ind., west coast of Am.

Typical specimens of this fine species were collected, some more than 12 cm. long. The species has a characteristic rusty-brown color and is coarser than *P. Paronia.* In the median portion the frond has usually ten layers of cells, 80 × 40 μ in section, and a single layer of cortical cells, 14 × 9 μ. The very large oosporangia vary from 110–125 μ × 70–84 μ.

SARGASSUM, Ag.

S. CYMOSUM, Ag. Spec. i. 20 (1821); J. Ag. Sp. Sarg. Austr. 109 (1889). — ALBEMARLE ISL.: Tagus Cove and Turtle Pt., March, 1899, *Snodgrass* & *Heller.* WENMAN ISL.: Dec. 1898, *Snodgrass* & *Heller.* Further distrib. warmer Atlantic coasts of Asia and Africa.

In the determination of this species I have followed J. Ag. l. c. (1889) and the Albemarle and Wenman Island specimens seem to me to belong to *S. cymosum* as that species is limited by J. Agardh. The species has, however, been variously interpreted by different writers.

S. GALAPAGENSE, Grunow in Piccone (1), 48, t. 1. — CHATHAM ISL.: *Marcacci*, acc. to Piccone, l. c. Endemic.

S. GALAPAGENSE, var. SETIFOLIA, Grunow in Piccone (1), 48, t. 2, f. 1, 2 (1886). — ALBEMARLE ISL.: Tagus Cove and Turtle Pt., Feb. 1899, *Snodgrass* & *Heller.* CHATHAM ISL.: March, 1884, *Marcacci*, acc. to Piccone, l. c. Endemic.

As is remarked by J. Agardh, l. c., 122, this variety is closely related to *S. piluliferum* (Turn.) Ag., of California and Japan, but not to *S. acinaria* (Turn.) J. Ag., as suggested by Grunow.

? S. GRAMINIFOLIUM, J. Ag. Spec. i. 103 (1848). *Fucus graminifolius*, Turn. Hist. Fuc. Pl. 210 (1819). — WENMAN ISL.: *Snodgrass* & *Heller.* Further distrib. China Sea.

To this species is referred with much doubt a single specimen of a *Sargassum* with the upper fructiferous portion only. The receptacles are racemose, the conceptacles contain antheridia and oogonia, and the leaves are long, narrow, serrate, with but few cryptostomata.

S. LENDIGERUM, var. FOLIOSA, Grunow in Piccone (1), 49. — CHATHAM ISL.: March, 1884, *Marcacci*, acc. to Piccone, l. c.

S. LENDIGERUM, var. FURCIFOLIA, Grunow, l. c. 50. — CHATHAM ISL.: March, 1884, *Marcacci*, acc. to Piccone, l. c.

Of the var. *foliosa*, Grunow described two forms, *subdelicatula* and *rigidiuscula* and of var. *furcifolia* a forma *denticulata.* J. Agardh, l. c.

122, remarks that the var. *furcifolia* is hardly different from *S. galapagense*. *S. lendigerum* is a species known mainly from the figure in Turner, Hist. Fuc. Pl. 48 (1808), with which agree the specimens from Bermuda referred to *S. lendigerum* by J. Agardh, l. c. 110, and the species appears to be limited to the Atlantic. None of the specimens of Snodgrass & Heller can be referred to *S. lendigerum*, but it is probable that some of the forms of Grunow's var. *foliosa* may be the same as specimens which I have included under *S. cymosum*.

SPATOGLOSSUM (KUETZ.), J. AG. Emend.

S. SCHROEDERI, KUETZ. Tab. Phyc. ix. 21, t. 51, f. 1 (1859); Piccone (2), 17. *Zonaria*, Ag. Syst. 265 (1824). — CHARLES ISL.: *Marcacci*, acc. to Piccone, l. c. Further distrib. Brazil, W. Ind.

ZONARIA, J. G. Agardh.

Z. LOBATA, Ag. Syst. 265 (1824); Piccone (1), 89, & (2), 40. — CHARLES and CHATHAM IDS.: March, 1884, *Marcacci*. Further distrib. trop. Atlantic coast of America, Teneriffe, Cape of Good Hope.

RHODOPHYCEAE.

AMPHIROA, Lamx.

A. DILATATA, Lamx. Hist. Polyp. Flex. 299 (1816); Piccone (1), 66, & (2), 46. — ALBEMARLE ISL.: Elizabeth Bay, *Snodgrass* & *Heller*. CHARLES ISL.: *Snodgrass* & *Heller*. Further distrib. Australia.

A. ORBIGNIANA, Decaisne, Ann. Sci. Nat. Bot. ser. 2, xviii. 124 (1842). — ALBEMARLE ISL.: Turtle Pt., March, 1899, *Snodgrass* & *Heller*. Further distrib. Pacific Coast of Am.

The specimens are covered with parasites and not in typical condition, but they appear to belong to this species.

A. PERUANA, Aresch. Phyc. Extraeur. 41 (1854). — WENMAN ISL.: Dec. 1898, *Snodgrass* & *Heller*. Further distrib. Florida, Peru.

CALLOPHYLLIS, Kütz.

C. sp. — ALBEMARLE ISL: Iguana Cove, *Snodgrass* & *Heller*.

Two small specimens with cystocarps which are of large size, some of them with marked papillae around the carpostome. The species is apparently near *C. furcata*, Farl. but the specimens are incomplete and cannot be referred with certainty to that species.

CHONDRUS, Stack.

C. CANALICULATUS, Grev., Alg. Brit., lv. (1830). *Sphaerococcus canaliculatus* C. Ag. Spec. I, 260 (1822). — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*. Further distr. West coast of S. Am.

A single specimen which can be referred to this species. There are a few specimens which may perhaps belong to this species but they certainly are not normal. They have the structure of the frond and the sporidia of *Chondrus*, but, in the absence of cystocarps, there is a possibility that they may belong to the genus *Iridaea*. Their habit, however, is that of *Chondrus*, but the fronds are occasionally perforate, which may however be accidental rather than normal.

CORALLINA, Lamx.

? C. BERTERII, Mont. in Harvey, Ner. Austr. ii. 103 (1858), & Fl. Chili. viii. 318. — ALBEMARLE ISL.: Elizabeth Bay, *Snodgrass* & *Heller*. Further distrib. Chili.

Much covered with parasites and not in favorable condition to be determined.

DASYA, C. Ag.

D. **Stanfordiana**, nov. sp.

Frons cylindrica, ad 15 cm. longit., basem versus 1.25 mm. latit., irregulariter composito-pinnata, usque apicem dense corticata, ramis superne elongatis, ramellis subaequalibus, brevibus, circa 32 mm. longit., vestitis. Ramelli monosiphonii, spiraliter inserti, unilateraliter cymosi, cellulis 56 μ × 28 μ, cellula terminali conica, inter ramificationes bicellulis. Stichidia primum ovato-lanceolata demum cylindrico-apiculata, 168 μ × 28 μ bene evoluta, sporidiis 20-seriebus et ultra ordinatis, cellulis pericentralibus 5–8. Color roseo-purpureus, substantia gelatinosa. Chartae arcte adhaeret. — WENMAN ISL.: Dec. 21, 1898, *Snodgrass* & *Heller*. ALBEMARLE ISL.: Turtle Pt., March, 1899, *Snodgrass* & *Heller*.

A species resembling *D. pacifica*, Harv. in its long stichidia but differing in its long slender habit quite unlike that of *D. arbuscula* J. Ag. to which *D. pacifica* bears a strong resemblance and in its branches densely corticated to the apex. In the authentic specimen of *D. pacifica* which I have examined the pericentral cells were easily seen at some distance from the apex, whereas in *D. Stanfordiana* they can be seen only at the extreme tip on account of the thick cortex. The monosiphonous ramelli and the exposed sporangia agree with those of *Dasya*

as defined by Falkenberg, but there is a free cell between those from which the divisions of the ramelli arise. With this exception, the plan of ramification follows the type shown in Falkenberg, fig. 10, A. The divisions of the ramelli are not divergent but penicillate with a tendency to be slightly convergent. The older stichidia are very long and the consecutive series of sporidia are often as many as 20 and sometimes more, showing 3–4 sporangia on side view. The branches are radial and not dorsiventral, although in drying they have the appearance of being flattened in the upper part. In section of the larger branches it was not possible to recognize any definite axial or pericentral cells in the material examined, but they had probably collapsed in the preparation.

DILSEA, Stack.

D. sp. — ALBEMARLE ISL.: Iguana Cove, Dec. 30, 1898, *Snodgrass & Heller.*

A single imperfect specimen with sporangia; the material insufficient for specific determination.

GALAXAURA, Lamx.

G. MARGINATA, Lamx. Hist. Polyp. Flex. 264 (1816). *Corallina marginata*, Ell. & Sol. 122, t. 22, f. 6 (1786). *Brachycladia*, Sonder et Auct. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass & Heller.* Further distrib. tropical seas.

GELIDIUM, Lamx.

G. CRINALE, J. Ag. Epicr. 546 (1876). *Fucus crinalis*, Turn. Hist. Fuc. Pl. 198 (1819). *Gelidium corneum*, var. *crinale*, Auct.; Piccone (2) 39. — CHARLES ISL.: *Marcacci*, acc. to Piccone, l. c. Further distrib. general.

G. SERRULATUM, J. Ag. Oefv. Kongl. Vet. Akad. Förh. iv. 11 (1847). — WENMAN ISL.: Dec. 1898, *Snodgrass & Heller.* Further distrib. Venezuela.

Several specimens of a *Gelidium* were collected at Wenman Isl. with well-developed bilateral conceptacles characteristic of the genus. One would have expected from this locality rather *G. filicinum*, Bory, Coquille, 162 (1829), than *G. serrulatum* since the former species was first described from Chili, whereas the latter is an Atlantic species. I have been able to compare the Wenman Isl. specimens with an authentic specimen of *G. filicinum* from herb. Bory through the kindness of

Dr. E. Bornet and with an authentic specimen of *G. serrulatum* from herb. Agardh. It seems to me that there can be no doubt that they agree perfectly with *G. serrulatum* and differ from *G. filicinum* although the two species are certainly closely related.

G. FILICINUM, Bory, Coquille, 162 (1829). — ALBEMARLE ISL.: Iguana Cove, Dec. 30, 1898, *Snodgrass* & *Heller*. Further distrib. west coast of South America.

But a single specimen was found which may be referred to this species.

GIGARTINA, Stackh.

G. LESSONII, J. Ag. Spec. Alg. ii. 268 (1851). *Sphaerococcus Lessonii*, Bory, Coquille, 171 (1829), excl. syn. — ALBEMARLE ISL.: Elizabeth Bay and Iguana Cove, Feb. 1899, *Snodgrass* & *Heller*. Further distrib. Peru, Chili.

Most of the specimens were small and sterile, but one had conceptacles which in this species are borne at the base of small branchlets rather than in well-marked papillae as in most species of the genus.

GRACILARIA, Grev.

? G. RUGULOSA, Mont. Pole Sud, 155, Pl. 13, fig. 1 (1845) sub *Hypnea*. — ALBEMARLE ISL.: Iguana Cove. *Snodgrass* & *Heller*. Further distrib. Australia.

Two specimens with cruciate sporidia which agree well with the description and figure of the habit of this species. Montagne's figure shows sporidia which certainly are not zonate as in *Hypnea*, but may be tripartite, although the figure is not quite clear on that point. The Galapagos alga has cruciate sporidia and hence, in spite of its habit, can be referred only with doubt to Montagne's species.

GYMNOGONGRUS, Mart.

G. GRIFFITHSIAE, var. GALAPAGENSIS, Picc. & Grun. in Piccone (1), 60, & (2), 31. — CHARLES and CHATHAM IDS.: *Marcacci*, acc. to Piccone, ll. cc. Endemic.

G. MELANOTHRIX, Grunow in Piccone (1), 60, & (2), 31. *Gigartina melanothrix*, Bory, Coquille, 152, t. 19, f, 3 (1829.) — CHARLES and CHATHAM IDS.: *Marcacci*, acc. to Piccone, ll. cc. Further distrib. Chili.

G. VERMICULARIS, J. Ag. Spec. Alg. ii. 323 (1851.) *Fucus vermicularis*, Turn. Hist. Fuc. Pl. 221 (1819); Piccone (1), 61. —

CHATHAM ISL.: March, 1884, *Marcacci*, acc. to Piccone, l. c. Further distrib. western So. Am.

HERPOPHYLLON, gen. prov.

Frons membranacea, prostrata, subcircularis vel vague expansa, centro radicibus adfixa. Structura interne cellulis oblongis vel radiatim elongatis, parietibus distincte colloideis cellulis superficiem versus rotundatis in stratum unicum corticale transformatis constituta. Sporangia cruciatim divisa, in soris verrucaeformibus supra frondem sparsa. Cystocarpia ignota.

H. **coalescens**, spec. prov.

Frondes circa 40 mm. diametro, aggregatae, a marginibus coalescentes, thallum indefinite expansum formantes. Sporangia in soris verrucaeformibus ad superficiem superiorem frondis inter paraphyses allata, anguste ovales, 56 μ × 15–20 μ. ALBEMARLE ISL.: Tagus Cove, March, 1899, *Snodgrass* & *Heller*.

The provisional name given above is adopted to designate a curious alga the cystocarpic fruit of which is unknown and which in other respects does not agree with any genus known to me. When removed from the fluid in which it was preserved it appeared like an irregular, rather cartilaginous pellicle adherent to the substratum by groups of coarse rhizoids. On a closer examination the surface was seen to be ridged, or veined, and apparently the larger specimens are composed of smaller individual fronds which have united, each tuft of rhizoids being in the centre of a frond. The smaller specimens were like the larger, but none seen was so small as to be composed of a single frond. The microscopic structure of the ridges show that they are the lines of union of two different fronds, or, if the whole is to be regarded as a single plant, of its different lobes. The internal structure consists of a compact tissue of large cells about 56 μ in transverse section but more or less elongated in radial sections, some being 140 μ long. They cannot, however, be called filaments. Near the upper and lower surfaces and near the margin the cells become more nearly spherical. The cell-walls are markedly colloidal and near the margin seem to be imbeded in a solid colloidal matrix. The cortical cells are small and arranged in a single layer, especially on the lower surface, those on the upper surface more frequently dividing anticlinically, especially where the sporidia are forming. The latter are in spots on the upper surface, which are sometimes not much raised but at other times form well-marked warts. The sporangia are borne between paraphyses about 60 μ long and 3 μ broad, formed by the outgrowth of cortical cells. It is to be regretted that more is not known as to the mode of growth of the frond. As it is, this alga, although at first sight it suggested *Peyssonnelia rugosa* Harv. in its habit, cannot be referred to that genus, for the frond in section is seen to be symmetrical except that the cortical cells on the under side are more uniformly in a single layer than above, but quite different from the well-defined base of *Peyssonnelia*. Nor can it be considered a prostrate form of *Kallymenia* or *Constantinea*, to which it appears to be more nearly related. It might be suggested that the specimens were abnormal developments of some species with erect fronds, but the abundance of sporidia appear to indicate that the growth was normal.

HYPNEA, Lamx.

H. PANNOSA, J. Ag. Oefv. Kongl. Vet. Akad. Förh. iv. 14 (1847). — ALBEMARLE ISL.: Tagus Cove, Feb. 1889, *Snodgrass* & *Heller*. Further distrib. tropical seas.

LAURENCIA, Lamx.

L. OBTUSA (Huds.) Lamx., var. GRACILIS, Harv. Ceyl. Alg. 26; Piccone (1), 80. — CHATHAM ISL.: March, 1884, *Marcacci*, acc. to Piccone, l.c. Further distrib. general.

LITHOPHYLLUM, Phil.

* L. FARLOWII, Heydrich in Engler, Bot. Jahrb. xxviii. 532, t. 1, f. 6 (1901). — CHARLES ISL.: *Hassler Exped.* Endemic.

LITHOTHAMNIUM, Phil.

L. CRASSUM, Phil. Weigm. Arch. 1837, 2, p. 388; Piccone (2), 45. — CHARLES ISL.: *Marcacci*, acc. to Piccone, l. c.

No specimens of this species have been seen, and I am unable to say whether the alga collected by Marcacci on the Pisani Exp. is the same as the *L. crassum*, originally described from the Mediterranean, as that species is now understood.

MELOBESIA, Lamx.

M. CORTICIFORMIS, Kütz. Spec. Alg. 696 (1849). — WENMAN ISL.: on *Gelidium serrulatum*, Dec. 1888, *Snodgrass* & *Heller*. Further distrib. general.

M. PUSTULATA, Lamx. Hist. Polyp. Flex. 315, t. 12, f. 2 (1816); Piccone (1), 65. — CHATHAM ISL.: (on *Zonaria lobata*) *Marcacci*, acc. to Piccone, l. c. Further distrib. general.

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* Name changed to *L. claudescens*, Heydrich, Ber. deutsch. bot. Ges. xix, 430, as it is antedated by *L. Farlowii*, Foster.

OCHTODES, J. Ag.

O. FILIFORMIS J. Ag. Bid. Flor. Syst., 5 (1871), Flor. Morph. Pl. 31, p. 1–8 (1879) ; *Sphaerococcus filiformis* J. Ag. Spec. Alg. ii, 664 (1851). — WENMAN ISL. : Dec. 1898, *Snodgrass* & *Heller.* Further distrib. West Indies.

The occurrence of this characteristic but not common West Indian species at the Galapagos is interesting. The few specimens collected have an abundance of cystocarps whose structure together with the peculiar axial structure of the fronds leaves no doubt as to the genus, nor am I able, even in their habit, to recognize any specific distinction between West Indian and Pacific specimens.

PEYSSONNELIA, Decaisne.

P. RUBRA J. Ag. Spec. Alg. ii. 502 (1851). *Zonaria rubra* Grev. Trans. Linn. Soc. xv. 340, Pl. III. f. 3 (1827). — ALBEMARLE ISL. : Elizabeth Bay, *Snodgrass* & *Heller.* Further distrib. Mediterranean, Pacific Islands.

Specimens with sporangia, the color altered by the preserving fluid.

PLOCAMIUM, Lyngb.

P. COCCINEUM Lyngb. Tent. Hydr. 39, Pl. 9, B. (1819). *Fucus coccineus* Huds. Fl. Angl. ed. 2, 586 (1778). — WENMAN ISL., Dec. 1898, *Snodgrass* & *Heller.* Distribution, general.

RHODYMENIA (Grev.) J. Ag. Emend.

R. FLABELLIFOLIA, Mont. Bonite, 105 (1844). *Sphaerococcus flabellifolius,* Bory, Coquille, 174, t. 17 (1829). — ALBEMARLE ISL. : Elizabeth Bay and Iguana Cove, Dec. 30, 1898, *Snodgrass* & *Heller.* Further distrib. Peru, Chili.

All the specimens from Elizabeth Bay and some of those from Iguana Cove have the typical habit of this species. Some of the specimens from the latter locality are more branched than the type but appear to belong to the same species.

HEPATICAE.

By A. W. EVANS.

Bryopteris, Lindenb.

B. filicina (Sw.) Nees, Syn. Hep. 284 (1845). *Jungermannia filicina*, Wilson in Hook. f. (3), 165. — James Isl.: *Darwin.* Widely distributed in tropical America.

B. galapagana, Gottsche, Ann. Sci. Nat. ser. 4, viii. 341 (1857). Listed by Wilson in Hook. f. (3), 165, as *Jungermannia filicina*, var. *tenuis.* — Charles and James Ids.: *Darwin.* Also collected by *Andersson.* The species is endemic but is very close to the following.

B. tenuicaulis, Tayl. Syn. Hep. 285 (1845). — Abingdon Isl.: *Snodgrass & Heller.* Chatham Isl.: *Baur.* Known also from the Andes.

Frullania, Raddi.

F. aculeata, Tayl. Lond. Jour. Bot. v. 407 (1846). — Galapagos Ids.: *Darwin.* Also found in the Andes. *Jungermannia pungens*, Wils. in Hook. f. (3), 165, collected on Charles Isl. by *Darwin*, is apparently a synonym of this species.

F. atrata (Sw.) Nees, Syn. Hep. 463 (1845). — Charles Isl.: *Darwin;* 8 Apr. 1888, *Lee.* Duncan Isl.: *Baur.* Widely distributed in tropical America.

F. galapagona, Ångstr. Oefversigt Kongl. Vetensk.-Akad. Förhandl. xxx. no. 5, 116 (1873). — Galapagos Ids.: *Andersson.* Endemic.

F. tamarisci (L.) Dum. Recueil d'Obs. 13 (1835). — This widely distributed species of Europe and North America is quoted by Wilson from Charles Isl.: *Darwin.* The species would hardly be expected from the Galapagos Islands.

F. vaginata (Sw.) Dum. l. c. — Charles and James Ids.: *Darwin.* A Javan species. Its occurrence on the Galapagos Ids. is considered doubtful by Schiffner (cf. Conspect. Hepat. Arch. Ind. 348).

Lopholejeunea (Spruce) Schiffn.

L. Anderssonii, Steph. Hedwigia, xxxv. 108 (1896). — Galapagos Ids.: *Andersson.* Endemic.

MARCHESINIA, S. F. Gray.

M. BRACHIATA (Sw.) Schiffn. in Engl. & Prantl. Nat. Pflanzenf. i. Abt. 3, 128 (1893). — DUNCAN ISL.: *Baur*. Widely distributed in tropical America. *Phragmicoma galapagana*, Ångstr. l. c. 114, and *P. nigrescens*, Ångstr. l. c. 115, are very close to this extremely variable species. Both were collected by Andersson.

NOTOTHYLAS, Sull.

N. ORBICULARIS (Schwein.) Sull. Mem. Am. Acad. new ser. iii. 64, t. 4 (1846). — ALBEMARLE ISL.: Tagus Cove, *Snodgrass* & *Heller*. Widely distributed in eastern N. Am.; also European.

OMPHALANTHUS, Lindenb. & Nees.

O. FILIFORMIS (Sw.) Nees, Syn. Hep. 304 (1845). — CHARLES and JAMES IDS.: *Darwin*. Widely distributed in tropical America, especially in mountainous regions.

PELTOLEJEUNEA (Spruce) Schiffin.

P. GALAPAGONA, Steph. Hedwigia, xxxv. 123 (1896). — CHARLES ISL.: *Andersson*. Endemic.

PLAGIOCHASMA, Lehm. & Lindenb.

P. ? — ALBEMARLE ISL.: Iguana Cove, Tagus Cove, and mountain east of Tagus Cove, *Snodgrass* & *Heller*. CHARLES ISL.: *Baur*, no. 380 (hb. Harvard Univ.). NARBOROUGH ISL.: south side, 615 m. alt., *Snodgrass* & *Heller*. The specimens are all sterile and indeterminable but seem referable to a single species.

PLAGIOCHILA, Dum.

P. ANDERSSONII, Ångstr. l. c. 114; Rose (1), 138. — GALAPAGOS IDS.: *Andersson*. CHATHAM ISL.: *A. Agassiz*. Further distrib. Cocos Island of the Pacific.

P. SPINIFERA, Ångstr. l. c. — GALAPAGOS IDS.: *Andersson*. Endemic.

RADULA, Dum.

R. RETROFLEXA, Tayl. Lond. Jour. Bot. v. 378 (1846). — GALAPAGOS IDS.: *Andersson*. Also known from the "Pacific Islands," *Nightingale*.

RICCIA, Mich.

R. ? Sterile and indeterminable. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*.

MUSCI.

By W. G. FARLOW.

Calymperes, Sw.

C. Sprucei, Bescherelle, Ann. Sci. Nat. Bot. ser. 8, i. 304 (1895). — Bindloe Isl.: *Snodgrass & Heller*. Several specimens of this species were collected. There are no capsules but an abundance of septate gemmae borne on the long club-shaped prolongation of the ribs. As there appeared to be a slight difference between the cell-structure of these specimens and Spruce's no. 20, on which Bescherelle founded his *C. Sprucei*, material of the Galapagos plant was submitted to M. Bescherelle, the learned author of the Essai sur le genre Calymperes, who has been so kind as to verify the identification.

Campylopus, Brid.

C. Anderssonii, Jaeg. Adumb. i. 140 (1870). *C.* sp. Anderss. (1), 125. *Dicranum Anderssonii*, C. Muell. Bot. Zeit. xiv. 169 (1856); Anderss. (2), 37. — Charles Isl.: *Andersson*. Endemic. To judge from the description this must be quite different from the following species.

C. lamellatus, Mont. Ann. Sci. Nat. Bot. ser. 2, ix. 52 (1838) *Dicranum lamellatum*, C. Muell. Syn. Musc. i. 411 (1849). — Albemarle Isl: mountain east of Tagus Cove, alt. 770 m., 1 Feb., and June, 1899, *Snodgrass & Heller*. Further distrib. Bolivia. To this species may be referred a moss collected in small quantity with a marked polytrichoid habit suggesting *C. polytrichoides*, De Not.; Renauld & Cardot, Musci. Eur. no. 114, and *C. leucotrichus*, Sull. & Lesq. Musc. Am. Bor. no. 73 (1856). The stems are from 1½ to 4 cm. high, nearly simple, but in a few cases with lateral innovations just below the tips, which are gemmiferous but without traces of antheridia or archegonia. The erect infolded leaves terminate in long hyaline papilliferous hairs. The ribs which occupy the greater part of the leaves have a large number, about 30, of dorsal laminae composed of three or four cells seen in section, the terminal cell being obtuse. In well-developed leaves there is in the costal region a single internal layer of squarish colorless cells, 16–18 μ by 14 μ in section, but in older leaves there is developed inside these colorless cells a layer of very narrow small cells. The union by Mitten, Musci. Austro-Americani, of *C. lamellatus*, *C. polytrichoides*, *C. leucotrichus* and a number of other species into a single species (*C. introflexus* (Hedw.) Mitt.) is incorrect so far as *C. introflexus*, at least, is concerned, as was shown by Duby and C. Mueller, but probably Mitten was justified in uniting some of the species referred to. I am unable to ascertain whether *C. lamellatus* or *C. polytrichoides* was first published since both species were described in 1838. The present species may be referred to *C. lamellatus*, Mont., to which it is at least very closely related, but the scanty and sterile material prevents an accurate determination. The specimens in their leaf structure agree well with the *C. lamellatus* of the Wilkes Expedition in herb. Sullivant.

DALTONIA, Hook. & Tayl.

D. ROBUSTA, Ångstr. Oefv. 1873, no. 5, p. 117. — GALAPAGOS IDS.: *Andersson*. Endemic.

MACROMITRIUM, Brid.

M. SCABRISETUM, Wils. in Hook. f. (3), 165; Anderss. (1), 125, & (2), 37. Reduced by Mitten, Jour. Linn. Soc. xii. 210, to *M. longifolium*, Brid. Bryol. Univ. i. 309, 738 (1826), which is *Orthotrichum longifolium*, Hook. Musc. Exot. t. 44 (1818). — ABINGDON ISL.: alt. 250 m., 25 June, 1899, *Snodgrass & Heller*. CHARLES ISL.: *Darwin*. The material secured by Messrs. Snodgrass & Heller consists of a single small specimen with lower leaf cells papillose and rather transparent. The specimen was sterile and the absence of the seta made accurate determination impossible, and furthermore the leaves themselves were not in very good condition, being old and more or less torn.

PAPILLARIA, C. Muell.

P. NIGRESCENS, Jaeg. Adumb. vii. 265 (1876). *Neckera nigrescens*, Schw. Suppl. 3, i. t. 244 (1828); *Anderss.* (1), 125, & (2), 37. — CHARLES ISL.: *Andersson*. ? JAMES ISL.: *Darwin* ("Neckera vel Pilotrichum sp.?" Hook. f. (3), 165). Neither seen by the writer. Further distrib. general in North and South America.

PILOTRICHELLA, C. Muell.

P. ANDERSSONII, Jaeg. Adumb. vii. 262 (1876). *Neckera Anderssonii*, C. Muell. Bot. Zeit. xiv. 170 (1856); Anderss. (2), 37. — CHARLES ISL.: *Andersson*. Endemic. Not seen by the writer.

P. NIGRICANS, Besch. Prodr. Bry. Mex. 79. *Hypnum nigrescens*, Hook. in Kunth, Syn. Pl. Equin. i. 64 (1822). — GALAPAGOS IDS.:

Andersson, vid. Ångström in Oefv. k. Vet. Akad. Förh. xxx. 118 (1873).

SCHLOTHEIMIA, Brid.

S. JAMESONI, Brid. Bryol. Univ. i. 742 (1826). *Orthotrichum Jamesoni*, Arn. Trans. Wern. Soc. v. 201 (1824). — ALBEMARLE ISL.: Iguana Cove, 30 Dec. 1898, *Snodgrass* & *Heller*. Further distrib. Brazil. The specimens referred to this species were sterile. The leaf structure agrees so well with that of *S. Jamesoni* in herb. Taylor that the name may be retained here. The relations of *S. Jamesoni* to *S. nitida*, Schwaeg., are not well defined and in herb. Sullivant the *S. Jamesoni* of the Wilkes Expedition was later considered to be *S. nitida*.

FILICES.

ACROSTICHUM, L.

A. AUREO-NITENS, Hook. Ic. Pl. x. t. 933 (1854), & Sp. Fil. v. 270 (1864); Anderss. (1), 131, & (2), 42; Hook. & Bak. Syn. Fil. 421 (1868). *Hemionitis pinnata*, Hook. f. (3), 167, excellently described, yet neglected by most subsequent authors; Andersson (1), 129, & (2), 40. *Neurocallis aureo-nitens*, Moore, Ind. Fil. 5 (1857). *Chrysodium aureo-nitens*, Christ, Farnkr. 47 (1897). *Elaphoglossum aureo-nitens*, Diels in Engl. & Prantl, Nat. Pflanzenf. i. Abt. 4, 334, f. 174 H. (1899). — GALAPAGOS IDS.: *Cuming*, no. 109. ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 17 (hb. Gr.); on mountain east of Tagus Cove, alt. 920 m., *Snodgrass* & *Heller*, no. 229 (hb. Gr.). CHARLES ISL.: *Darwin*; *Andersson*; *Lee* (hb. U. S. Nat. Mus.); *Baur*, no. 356 (hb. Gr.). CHATHAM ISL.: *Capt. Wood*. NARBOROUGH ISL.: southern part, *Snodgrass* & *Heller*, no. 319 (hb. Gr.). Further distrib. Ecuador, acc. to Diels. l. c. A fern of remarkable and characteristic habit, the sterile fronds spatulate and entire, the fertile pinnate.

A. AUREUM, L. Sp. 1069 (1753); Schk. Fil. t. 1; Hook. Sp. Fil. v. 266. *Chrysodium aureum*, Mett. Fil. Lips. 21 (1856). — GALAPAGOS IDS.: *Capt. Wood*, acc. to Hook. l. c. Further distrib. general in trop. and subtrop. reg.

A. MUSCOSUM, Sw. Fl. Ind. Occ. 1591 (1806), & Syn. Fil. 10; Hook. Sp. Fil. v. 231. *A. Langsdorfii*, Hook. & Grev. Ic. t. 164 (1831). *Elaphoglossum muscosum*, Moore, Ind. Fil. 12, 362 (1862). *Olfersia muscosa*, Presl, Tent. Pterid. 233 (1836). *O. Langsdorfii*, Presl, l. c. 234; Hook. f. (3), 167; Anderss. (1), 128, & (2), 39. —

JAMES ISL.: *Darwin*, acc. to Hook. f., l. c. Not secured by subsequent collectors in the Galapagos Ids. Further distrib. Mex. (acc. to Hook. & Bak.), W. Ind. to Peru and Brazil.

A. VISCOSUM, Sw. Syn. Fil. 10, 193 (1806); Hook. & Grev. Ic. t. 64; Hook. Sp. Fil. v. 220. *Elaphoglossum viscosum*, Schott acc. to Moore, l. c. 369. *Olfersia viscosa*, Presl, l. c. *O. viscida*, Presl acc. to Hook. f. (3), 167; Anderss. (1), 129, & (2), 39.—JAMES ISL.: *Darwin*, acc. to Hook. f., l. c. Not since collected on the Galapagos Ids. Further distrib. Mex., W. Ind., S. Am., E. Ind., etc.

ADIANTUM, L.

A. AETHIOPICUM, L. Sp. ed. 2, 1560 (1763); Hook. Sp. Fil. ii. 37, t. 77 A; Moore, Ind. Fil. 19.—GALAPAGOS IDS.: acc. to Moore, l. c. Further distrib. general in trop.

A. CONCINNUM, Humb. & Bonpl. in Willd. Sp. v. 451 (1810); HBK. Nov. Gen. & Sp. i. 20, t. 668 (1815); Hook. Sp. Fil. ii. 42. *A cuneatum*, Hook. f. (3), 168; Anderss. (1), 129, & (2), 40; not Langsd. & Fisch.—ABINGDON ISL.: alt. 520 m., *Snodgrass* & *Heller*, no. 818 (hb. Gr.). ALBEMARLE ISL.: north mountain, Elizabeth Cove, *Snodgrass* & *Heller*, no. 282 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 15 (hb. Gr.); Tagus Cove, abundant from the sea-level to 950 m. alt., *Snodgrass* & *Heller*, no. 182 (hb. Gr.). JAMES ISL.: *Scouler*; *Darwin*; common on lava rocks, *Snodgrass* & *Heller*, no. 361 (hb. Gr.). NARBOROUGH ISL.: southern part, abundant in shade, alt. 310 to 620 m., *Snodgrass* & *Heller*, no. 320 (hb. Gr.). The last number has broader, flatter sori and its identity with the others is somewhat doubtful. Further distrib. Mex., W. Ind. to Peru and Brazil.

A. HENSLOVIANUM, Hook. f. (3), 169; Anderss. (1), 129, & (2), 40; Hook. & Bak. Syn. Fil. 118; Rob. & Greenm. (1), 149. *A. sessilifolium*, Hook. Sp. Fil. ii. 44, t. 85.—CHARLES ISL.: *Darwin*. CHATHAM ISL.: southwest end, middle region, *Baur*, no. 361 (hb. Gr.). JAMES ISL.: *Darwin*. Further distrib. Andean S. Am.

A. INCISUM, Presl, Rel. Haenk. i. 61, t. 10, f. 3 (1830), & Tent. Pterid. 157; Hook. Syn. Fil. ii. 28; Moore, Ind. Fil. 28.—GALAPAGOS IDS.: acc. to Moore, l. c., and Christ, Farnkr. 138. Further distrib. Pacific slope of northern S. Am. and S. Mex.

A. KAULFUSSII, Kunze, Linnaea, xxi. 221 (1848); Moore, Ind. Fil. 29.—"CHATHAM ISL." [presumably of the Galapagos group] acc. to Moore, l. c. Further distrib. northern S. Am., Mex., W. Ind.

A. PARVULUM, Hook. f. (3), 168; Anderss. (1), 129, & (2), 40; Hook. Sp. Fil. ii. 44. — CHARLES ISL.: *Darwin.* A species not secured by any subsequent collector, and omitted by Hook. & Bak. from their Syn. Fil. Endemic so far as known.

A. PATENS, Willd. Sp. v. 439 (1810); Hook. Sp. Fil. ii. 29, t. 87 A; Hook. & Bak. Syn. Fil. 126. — GALAPAGOS IDS.: acc. to Moore, l. c., and Hook. & Bak. l. c. Further distrib. Mex. to Ecuador.

A. PRIONOPHYLLUM, HBK. Nov. Gen. & Sp. i. 20 (1815); Hook. Sp. Fil. ii. 22; Moore, Ind. Fil. 34. — CHATHAM ISL.: acc. to Moore. Further distrib. Mex., W. Ind., northern S. Am.

ASPIDIUM, Sw.

A. CORIACEUM, Sw. Syn. Fil. 57 (1806); Willd. Sp. Pl. v. 268; Hook. Sp. Fil. iv. 32; Schk. Fil. 50, t. 50. *A. capense*, Willd. l. c. 267 (1810); Hook. & Bak. Syn. Fil. 254. *Polypodium coriaceum*, Sw. Prodr. 133 (1788). *Polystichum coriaceum*, Schott, Gen. Fil. t. 9 (1834); Presl, Tent. Pterid. 84; Hook. f. (3), 171; Anderss. (1), 131, & (2), 41. — JAMES ISL.: *Darwin*, acc. to Hook. f., l. c. Apparently not since collected on the islands. Further distrib. W. Ind., S. Am., Trop. of Old World.

A. SEMICORDATUM, Sw. Syn. Fil. 45 (1806); Hook. Sp. Fil. iv. 16, which see for synon. — GALAPAGOS IDS.: *Capt. Wood*, acc. to Hook. l. c. Further distrib. W. Ind. to Peru and Brazil. Varieties also in the Old World.

ASPLENIUM, L.

A. ANISOPHYLLUM, var. LATIFOLIUM, Hook. Sp. Fil. iii. 111 (1860). *A. nigrescens*, Hook. f. (1), 170; Anderss. (1), 131, & (2), 41; not Bl. *A. nubilum*, Moore, Ind. Fil. 150 (1859). — GALAPAGOS IDS.: *Capt. Wood.* JAMES ISL.: *Darwin.* Further distrib. trop. Am. and Afr.

A. AURITUM, Sw. Fl. Ind. Occ. 1616 (1806); Hook. Sp. Fil. iii. 178; Rob. & Greenm. (1), 149. *A. marinum*, var. *auriculatum*, Hook. f. (3), 170; Anderss. (1), 131, & (2), 41. — ABINGDON ISL.: *Snodgrass & Heller*, no. 835 (hb. Gr.). CHARLES ISL.: *Darwin;* in the upper shaded region, *Andersson; A. Agassiz.* CHATHAM ISL.: *Capt. Wood*, acc. to Hook. l. c.; on the southwest end, middle region, *Baur*, no. 354 (hb. Gr.). JAMES ISL.: *Darwin.* Further distrib. Mex., W. Ind., S. Am. Capt. Wood's plant stated by Hook. to be partially bipinnate, is probably the one which Moore (Ind. Fil. 170) doubtfully refers to the Galapagos Ids. under the name of *A. sulcatum*, Lam.

Var. MACILENTUM, Moore, Ind. Fil. 115 (1859); Hook. Sp. Fil. iii. 179. — Galapagos Ids.: acc. to Moore, l. c. Further distrib. Mex., W. Ind., trop. S. Am.

A. CICUTARIUM, Sw. Prodr. 130 (excl. syn.); Hook. Sp. Fil. iii. 198; Lowe, Nat. Hist. Ferns, v. t. 20; Rob. & Greenm. (1), 149. — Galapagos Ids.: *Capt. Wood,* acc. to Hook. l. c. Chatham Isl.: acc. to Moore, Ind. Fil. 119; southwest end, middle region, *Baur,* no. 359 (hb. Gr.). Further distrib. Mex., W. Ind., S. Am., Afr.

A. FORMOSUM, Willd. Sp. v. 329 (1810); Hook. Sp. Fil. iii. 143, & Fil. Exot. t. 16. *A. subalatum,* Hook. & Arn. Bot. Beech. 312, t. 71. *A. subulatum,* Hook. f. (3), 169; Anderss. (1), 130, & (2), 41. *A. farinosum,* Rob. & Greenm. (1), 149, by typographical error. — Abingdon Isl.: alt. 540 m., *Snodgrass* & *Heller,* no. 819 (hb. Gr.). Albemarle Isl.: Iguana Cove, *Snodgrass* & *Heller,* no. 27 (hb. Gr.); on mountain east of Tagus Cove, *Snodgrass* & *Heller,* no. 240 (hb. Gr.). Charles Isl.: *Darwin; Lee* (hb. U. S. Nat. Mus.); *Baur,* no. 366 (hb. Gr.). Chatham Isl.: *Capt. Wood; Cuming,* no. 108, acc. to Hook. Sp. Fil. iii. 143. Further distrib. general in trop. of both hemispheres.

A. FURCATUM, Thunb. Prodr. Fl. Cap. 172 (1800); Hook. Sp. Fil. iii. 165; Hook. f. (3), 169; Anderss. (1), 130, & (2), 41. *A. praemorsum,* Sw. Fl. Ind. Occ. iii. 1620 (1806); Moore, Ind. Fil. 155. — Galapagos Ids.: *Capt. Wood,* acc. to Hook. l. c. Albemarle Isl.: on mountain east of Tagus Cove, *Snodgrass* & *Heller,* no. 239 (hb. Gr.). James Isl.: *Darwin.* Further distrib. general in trop. of both hemispheres.

A. LAETUM, Sw. Syn. Fil. 79, 271 (1806); Hook. Sp. Fil. iii. 133. *A. abscissum,* Willd. Sp. v. 321 (1810), in part. — Chatham Isl.: *Capt. Wood,* acc. to Hook. l. c. Further distrib. general in trop. Am.

A. LUNULATUM, Sw. Syn. Fil. 80 (1806); Hook. & Bak. Syn. Fil. 202. *A. erectum,* Bory in Willd. Sp. v. 328 (1810); Hook. Sp. Fil. iii. 126. — Charles Isl.: *Lee* (hb. U. S. Nat. Mus.). Widely distrib. in trop. reg.

A. RHIZOPHYLLUM, Kunze, Linnaea, ix. 71; Hook. Sp. Fil. iii. 200; not L. *A. Macraei,* Hook. & Grev. Ic. 217 (acc. to Hook. l. c.), and hence (if rightly identified) Hook. f. (3), 169, and Anderss. (1), 131, & (2), 41. ? *A. myriophyllum,* var. *divaricatum,* Moore, Ind. Fil. 148, as to pl. Galapagos. *Darea rhizophylla* & *Caenopteris rhizophylla,* Hook. & Grev. Ic. t. 193. — Galapagos Ids.: *Capt. Wood,* acc. to Hook. l. c. James Isl.: *Darwin,* acc. to Hook. f., l. c. Not secured on the

Islands by recent collectors. Further distrib. general in trop. of both hemispheres.

A. RUTACEUM, Metten. Asplen. 129, t. 5, f. 32–33 (1859); Hook. Sp. Fil. iii. 203; Hook. & Bak. Syn. Fil. 220. — GALAPAGOS IDS.: acc. to Hook. & Bak. l. c., where no collector is mentioned. Further distrib. trop. S. Am.

A. SERRA, Langsd. & Fisch. Fil. 16, t. 16 (1810–1818); Hook. Sp. Fil. iii. 154; Moore, Ind. Fil. 166. — GALAPAGOS IDS.: acc. to Moore, l. c. Further distrib. trop. Am.

A. SERRATUM, L. Sp. 1079 (1753); Hook. Sp. Fil. iii. 81; Moore, Ind. Fil. 167. — GALAPAGOS IDS.: *Capt. Wood*, acc. to Hook. l. c. CHATHAM ISL.: acc. to Moore. Further distrib. general in trop. Am.

BLECHNUM, L.

B. OCCIDENTALE, L. Sp. ed. 2, 1534 (1763); Sw. Syn. Fil. 113; Jacq. Ic. Pl. Rar. iii. t. 644; Hook. f. (3), 169; Anderss. (1), 130, & (2), 41; Caruel (1), 625. *B. orientale*, L. Sp. 1077 (1753), through clerical or typographical transposition of names with the Asiatic species, an error corrected by Linnæus himself. — GALAPAGOS IDS.: acc. to Moore, Ind. Fil. 200. ABINGDON ISL.: *Snodgrass* & *Heller*, no. 815 (hb. Gr.). ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 19 (hb. Gr.), 134 (hb. Gr.); mountains east of Tagus Cove, alt. 920 m., *Snodgrass* & *Heller*, no. 247 (hb. Gr.). CHARLES ISL.: in the upper shaded regions, *Andersson*. CHATHAM ISL.: *Chierchia; Baur*, no. 360 (hb. Gr.). JAMES ISL.: *Darwin*. Further distrib. Mex. to Chili, Brazil, and W. Ind.

Var. CAUDATUM, Hook. Sp. Fil. iii. 51 (1860). — GALAPAGOS IDS.: *Capt. Wood*, acc. Hook. l. c., and Moore, l. c. Further distrib. general in trop. Am. and Philippines.

CHEILANTHES, Sw.

C. MICROPHYLLA, Sw. Syn. Fil. 127 (1806); Hook. Sp. Fil. ii. 84, t. 98 A; Hook. & Bak. Syn. Fil. 135; Rob. & Greenm. (1), 149. *C. heterotrichus*, Anderss. (1), 129, & (2), 40, to judge from character. — GALAPAGOS IDS.: acc. to Moore, Ind. Fil. 247. — ALBEMARLE ISL.: on hillsides near Iguana Cove, *Snodgrass* & *Heller*, nos. 20 (hb. Gr.), 114 (hb. Gr.). CHARLES ISL.: in shade near summit of mountain, *Andersson* (*C. heterotrichus*). CHATHAM ISL.: southwest end, middle region, *Baur*, no. 369 (hb. Gr.). Further distrib., Mex., Andes of Peru, and Bolivia.

C. MYRIOPHYLLA, Desv. Berl. Mag. v. 328, & Jour. Bot. ii. 44, t. 13, f. 1 (1813); Hook. Sp. Fil. ii. 100. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 12 (hb. Gr.). Further distrib. Mex., Andes, Ecuador to Peru.

CYSTOPTERIS, Bernh.

C. FRAGILIS, Bernh. in Schrad. Neues Jour. Bot. i. pt. 2, 26, 49, t. 2, f. 9 (1806); Wolf (1), 283. — CHARLES ISL.: acc. to Wolf. Of wide distribution. I can but question the identity of the Galapageian plant. The species has been secured on the islands by no other collector, and the specimen in question received only a field determination.

GLEICHENIA, Sm.

G. DICHOTOMA, Hook. Sp. Fil. i. 12 (1846); Moore, Ind. Fil. 375. — GALAPAGOS IDS.: acc. to Moore, l. c. Further distrib. general in trop.

GYMNOGRAMME, Desv.

G. CHAEROPHYLLA, Desv. Jour. Bot. i. 26 (1813); Hook. Sp. Fil. v. 136; Hook. & Grev. Ic. Fil. t. 45; Hook. f. (3), 168 (*Gymnogramma*); Anderss. (2), 40. — CHARLES ISL.: *Darwin*. It is not unlikely that a reëxamination will show this specimen to be *G. leptophylla*, Desv., since discovered upon the same island by Baur. Further distrib. Mex., W. Ind., Brazil, Paraguay.

G. LEPTOPHYLLA, Desv. Jour. Bot. i. 26 (1813); Hook. Sp. Fil. v. 136; Hook. & Grev. Ic. Fil. t. 25; Rob. & Greenm. (1), 149. — CHARLES ISL., *Baur*, no. 368 (hb. Gr.). Further distrib. trop. of both hemispheres.

G. TARTAREA, Desv. Berl. Mag. v. 305; Hook. Sp. Fil. v. 148; Caruel (1), 625. *G. dealbata*, Link, Hort. Berol. ii. 52 (1833). *G. calomelanos*, Rob. & Greenm. (1), 149, not Kaulf. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 25 (hb. Gr.); on mountain east of Tagus Cove, *Snodgrass* & *Heller*, no. 241 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 782 (hb. Gr.), the yellow-powdered form, ? no. 778 (hb. Gr.), a juvenile state. CHATHAM ISL., *Chierchia*; southwest end, upper region, *Baur*, no. 362 (hb. Gr.). Further distrib. Mex., W. Ind., S. Am., Tropics of the Old World.

HYPOLEPIS, Bernh.

H. REPENS, Presl, Tent. Pterid. 162 (1836); Hook. Sp. Fil. ii. 64, t. 90 B. *Lonchitis repens*, L. Sp. 1078 (1753). *Cheilanthes repens*, Kaulf. Enum. Fil. 215 (1824). — GALAPAGOS IDS.: *Capt. J. Wood*, acc. to Hook. l. c. Further distrib. Mex., W. Ind., trop. S. Am.

NEPHRODIUM, Rich.

N. BRACHYODON, Hook. Sp. Fil. iv. 83 (1862); Hook. & Bak. Syn. Fil. 295. — GALAPAGOS IDS.: *Capt. Wood.* Further distrib. W. Ind., S. Am., Malayan Penin. & Isles.

N. MACROPHYLLUM, Bak. Syn. Fil. 300. *Aspidium macrophyllum*, Sw. Syn. Fil., 43, 239 (1806); Hook. Sp. Fil. iv. 55. — GALAPAGOS IDS.: *Capt. Wood*, acc. to Hook., l. c. Further distrib. general in trop. Am.

N. MOLLE, Desv. Mém. Soc. Linn. vi. 258 (1827); Schott, Gen. Fil. t. 10 (1834); Hook. f. (3), 171; Anderss. (1), 131, & (2), 41; Hook. & Bak. Syn. Fil. 293. *Aspidium molle*, Sw. Syn. Fil. 49 (1806); Rob. & Greenm. (1), 149. — ALBEMARLE ISL.: common at 460 m., above Iguana Cove, *Snodgrass* & *Heller*, nos. 18 (hb. Gr.), 115 (hb. Gr.). CHARLES ISL.: *Lee* (hb. U. S. Nat. Mus.); *Baur.* JAMES ISL.: *Darwin.* Further distrib. Mex., W. Ind., S. Am., Tropics of Old World.

N. UNITUM, R. Br. Prodr. Fl. Nov. Holl. 148 (1810); Hook. & Bak. Syn. Fil. 289; Caruel (1), 625. — CHATHAM ISL.: *Chierchia.* Further distrib. tropics of both hemispheres.

N. VILLOSUM, Presl, Rel. Haenk. i. 38 (1830); Hook. Sp. Fil. iv. 134. *Polypodium villosum*, L. Sp. 1093 (1753). *P. Berteroanum*, Hook. Sp. Fil. iv. 269, not Spreng. *Aspidium villosum*, Sw. Syn. Fil. 56 (1803). — CHATHAM ISL.: *Capt. Wood*, acc. to Hook. l. c. Further distrib. W. Ind. to Chili.

NEPHROLEPIS, Schott.

N. ACUTA, Presl, Tent. Pterid. 79 (1836); Hook. Sp. Fil. iv. 153. *Aspidium acutum*, Schk. Fil. 32, t. 31. — BINDLOE ISL.: *Snodgrass* & *Heller*, no. 779 (hb. Gr.). Although the specimens are sterile the somewhat characteristic venation and contour of the pinnae leave no doubt of the identity of this species, which appears not to have been hitherto recorded from the islands. Further distrib. Mex., W. Ind., S. Am., Tropics of Old World.

N. PECTINATA, Schott, Gen. Fil. under t. 3 (1834). *N. cordifolia* β, Hook. & Bak. Syn. Fil. 300 (1868). *N. cordifolia*, Rob. & Greenm. (1), 149. *Aspidium pectinatum*, Willd. Sp. Pl. 223 (1810). *Nephrodium pectinatum*, Hook. f. (3), 170; Anderss. (1), 131, & (2), 41.

— CHATHAM ISL.: southwest end, upper region, *Baur*, no. 357 (hb. Gr.). JAMES ISL.: *Darwin*. This smooth-stiped fibrous-rooted fern, with short blunt oblong pinnae, cannot be very satisfactorily placed (as by Hook. & Bak. l. c.) in *N. cordifolia*, Presl, which has tuberous roots, scaly stipes, and triangular often acutish pinnae. Further distrib. Panama, W. Ind., S. Am.

NOTHOCHLEANA, R. Br.

N. SULPHUREA, J. Sm. in Seem. Bot. Herald, 233 (1852–57); Hook. & Bak. Syn. Fil. 373. *N. candida*, Hook. Sp. Fil. v. 110 (1864). *N. argentea*, Lowe, Ferns Brit. and Exot. i. t. 55 (1856). *Pteris lutea*, Hook. f. (3), 168; Anderss. (1), 129, & (2), 40; not Cav. — GALAPAGOS IDS.: *Cuming*, no. 110, acc. to Hook. l. c. ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 23 (hb. Gr.); north mountain, Elizabeth Bay, *Snodgrass* & *Heller*, no. 287 (hb. Gr.); mountain east of Tagus Cove, altitude 925 m., *Snodgrass* & *Heller*, no. 246 (hb. Gr.). JAMES ISL.: *Scouler*. Further distrib. S. W. United States, Mex., Andean S. Am.

PELLAEA, Link.

P. GERANIAEFOLIA, FÉE, Gen. Fil. 130 (1850–1852); Hook. Sp. Fil. ii. 132; Hook. & Bak. Syn. Fil. 146. *Pteris geraniifolia*, Hook. Ic. Pl. x. t. 915 (1854). — GALAPAGOS IDS.: *Douglas*, acc. to Hook. Sp. Fil. ii. 133. Further distrib. general in trop. reg. Strikingly like *Pteris pedata*, L., but with free veinlets.

POLYPODIUM, L.

P. ANGUSTIFOLIUM, Sw. Syn. Fil. 27 (1806); Raddi, Fil. Bras. 14, t. 24, f. 2. *P. ensifolium*, Willd. Sp. Pl. v. 152 (1810). *Marginaria ensifolia*, Presl, Tent. Pterid. 188 (1836); Hook. f. (3), 167; Anderss. (1), 127, & (2), 39. — JAMES ISL.: *Darwin*, acc. to Hook. f., l. c., an occurrence apparently doubted by Andersson and not subsequently confirmed by more recent collectors. Further distrib. Mex., W. Ind., S. Am.

P. AUREUM, L. Sp. 1087 (1753); Lowe, Ferns Brit. and Exot. ii. t. 56; Hook. Sp. Fil. v. 16. *Pleopeltis aurea*, Presl, Tent. Pterid. 193 (1836); Hook. f. (3), 167; Anderss. (1), 128, & (2), 39. — GALAPAGOS IDS.: *Capt. Wood*, acc. to Hook. l. c. JAMES ISL.: *Darwin*, acc. to Hook. f. (3), 167. This large and conspicuous species, secured by these early collectors on the Galapagos Ids., has curiously eluded the subsequent and much more thorough collectors who have visited the archipelago. Further distrib. Southern U. S., Mex., W. Ind., S. Am., "Australia."

P. CRASSIFOLIUM, L. Sp. 1083 (1753); Hook. Sp. Fil. v. 62; Hemsl. Biol. Cent.-Am. Bot. iii. 656; N. E. Brown, Trans. Linn. Soc. ser. 2, vi. 85. *P. coriaceum*, Raddi, Fil. Bras. 16, t. 25. *Pleuridium crassifolium*, Fée, Gen. Fil. 273. — Galapagos Ids.: *Capt. Wood* (hb. Kew). Further distrib. Mex., W. Ind. to Peru and Brazil.

P. INCANUM, Sw. Prodr. 131 (1788); Hook. Sp. Fil. iv. 208. *Acrostichum polypodioides*, L. Sp. 1068 (1753). *Marginaria incana*, Presl, Tent. Pterid. 188 (1836); Hook. f. (3), 166; Anderss. (1), 127, & (2), 38 (where *P. incisum*, Sw., is incorrectly given as a synonym, instead of the correct *P. incanum*, Sw.). — Galapagos Ids.: *Capt. Wood*, acc. to Hook. l. c. Albemarle Isl.: *Macrae*. Charles Isl.: *Darwin*. Further distrib. U. S., Mex., W. Ind., S. Am., Trop. Afr.

P. LANCEOLATUM, L. Sp. 1082 (1753). *P. macrocarpum*, Willd. Sp. Pl. v. 147 (1810). *P. lepidotum*, Willd. in Schlecht. Adumbr. 17, t. 8 (1825). *Pleopeltis ensifolia*, Hook. Ex. Fl. t. 62 (1823). *P. macrocarpa*, Kaulf. Enum. Fil. 245 (1824); Hook. f. (3), 167; Anderss. (1), 127, & (2), 39. *P. lanceolata*, Presl, Tent. Pterid. 193 (1836); Anderss. (1), 128, & (2), 39. *P. lepidota*, Hook. f. (3), 167; Anderss. (1), 126, & (2), 39. *Lepicystis lanceolata*, Diels in Engl. & Prantl, Nat. Pflanzenf. i. Abt. 4, 323 (1899). — Albemarle Isl.: Iguana Cove, on trees and bushes, alt. 310 m., *Snodgrass* & *Heller*, no. 127 (hb. Gr.). Charles Isl.: *Darwin*; in upper shaded region, *Andersson*. James Isl.: *Darwin*. Further distrib. trop. of both hemispheres.

P. LEPIDOPTERIS, Kunze, Linnaea, xiii. 132 (1839); Hook. Sp. Fil. iv. 211; Hook. & Bak. Syn. Fil. 346. *Acrostichum lepidopteris*, Langsd. & Fisch. Fil. i. 5, t. 2 (1810). — Galapagos Ids.: *Capt. Wood*, acc. to Hook., l. c. Further distrib. general in trop. Am.

P. LORICEUM, L. Sp. 1086 (1753); Hook. Sp. Fil. v. 21. *P. punctulatum*, Hook. Ic. Pl. t. 720 (1848). *Goniophlebium loriceum*, J. Sm. in Hook. Gen. Fil. under t. 51 (1838); Moore, Ind. Fil. 392. — Galapagos Ids.: acc. to Moore, l. c. Further distrib. general in trop. Am.

P. PALEACEUM, Hook. f. (3), 166; ? Anderss. (1), 126, & (2), 38; ? Hook. Sp. Fil. iv. 261. ? *Alsophila* sp., Rob. & Greenm. (1), 149. — Charles Isl.: *Darwin*. James Isl.: *Darwin*. Endemic. To this species I would doubtfully refer specimens from Albemarle Isl., Iguana Cove, *Snodgrass* & *Heller*, no. 113 (hb. Gr.); Charles Isl., *Andersson*, *Lee* (hb. U. S. Nat. Mus.); and Chatham Isl., southwest end, middle region, *Baur*, no. 367 (hb. Gr.). These plants, none of which is in quite satisfactory condition, represent one, or perhaps several, very nearly related species close to *P. punctatum*, Thunb., and *P. prasinum*, Baker.

P. PECTINATUM, L. Sp. 1085 (1753); Hook. Sp. Fil. iv. 203; Caruel (1), 625. *P. paradisiae*, Langsd. & Fisch. Ic. Fil. 11, t. 11 (1810–1818); Hook. f. (3), 165; Andersson (1), 126, & (2), 38; Rob. & Greenm. (1), 149. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass & Heller*, nos. 14 (hb. Gr.), 133 (hb. Gr.). CHARLES ISL.: *Andersson*. CHATHAM ISL.: *Chierchia;* southwest end, middle region, *Baur*, no. 358 (hb. Gr.). JAMES ISL.: *Darwin*. Further distrib. Mex., W. Ind., S. Am.

P. PERCUSSUM, Cav. Prael. 243 (1801); Sw. Syn. Fil. 26; Hook. Fil. Exot. t. 59, & Sp. Fil. v. 55. *P. cuspidatum*, Presl, Rel. Haenk. 20, t. 1, f. 3 (1825). *Lepicystis percussa*, Diels in Engl. & Prantl, Nat. Pflanzenf. i. Abt. 4, 323 (1899). — GALAPAGOS IDS.: *Capt. Wood*, acc. to Hook. Fil. Exot. l. c. This fern does not appear to have been secured by any recent collector in the Galapagos Islands. Further distrib. trop. S. Am.

P. PHYLLITIDIS, L. Sp. 1083 (1753); Hook. Sp. Fil. v. 38. *Campyloneurum Phyllitidis*, Presl, Tent. Pterid. 190 (1836); Hook. f. (3), 167; Anderss. (1), 128, & (2), 39. — JAMES ISL.: *Darwin*, acc. to Hook. f. (3), 167. Not secured by more recent collectors. Further distrib. trop. Am.

P. PLEIOSORUM, Hook. f. (3), 166 (as *pleiosoros*); Anderss. (1), 126, & (2), 38 (as *Pleiosoros*). — JAMES ISL.: *Darwin*. Endemic. This species is briefly referred to by Hook. Sp. Fil. iv. 262, where treated as an unrecognizable and therefore negligible synonym. Nevertheless the original characterization is sufficiently detailed to show that the plant was distinct from any other Galapageian species unless it be *P. paleaceum*, Hook. f. The type will probably be found in the Darwin collection at the herbarium of Cambridge University.

P. RUDE, Kunze, Linnaea, xiii. 133 (1839); Hook. Sp. Fil. iv. 243. — GALAPAGOS IDS.: *Capt. Wood* ? acc. to Hook. l. c. Further distrib. Mex. to Peru and Brazil.

P. SQUAMATUM, L. Sp. 1086 (1753); Hook. Sp. Fil. iv. 209; Rob. & Greenm. (1), 149. *P. tridens*, Kunze, Farnkr. i. 23, t. 13, f. 1 (1840). *Pleopeltis* sp., Anderss. (1), 128, & (2), 39. *Lepicystis squamata*, J. Sm. acc. to Diels in Engl. & Prantl, Nat. Pflanzenf. i. Abt.

4, 323 (1899). — GALAPAGOS IDS.: *Cuming*, no. 112; *Habel*, acc. to Hemsl. in litt.* ABINGDON ISL.: *Snodgrass* & *Heller*, no. 814 (hb. Gr.). ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 13 (hb. Gr.); mountain east of Tagus Cove, alt. 770 m., *Snodgrass* & *Heller*, nos. 238 (hb. Gr.), 900 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 783 (hb. Gr.). CHARLES ISL.: in the upper region, *Andersson*; *Lee* (hb. U. S. Nat. Mus.); *Baur*, no. 373 (hb. Gr.). CHATHAM ISL.: *Capt. Wood*; *Snodgrass* & *Heller*, no. 499 (hb. Gr.); southwestern end, upper region, *Baur*, no. 370 (hb. Gr.). DUNCAN ISL.: *Baur*, no. 372 (hb. Gr.); *Snodgrass* & *Heller*, no. 705 (hb. Gr.). NARBOROUGH ISL.: southern part, *Snodgrass* & *Heller*, no. 324 (hb. Gr.). Further distrib. Peru to Mex., and W. Ind.

P. sp. — WENMAN ISL.: acc. to ms. notes of Mr. Heller an undetermined species of *Polypodium* was observed in a cave. The matter is of interest since it is the first case in which a fern has been reported from any of the lower islands.

PTERIS, L.

P. AQUILINA, var. ESCULENTA, Hook. f. Fl. Nov. Zeal. ii. 25 (1855); Hook. Sp. Fil. ii. 197. *P. esculenta*, Forst. Prodr. 79 (1786); Schkuhr, Fil. t. 97. *P. aquilina*, var. *caudata*, Rob. & Greenm. (1), 149, not Hook. — GALAPAGOS IDS.: *Capt. Wood*, acc. to Hook. ALBEMARLE ISL.: 320 m. above Iguana Cove, *Snodgrass* & *Heller*, nos. 866, 867 (both in hb. Gr.). CHATHAM ISL.: southwest end, upper region, *Baur*, no. 363 (hb. Gr.). Generally distrib. in trop.

P. INCISA, Thunb. Fl. Cap. 733 (1823); Hook. Sp. Fil. ii. 230. *P. pallida*, Raddi, Fil. Bras. 49, t. 71 (1825). — GALAPAGOS IDS.: *Capt. Wood*, acc. to Hook. l. c.

P. PEDATA, L. Sp. ed. 2, 1532 (1763); Hook. & Bak. Syn. Fil. 167. *Litobrochia pedata*, Presl, Tent. Pterid. 149 (1836); Hook. f. (3), 168; Anderss. (1), 129, & (2), 40 (*Lithobrachia*). — GALAPAGOS IDS.: *Cuming*, no. 107; *Capt. Wood*. ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 22 (hb. Gr.), 24 (hb. Gr.). CHARLES ISL.: *Darwin*; *Andersson*; *Lee* (hb. U. S. Nat. Mus.); *Baur*, no. 364 (hb. Gr.). CHATHAM ISL.: southwest end, middle region, *Baur*, no. 365 (hb. Gr.). JAMES ISL.: *Darwin*; *Douglas*; *Cuming*. Further distrib. W. Ind., S. Am.

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* This and all following references to the plants of Dr. Habel are based upon a list preserved at the Royal Gardens, Kew, a transcription of this list having been kindly forwarded to the writer by Mr. W. Botting Hemsley.

P. PROPINQUA, var. CUMINGIANA, Ag. Sp. Gen. Pterid. 65; Hook. Sp. Fil. ii. 223. — GALAPAGOS IDS.: *Capt. Wood*, acc. to Hook. l. c. Further distrib. Venezuela to Mex., Trinidad. This species is by Hook. & Bak. (Syn. Fil. 171) regarded as a form of *P. aculeata*, Sw.

TAENITIS, Sw.

T. ANGUSTIFOLIA, R. Br. Prodr. 154, in note (1810); Hook. Sp. Fil. v. 187; Hook. & Bak. Syn. Fil. 396. *Vittaria costata*, Kunze, Analect. Pterid. 29, t. 18, f. 2, also f. 1, *a* and *b* (1837). — GALAPAGOS IDS.: *Capt. Wood*, acc. to Hook. l. c. Further distrib. W. Ind., Colombia to Brazil.

SALVINIACEAE.

AZOLLA, Lam.

A. CAROLINIANA, Willd. Sp. Pl. v. 541 (1810); Anderss. (1), 132, & (2), 42. — CHARLES ISL.: in cold spring of the middle region, *Andersson*.

SALVINIA, L.

S. sp., Wolf (1), 284. — CHARLES ISL.: in brooks near hacienda, acc. to Wolf, l. c.

LYCOPODIACEAE.

LYCOPODIUM, L.

L. CLAVATUM, L. Sp. 1101 (1753); Rob. & Greenm. (1), 149. — CHATHAM ISL.: southwest end, upper region, *Baur*, no. 375 (hb. Gr.). Of wide distribution.

L. DICHOTOMUM, Jacq. Enum. Vindob. 314 (1762), & Hort. Vindob. iii. 26, t. 45; Spring, Monog. Lycop. i. 41, ii. 18; Hemsl. Biol. Cent.-Am. Bot. iii. 701; Bak. Fern-allies, 16. *L. pithyoides*, Schlecht. & Cham. Linnaea, v. 623 (1830). *L. mandiocanum*, Raddi, Fil. Bras. 77, t. 4 (1825). — GALAPAGOS IDS.: acc. to Hemsley and Baker, ll. cc. Further distrib. trop. Am., Madagascar.

Dr. Wolf (1), 283, refers to two undetermined species of *Lycopodium* found on Charles Isl.

POTAMOGETONACEAE.

POTAMOGETON, Tourn.

P. PECTINATUS, L. Sp. 127 (1753); Schumann in Mart. Fl. Bras. iii. pt. 3, 693 (where widely interpreted). — ALBEMARLE ISL. Iguana Cove, *Snodgrass* & *Heller*, no. 941a (hb. Gr.). Of cosmopolitan distribution.

As to robustness and leaf-breadth the Galapageian specimen at hand appears to correspond most closely to the form *P. Vaillantii*, Roem. & Schultes, Syst. iii. 514 (1818), which is intermediate between the very slender typical form and the broad-leaved form which is often separated as *P. zosteraceus*, Fries, Nov. ed. 2, 51 (1828).

RUPPIA, L.

R. MARITIMA, L. Sp. 127 (1753); Reichenb. Ic. Fl. Germ. vii. t. 17, f. 26. — ALBEMARLE ISL.: Elizabeth Bay, lava fields near the beach, *Snodgrass* & *Heller*, no. 938 (hb. Gr.). Widely distributed. A species not hitherto recorded on the Galapagos Ids.

NAJADACEAE.

NAJAS, L.

N. MARINA, L., var. LATIFOLIA A. Br. ex Schum. in Mart. Fl. Bras. iii. pt. 3, 725 (1894); Rendle, Trans. Linn. Soc. ser. 2, v. 396, t. 39, f. 11. *N. latifolia*, A. Br. Journ. Bot. ii. 276 (1864). — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 940a (hb. Gr.). The material at hand varies somewhat as to the arming of the internodes. In some instances they are smooth, as in var. *latifolia*; in other stems they are rather conspicuously armed. There is reason to believe that this is individual variation, and the long rather broad many-toothed leaves are those of var. *latifolia*. Further distrib. trop. S. Am.

GRAMINEAE.

ANTHEPHORA, Schreb.

A ELEGANS, Schreb. Beschr. Gräs. ii. 105, t. 44 (1810); Kunth, Enum. i. 169; Anderss. (1), 141, & (2), 48; Rob. & Greenm. (1), 149. ? *A. cuspidata*, Anderss. ll. cc. (differences not greater than in the various continental forms, which have not yet been successfully separated). — CHARLES ISL.: *Andersson* (hb. Gr.); *Baur*, no. 346 (hb. Gr.), subglabrous form. CHATHAM ISL.: northern part, *Baur*, no. 347 (hb. Gr.), also subglabrous. INDEFATIGABLE ISL.: *Andersson* (form *A. cuspidata*). JAMES ISL.: on James Bay, *Snodgrass* & *Heller*, no. 397 (hb. Gr.). Further distrib. Mex., W. Ind., S. Am.

ARISTIDA, L.

A. DIVULSA, Anderss. (1), 143, & (2), 49; Rob. & Greenm. (1), 149. — ALINGDON ISL.: *Snodgrass* & *Heller*, no. 809 (hb. Gr.). BINDLOE

ISL.: *Baur*, no. 336 (hb. Gr.). CHATHAM ISL.: in stony places, *Andersson* (hb. Gr.). Endemic.

A. REPENS, Trin. Mém. Acad. Pétersb. ser. vi. i. 87 (1831); Trin. & Rupr. ibid. vii. 128 (1842); Hook. f. (3), 175; Anderss. (1), 146, & (2), 50. — JAMES ISL.: *Douglas.* Not secured or at least not recognized on the islands since.

A. SUBSPICATA, Trin. & Rupr. l. c. 125 (1842); Steud. Syn. 137; Hook. f. (3), 174; Anderss. (1), 143, & (2), 49; Rob. & Greenm. (1), 149. *A. caudata*, Anderss. (1), 144, & (2), 49; Rob. & Greenm. (1), 149. *A. compacta*, Anderss. (1), 145, & (2), 50. — GALAPAGOS IDS.: *Habel.* ALBEMARLE ISL.: *Macrae* (hb. Gr.); in dry upper region, *Andersson* (hb. Gr.); eastern portions, Cowley Bay, *Baur*, no. 340 (hb. Gr.); Elizabeth Cove, *Snodgrass* & *Heller*, no. 273, past flowering and doubtful (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 152 (hb. Gr.); everywhere on sandy hillsides in tufa soil, the most abundant grass above the cove, *Snodgrass* & *Heller*, no. 209 (hb. Gr.). BARRINGTON ISL.: *Baur*, no. 339 (hb. Gr.); *Snodgrass* & *Heller*, no. 488 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 776 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson* (hb. Gr.); south of Conway Bay, *Baur*, no. 338 (hb. Gr.); northern part, *Snodgrass* & *Heller*, no. 653 (hb. Gr.). JAMES ISL.: James Bay, on sand in crevices of lava rock near beach, *Snodgrass* & *Heller*, no. 396 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 581 (hb. Gr.); south, *Snodgrass* & *Heller*, no. 566 (hb. Gr.). Further distrib. S. Am.

A. VILLOSA, Rob. & Greenm. (1), 144, 149. — DUNCAN ISL.: *Snodgrass* & *Heller*, no. 699 (hb. Gr.). JERVIS ISL.: *Baur*, no. 337 (hb. Gr.). Endemic.

BOUTELOUA, Lag.

B. PILOSA, Benth. acc. to Wats. Proc. Am. Acad. xviii. 179 (1883). *Eutriana pilosa*, Hook. f. (3), 173; Anderss. (1), 147, & (2), 52 (where by Darwin is meant Macrae), t. 1, f. 3; Rob. & Greenm. (1), 149. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 807 (hb. Gr.). ALBEMARLE ISL.: *Macrae;* in lower rocky region, *Andersson;* southern portion, *Baur*, no. 352 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, nos. 36 (hb. Gr.), 109 (hb. Gr.), 139 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 216 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 490 (hb. Gr.). CHATHAM ISL.: *Andersson.* INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 655 (hb. Gr.). JAMES ISL.: on sea bluffs of James Bay, *Snodgrass* & *Heller*, no. 393

(hb. Gr.). JERVIS ISL.: *Baur*, no. 351 (hb. Gr.): SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 567 (hb. Gr.); south, *Snodgrass* & *Heller*, no. 611 (hb. Gr.). Endemic.

CENCHRUS, L.

C. DISTICHOPHYLLUS, Griseb. Cat. Pl. Cub. 234 (1866); Rob. & Greenm. (1), 148. — ALBEMARLE ISL.: southern portion, *Baur*, no. 335 (hb. Gr.), wrongly recorded as from Hood Isl. by Rob. & Greenm. l. c. Sterile and hence doubtful. Further distrib. Cuba.

C. GRANULARIS, Anderss. (1), 140, & (2), 47; Rob. & Greenm. (1), 148. — ALBEMARLE ISL.: Tagus Cove, from beach to 180 m. altitude, not abundant, *Snodgrass* & *Heller*, no. 211 (hb. Gr.). CHARLES ISL.: *Baur*, no. 324 (hb. Gr.); *Snodgrass* & *Heller*, no. 456 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.); *A. Agassiz* (hb. U. S. Nat. Mus.). SEYMOUR ISL.: south, *Snodgrass* & *Heller*, nos. 593, 594 (both in hb. Gr.). Endemic.

C. PLATYACANTHUS, Anderss. (1), 139, & (2), 47; Rob. & Greenm. (1), 148. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 808 (hb. Gr.). ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 37 (hb. Gr.), 112 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 484 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 781 (hb. Gr.). CHARLES ISL.: abundant, *Andersson; Snodgrass* & *Heller*, no. 456a (hb. Gr.). CHATHAM ISL.: *Andersson ; Snodgrass* & *Heller*, no. 550 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 636 (hb. Gr.). HOOD ISL.: *Baur*, no. 323 (hb. Gr.); *Snodgrass* & *Heller*, no. 716 (hb. Gr.). INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 678 (hb. Gr.). JAMES ISL.: James Bay, common along the coast in sand and lava soil, *Snodgrass* & *Heller*, no. 395 (hb. Gr.). NARBOROUGH ISL.: southern part, *Snodgrass* & *Heller*, no. 317 (hb. Gr.). Endemic so far as known but apparently spreading rapidly in the manner of an introduced weed.

C. sp. — CHATHAM ISL.: *Snodgrass* & *Heller*, no. 520 (hb. Gr.). A large soft-pubescent species nearly related to, if not identical with, some of the very diverse forms in the Hawaiian and other Pacific Islands now collected under *C. calyculatus*, Cav. and its supposed synonyms.

CHLORIS, Sw.

C. **anisopoda**, nov. sp., annua, radice fibrosa, culmis pluribus caespitosis suberectis tenuibus teretibus glaberrimis 5–7-nodosis; vaginis striatis margine ciliata excepta glaberrimis teretibus spicas inferiores fere in toto includentibus, foliae parte libera brevi lineari attenuata subpungente praeter basis longe ciliatae glaberrima, spicis inferioribus brevibus vaginis arcte involutis, supremis subternis erectis tenuibus flexuosis cum pedunculo gracile tereti bene exsertis, spiculis tenuibus teretibus adpressis bifloris, glumis sterilibus binis inaequalibus tenuibus lanceolati-linearibus attenuatis acutissimis uninervis translucentibus basi breviter fasciculatim barbatis, palea inferiori oblonga ciliata paullo infra apice subretusa cum arista recta tenui tenuissime barbata suae longitudini superante munita, palea superiori anguste lanceolata acutissima sed exaristata, flosculi sterilis pedicellati abortivi palea inferiori longe aristata. — *C.* sp. Rob. & Greenm. (1), 149. — CHARLES ISL.: *Baur*, no. 333 (hb. Gr.); *Snodgrass* & *Heller*, no. 459 (hb. Gr.). Endemic. Culms 2.5 to 4 dm. high; leaf blades 2 to 6 cm. long. Spikelets including the awns 6 mm. long, very slender. A species distinguished from most of its congeners by having inflorescences springing from nearly all the nodes, the lower inflorescences being almost or quite enveloped by the sheaths of the leaves.

C. ELEGANS, HBK. Nov. Gen. & Sp. i. 166, t. 49 (1815). — CHARLES ISL.: *Snodgrass* & *Heller*, no. 461 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 579 (hb. Gr.). Further distrib. southwestern U. S., Mex.

C. RADIATA, Sw. Prodr. 26 (1788); Rob. & Greenm. (1), 149. — CHARLES ISL.: *Baur*, no. 344 (hb. Gr.). Further distrib. W. Ind., S. Am.

CHUSQUEA, Kunth.

C. sp. — ALBEMARLE ISL.: on mountain east of Elizabeth Cove, alt. 923 m., *Snodgrass* & *Heller*, no. 230 (hb. Gr.), old, dead, and leafless culms only, but the genus recognizable from the peculiar mode of branching. Belonging to a group of tropical S. American grasses of the bamboo tribe, not heretofore noted upon the islands.

ELEUSINE, Gaertn.

E. AEGYPTICA, Desf. Fl. Atl. i. 85 (1798), as *aegyptia*. *E. aegyptiaca*, Rob. & Greenm. (1), 149. *Dactyloctenium aegyptiacum*, Willd. Enum. Hort. Berol. 1029 (1809); Caruel (1), 621; Rose (1), 138. — CHATHAM ISL.: *Chierchia; A. Agassiz; Snodgrass* & *Heller*, no. 551 (hb. Gr.). HOOD ISL.: *Baur*, no. 326 (hb. Gr.). Of wide distribution.

E. INDICA, Gaertn. Fruct. i. 8 (1788); Kunth, Enum. i. 272; Anderss. (1), 147, & (2), 52; Rose (1), 138; Caruel (1), 621; Rob. & Greenm. (1), 149. — CHARLES ISL.: in grassy places, middle region, *Andersson; Baur*, no. 374 (hb. Gr.); *Snodgrass* & *Heller*, no. 462 (hb. Gr.). CHATHAM ISL.: *Chierchia; A. Agassiz;* southwest end, middle region, *Baur*, no. 349 (hb. Gr.); *Snodgrass* & *Heller*, no. 552 (hb. Gr.). Of wide distribution.

ERAGROSTIS, Host.

E. BAHIENSIS, Roem. & Sch. Mant. ii. 318 (1824). — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 110 (hb. Gr.). Further distrib. S. Am.

E. CILIARIS, Link, Hort. Berol. i. 192 (1827); Rob. & Greenm. (1), 149. *Poa ciliaris*, L. Syst. ed. 10, 875 (1760); Hook. f. (3), 175; Anderss. (1), 150, & (2), 52. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 810 (hb. Gr.). ALBEMARLE ISL.: *Macrae;* Tagus Cove, *Snodgrass* & *Heller*, no. 218 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 777 (hb. Gr.). CHARLES ISL.: *Darwin; Andersson; Baur*, without number (hb. Gr.); *Snodgrass* & *Heller*, no. 457 (hb. Gr.). CHATHAM ISL.: *Darwin; Andersson; Baur*, no. 328 (hb. Gr.); *Snodgrass* & *Heller*, nos. 542 (hb. Gr.), 559 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 724 (hb. Gr.). JAMES ISL.: Orchilla Bay, *Baur*, no. 329 (hb. Gr.). NARBOROUGH ISL.: Mangrove Point, in crevices of lava near the coast, abundant, *Snodgrass* & *Heller*, no. 306 (hb. Gr.). TOWER ISL.: *Baur*, no. 330 (hb. Gr.); *Snodgrass* & *Heller*, no. 800 (hb. Gr.). Further distrib. W. Ind., S. Am.

E. MAJOR, Host. Gram. iv. 14, t. 24 (1809). *Poa megastachya*, Koehl. Desc. Gram. 181 (1802); Anderss. (1), 150, & (2), 52; Rob. & Greenm. (1), 149. — GALAPAGOS IDS.: *Habel*. BARRINGTON ISL.: *Baur*, no. 332 (hb. Gr.); *Snodgrass* & *Heller*, no. 489 (hb. Gr.). CHARLES ISL.: dry places on the shore, *Andersson; Snodgrass* & *Heller*, no. 460 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.); *Snodgrass* & *Heller*, no. 558 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 643 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 715 (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 590 (hb. Gr.). TOWER ISL.: *Snodgrass* & *Heller*, no. 798 (hb. Gr.). Of wide distribution.

E. PILOSA, Beauv. Agrost. 71 (1812). *Poa pilosa*, L. Sp. 68 (1753); Hook. f. (3), 175; Anderss. (1), 150, & (2), 52. — GALAPAGOS IDS.:

Habel. JAMES ISL.: *Darwin*, acc. to Hook. f., l. c. Further distrib. trop. reg.

ERIOCHLOA, HBK.

E. DISTACHYA, HBK. Nov. Gen. & Sp. i. 95, t. 30 (1815); Kunth, Enum. i. 72. *Helopus brachystachys*, Trin. Sp. Gram. iii. t. 277 (1836). — CHATHAM ISL.: *Snodgrass* & *Heller*, no. 547 (hb. Gr.). Further distrib. continental S. Am. from Panama to Brazil.

LEPTOCHLOA, Beauv.

L. ALBEMARLENSIS, Rob. & Greenm. (1), 145, 149. — ALBEMARLE ISL.: southern portion, *Baur*, no. 341 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 58 (hb. Gr.). Endemic.

L. FILIFORMIS, Roem. & Sch. Syst. ii. 580 (1817); Hook. f. Fl. Brit. Ind. vii. 298; not Beauv. — DUNCAN ISL.: *Snodgrass* & *Heller*, no. 697 (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 580 (hb. Gr.). Widely distributed in the tropics of both hemispheres.

L. LINDLEYANA, Kunth, Rev. Gram. ii. 655, t. 215 (1829), & Enum. i. 525; Rob. & Greenm. (1), 149. *L. hirta*, Nees acc. to Steud. Syn. i. 209 (1855). *L. Hookeri*, Anderss. (2), 51. *Trichoneuron Hookeri*, Anderss. (1), 149, & (2), 51, t. 1, f. 2. *Calamagrostis pumila*, Hook. f. (3), 176. — ALBEMARLE ISL.: *Macrae* (hb. Gr.); in the most sterile places, *Andersson*; eastern portion, *Baur*, no. 242 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 212 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 790 (hb. Gr.). CHATHAM ISL.: *Andersson*. Endemic.

L. MUCRONATA, Kunth, Rev. Gram. i. 91 (1829). — BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 491 (hb. Gr.). Not previously observed on these islands. Further distrib. southern U. S., Mex.

L. VIRGATA, Beauv. Agrost. 71 (1812); Anderss. (1), 147, & (2), 51. — CHARLES ISL.: upper grassy region, *Andersson* (hb. Gr.). Further distrib. Mex., W. Ind., S. Am.

OPLISMENUS, Beauv.

O. SETARIUS, Roem. & Sch. Syst. ii. 481 (1817); Rob. & Greenm. (1), 148 (a species reduced by some to *O. undulatifolius*, Beauv.).— CHATHAM ISL.: southwest end, middle region, *Baur*, no. 322 (hb. Gr.). Further distrib. U. S., Mex., W. Ind., S. Am. The sterile and indeterminate *Oplismenus* mentioned by Caruel (1), 621, as collected on Chatham Isl. by *Chierchia* was probably of this species.

PANICUM, L.

P. COLONUM, L. Syst. ed. 10, 870 (1760); Trin. Ic. ii. t. 160; Hook. f. (3), 172. *Oplismenus colonus*, HBK. Nov. Gen. & Sp. i. 108 (1815); Anderss. (1), 136, & (2), 45. — CHARLES ISL.: *Darwin*; in somewhat swampy places, middle region, *Andersson*. Further distrib. general in trop. regions.

P. FASCICULATUM, Sw. Prodr. 22 (1788), & Fl. Ind. Occ. i. 145; Anderss. (1), 135, & (2), 44. — CHARLES ISL.: *Snodgrass* & *Heller*, no. 455 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson*; northern part, *Snodgrass* & *Heller*, no. 649 (hb. Gr.). JAMES ISL.: common on sea-bluff on James Bay, *Snodgrass* & *Heller*, no. 392 (hb. Gr.). Further distrib. W. Ind., S. Am.

P. FLUITANS, Retz. Obs. iii. 8 (1783); Anderss. (1), 136, & (2), 45. *P. paspalodes*, Pers. Syn. i. 81 (1805). *P. paspaloides*, auct. — CHATHAM ISL.: in stagnant pools, middle region, *Andersson* (hb. Gr.); *Snodgrass* & *Heller*, no. 557 (hb. Gr.). HOOD ISL.: on the margin of a mud lake, *Snodgrass* & *Heller*, no. 746 (hb. Gr.). Further distrib. W. Ind., S. Am., trop. of Old World.

P. FUSCUM, Sw. Prodr. 23 (1788); Anderss. (1), 135, & (2), 44; Caruel (1), 621; Rose (1), 138. — CHARLES ISL.: *Andersson*; *Chierchia*. CHATHAM ISL.: *Andersson*; *A. Agassiz*. Further distrib. W. Ind., S. Am. Andersson, ll. cc., distinguishes three forms or varieties: *vegetius*, *firmius*, and *pubescens*.

P. HIRTICAULUM, J. & C. Presl, Rel. Haenk. i. 308 (1830); Kunth, Enum. i. 101; *P. hirticaulon*, Anderss. (1), 134, & (2), 43 (incl. var. *majus*, typical, and var. *glabrescens*, not very clearly marked); Rose (1), 138; Rob. & Greenm. (1), 148; a species sometimes reduced to *P. ayennense*, Lam. — BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 492 (hb. Gr.). CHARLES ISL.: *Andersson*. CHATHAM ISL.: *Andersson*; (hb. Gr.); *A. Agassiz*; *Snodgrass* & *Heller*, no. 548 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 648 (hb. Gr.). HOOD ISL.: *Baur*, no. 345 (hb. Gr.); *Snodgrass* & *Heller*, no. 722 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson* (hb. Gr.); northern part, *Snodgrass* & *Heller*, no. 661 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 565 (hb. Gr.); south, *Snodgrass* & *Heller*, no. 612 (hb. Gr.). It is noteworthy that this grass, now widely distributed and apparently common in the archipelago, was not noted or collected by Darwin or any of the earlier collectors upon the islands, a fact which strongly suggests recent introduction. Further distrib. Mex.

Var. MINUS, Anderss. (1), 135, & (2), 44. — CHARLES ISL.: *Andersson; Snodgrass* & *Heller*, no. 454 (hb. Gr). CHATHAM ISL.: *Andersson.* Endemic.

P. MOLLE, Sw. Prodr. 22 (1788); Caruel (1), 621. — CHATHAM ISL.: *Chierchia*, acc. to Caruel, l. c. Further distrib. W. Ind., S. Am.; also introd. in trop. Asia.

P. MULTICULMUM, Anderss. (1), 133, & (2), 43. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 138 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 487 (hb. Gr.). CHARLES ISL.: in springs of the interior, *Andersson* (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller*, no. 696 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 634 (hb. Gr.). Endemic.

P. SANGUINALE, L. Sp. 57 (1753). — CHATHAM ISL.: *Snodgrass* & *Heller*, no. 513 (hb. Gr.). Not secured or reported by earlier explorers, hence probably a recent introduction. Cosmopolitan weed.

P. SEROTINUM, Trin. Gram. Panic. 166 (1826). *Paspalus serotinus*, Fluegge, Gram. Monogr. 145 (1810); *Paspalum serotinum*, Kunth, Enum. i. 47; Anderss. (1), 132, & (2), 42. *Digitaria serotina*, Michx. Fl. i. 46 (1803); Hook. f. (4), 261. — CHARLES ISL.: *Edmonston* acc. to Hook. f., l. c. There can be little doubt that there was some mistake in this identification, but as it has not been possible to see the specimen the error cannot be corrected here. The species is of the southern U. S.

PASPALUM, L.

P. CANESCENS, Anderss. (1), 132, & (2), 42; Rob. & Greenm. (1), 148. *Panicum fuscum*, Rob. & Greenm. (1), 148, not Sw. — ALBEMARLE ISL.: upper part, *Andersson;* southern portion, *Baur*, no. 343 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 198 (hb. Gr.). CHARLES ISL.: *Baur* (hb. Gr.). NARBOROUGH ISL.: southern part, *Snodgrass* & *Heller*, nos. 312 (hb. Gr.), 336 (hb. Gr.). Endemic.

P. CONJUGATUM, Berg. Act. Helv. vii. 129, t. 8 (1772); Anderss. (1), 132, & (2), 42; Rob. & Greenm. (1), 148. — CHARLES ISL.: grassy places in the more fertile parts, *Andersson.* CHATHAM ISL.: southwest end, middle region, *Baur*, no. 350 (hb. Gr.). Further distrib. Mex., W. Ind., S. Am., trop. Africa.

P. DISTICHUM, L. Amoen. Acad. v. 391 (1760); Burm. f. Fl. Ind. 23. *P. vaginatum*, Sw. Prodr. 21 (1788); Rob. & Greenm. (1), 148. — JAMES ISL.: Orchilla Bay, in a sterile state, *Baur*, no. 334 (hb. Gr.). Further distrib. trop. and subtrop. regions.

P. LONGE-PEDUNCULATUM, Le Conte (Le Comte), Journ. Phys. xci. 284 (1820); Kunth, Enum. i. 56; Hook. f. (3), 172; Anderss. (1), 132, & (2), 42. *Panicum fuscum*, Rob. & Greenm. (1), 148, not Sw. — ALBEMARLE ISL.: *Macrae;* Iguana Cove, *Snodgrass* & *Heller*, no. 131 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, nos. 199 (hb. Gr.), 891 (hb. Gr.), 892 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 775 (hb. Gr.). CHARLES ISL.: *Darwin; Andersson; Baur*, no. 148 (hb. Gr.). This species is by some reduced to *P. arenarium*, Schrad., which however is a later name. Further distrib. southern U. S. to Brazil.

P. PENICILLATUM, Hook. f. (3), 171; Anderss. (1), 132, & (2), 42. — CHARLES ISL.: *Darwin*. Not since collected. Endemic.

P. SCROBICULATUM, L. Mant. i. 29 (1767), as *scrobiculata;* Caruel, (1), 621. — CHATHAM ISL.: *Chierchia*, acc. to Caruel. General distrib. tropics of the Old World. It is remarkable, if Prof. Caruel was correct in his identification of this gerontogeous grass, that no trace of it has been found in the Galapagos Archipelago by any other collector.

P. sp. — INDEFATIGABLE ISL.: north, *Snodgrass* & *Heller*, no. 669 (hb. Gr.). Resembling *P. canescens*, Anderss., but with the leaves bearded upon both surfaces with a long and very fine villosity, and the spikelets considerably larger. Probably a new species.

PENNISETUM, Rich.

P. PAUPERUM, Nees acc. to Steud. Syn. 102 (1855). *P. exaltatum*, Hook. f. & Jacks. Ind. Kew. i. 112, 1893 (see reference under *Amphochaeta*). *Amphochaeta exaltata*, Anderss. (1), 137, & (2), 45, t. 1, f. 2. *Gymnothrix paupera*, Nees acc. to Steud. l. c. — ALBEMARLE ISL.: *Andersson* (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, nos. 16 (hb. Gr.), 141 (hb. Gr.); near Tagus Cove, altitude 150 to 1230 m., in large clumps 1 to 2 m. high, *Snodgrass* & *Heller*, no. 223 (hb. Gr.). Endemic, and so far as known confined to the single Island.

SETARIA, Beauv.

S. FLORIANA, Anderss. (1), 138, & (2), 46. — CHARLES ISL.: *Andersson*. Not seen by the writer. Endemic.

S. SETOSA, Beauv. Agrost. 51 (1812). *S. caudata*, R. & S. Syst. ii. 495 (1817), slender form. *S. Antillarum*, Kunth, Rev. Gram. i. 46 (1829); Anderss. (1), 138, & (2), 46; Rob. & Greenm. (1), 148. ? *S. Rottleri*, Hook. f. (3), 172; Anderss. (1), 139, & (2), 46; not Spr.

Chaetochloa setosa, Scribn. U. S. Div. Agros. Bull. iv. 39 (1897). *C. caudata*, Scribn. Rep. Mo. Bot. Gard. x. 52 (1899). *C. setosa* & *C. caudata*, Scribn. & Merrill, U. S. Div. Agros. Bull. xxi. 38, 39, f. 23, 24. *Panicum caudatum*, Lam. Ill. i. 171 (1791). *P. setosum*, Sw. Prodr. 22 (1788). — GALAPAGOS IDS.: *Douglas* (hb. Gr.). ALBEMARLE ISL.: *Macrae?* Iguana Cove, growing 4.5 to 6 dm. high and woody at base, very common under *Opuntia* thickets, *Snodgrass* & *Heller*, no. 35 (hb. Gr.); common everywhere from beach to 600 m. altitude, the most abundant grass, *Snodgrass* & *Heller*, no. 111 (hb. Gr.); north mountain, Elizabeth Cove, *Snodgrass* & *Heller*, no. 285 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 205 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 470 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.); southwest end, lower region, *Baur*, no. 321 (hb. Gr.); *Snodgrass* & *Heller*, no. 553 (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller*, no. 692 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 718 (hb. Gr.). JAMES ISL.: James Bay, *Snodgrass* & *Heller*, no. 394 (hb. Gr.). NARBOROUGH ISL.: *Snodgrass* & *Heller*, no. 349 (hb. Gr.). Further distrib. trop. regions.

S. n. sp.? Hook. f. (3), 172; Anderss. (1), 139, & (2), 47. — ALBEMARLE ISL.: *Macrae*. Not seen by the writer. Endemic so far as known.

SPOROBOLUS, R. Br.

S. DOMINGENSIS, Kunth, Enum. i. 214 (1833). — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 811 (hb. Gr.). ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 140 (hb. Gr.), 862 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 717 (hb. Gr.). Further distrib. Mex., W. Ind.

S. INDICUS, R. Br. Prodr. i. 170 (1810); Kunth, Enum. i. 211; Anderss. (1), 146, & (2), 50; Caruel (1), 621; Vasey, U. S. Dept. Agric. Div. Bot. Bull. iii. 21, t. 7 (1887). *S. tenacissimus*, Beauv. Agrost. 26 (1812). — CHARLES ISL.: in upper grassy places, *Andersson* (hb. Gr.). CHATHAM ISL.: *Chierchia*. Further distrib. U. S., Mex., S. Am., W. Ind., Old World.

S. VIRGINICUS, Kunth, Rev. Gram. i. 67 (1829), & Enum. i. 210; Anderss. (1), 146, & (2), 51. *Agrostis virginica*, L. Sp. 63 (1753). *Distichlis* sp. Rose (1), 138. — ALBEMARLE ISL.: Elizabeth Cove, *Snodgrass* & *Heller*, no. 271 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, nos. 151 (hb. Gr.), 210 (hb. Gr.). CHATHAM ISL.: on banks near salt water, *Andersson* (hb. Gr.); *A. Agassiz* (hb. Gr.); *Snodgrass*

& *Heller*, no. 549 (hb. Gr.). Further distrib. U. S., Mex., W. Ind., S. Am. Andersson notes two vegetative forms: *obesa* and *elata*.

STENOTAPHRUM, Trin.

S. GLABRUM, Trin. Fund. Agrost. 176 ("1820"). *S. americanum*, Schrank, Pl. Rar. Hort. Monac. t. 98 ("1819"); Rob. & Greenm. (1), 149. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 57 (hb. Gr.), 873 (hb. Gr.). CHATHAM ISL.: southwest end, middle region, *Baur*, no. 325 (hb. Gr.). Further distrib. tropical shores of both continents.

STIPA, L.

S. ROSTRATA, Anderss. (1), 142, & (2), 48. — CHATHAM ISL.: *Andersson* (hb. Gr.). Habit of *Aristida subspicata*, but differing rather conspicuously in its untwisted awns. Endemic.

CYPERACEAE.

CYPERUS, L.

C. ARISTATUS, Rottb. Descr. Nov. Pl. 23, t. 6, f. 1 (1786); C. B. Clarke, Jour. Linn. Soc. xxi. 91. *C. inflexus*, Muhl. Descr. Gram. 16 (1817); Hook. f. (3), 177; Anderss. (1), 153, & (2), 54. — ALBEMARLE ISL.: *Macrae* (var. *γ elongatus*, Hook. f., l. c. 178); Tagus Cove, *Snodgrass* & *Heller*, no. 173 (hb. Gr.). CHARLES ISL.: *Darwin* (var. *β acaulis*, Hook. f., l. c.). CHATHAM ISL.: *A. Agassiz* (hb. U. S. Nat. Mus.). JAMES ISL.: *Scouler*. NARBOROUGH ISL.: south, 615 m. altitude, *Snodgrass* & *Heller*, no. 345 (hb. Gr.). Further distrib. N. Am., Mex., W. Ind., S. Am., Old World.

C. BRACHYSTACHYS, Anderss. (2), 53, t. 13, f. 2; Rob. & Greenm. (1), 148, where by typographical error ascribed to Charles Island. *Mariscus brachystachys*, Hook. f. (3), 179; Anderss. (1), 152. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 832 (hb. Gr.). ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 39 (hb. Gr.); southern portion, *Baur*, no. 313 (hb. Gr.). CHARLES ISL.: *Darwin; Andersson*. CHATHAM ISL.: southwest end, middle region, *Baur*, no. 312 (hb. Gr.). JAMES ISL.: *Scouler*. TOWER ISL.: *Snodgrass* & *Heller*, no. 791 (hb. Gr.). Endemic. This species is known to me only from the descriptions of Hooker and Andersson, which do not altogether agree, and from Andersson's figure, which is not very satisfactory. The specimens of recent collections mentioned above are referred here with doubt.

Snodgrass & Heller's no. 132 (hb. Gr.) from Iguana Cove, Albemarle Isl., with irregular unequal rayed umbels and remarkably slender spikes, has closely similar floral structure and is probably an extreme form of this species.

C. CONFERTUS, Sw. Prodr. 20 (1788), & Fl. Ind. Occ. i. 115; Anderss. (2), 54; Rose (1), 138. *C. biuncialis*, Anderss. (1), 156, & (2), 56. *C. dissitiflorus*, Anderss. (1), 153, & (2), 54. — ALBEMARLE ISL.: *Andersson* (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 907 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 784 (hb. Gr.). CHARLES ISL.: *Andersson* (hb. Gr.); *A. Agassiz* (hb. Gr.); *Snodgrass* & *Heller*, nos. 403 (hb. Gr.), 458 (hb. Gr.). CHATHAM ISL.: *Andersson; A. Agassiz; Snodgrass* & *Heller*, no. 500 (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller*, no. 681 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 742 (hb. Gr.) INDEFATIGABLE ISL.: *Andersson.* JAMES ISL.: *Andersson.* Further distrib. trop. Am.

C. ESCULENTUS, L. Sp. 45 (1753); Kunth, Enum. ii. 61; Anderss. (1), 153, & (2), 54. *C. strigosus*, Anderss. ll. cc., at least in part? not L. — ALBEMARLE ISL.: Tagus Cove, in damp soil of cañons, *Snodgrass* & *Heller*, no. 179 (hb. Gr.). CHATHAM ISL.: in grassy places, *Andersson* (hb. Gr.). Further distrib. general.

C. FUGAX, Liebm. Mex. Halv. 8 (1850), & Vidensk Selsk. Skr. ser. 5, ii. 196 (1851); Rob. & Greenm. (1), 148. — CHATHAM ISL.: southwest end, middle region, *Baur*, no. 311 (hb. Gr.). Further distrib. Mex. A species referred by Hook. f. & Jacks. Ind. Kew. i. 693 to *Pycreus polystachyus.*

C. GALAPAGENSIS, Caruel (1), 621. — CHATHAM ISL.: *Chierchia.* Endemic.

C. GRANDIFOLIUS, Anderss. (1), 157, & (2), 56. — CHATHAM ISL.: in ditches filled with water in the interior part of the island, *Andersson.* Endemic.

C. LAEVIGATUS, L. Mant. ii. 179 (1771). — ALBEMARLE ISL.: Elizabeth Cove, *Snodgrass* & *Heller*, no. 260 (hb. Gr.). Further distrib. cosmop. trop.

C. LIGULARIS, L. Amoen. Acad. v. 391 (1760). *Mariscus rufus*, HBK. Nov. Gen. & Sp. i. 216, t. 67 (1815). — ALBEMARLE ISL.: Elizabeth Cove, *Snodgrass* & *Heller*, nos. 263 (hb. Gr.), 264 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 861 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 191 (hb. Gr.). Further distrib. W. Ind., trop. S. Am., trop. Africa.

C. MUTISII, Anderss. (2), 53, in two forms; Rob. & Greenm. (1), 148 (var. *plenus*). *Mariscus Mutisii*, HBK. Nov. Gen. & Sp. i. 216, t. 66 (1815); Kunth, Enum. ii. 124; Hook. f. (1), 178; Anderss. (1), 151. — ALBEMARLE ISL.: *Macrae;* Elizabeth Cove, *Snodgrass & Heller*, no. 262 immature and doubtful (hb. Gr.); Tagus Cove, abundant wherever a little soil occurs in cañons, lava fields, or on hillsides, *Snodgrass & Heller*, nos. 894 (hb. Gr.), 190 also immature and doubtful (hb. Gr.). CHARLES ISL.: *Andersson* (hb. Gr.). CHATHAM ISL.: dry places, middle region, *Andersson; Baur.* GARDNER ISL.: *Snodgrass & Heller*, no. 647 (hb. Gr.). NARBOROUGH ISL.: Mangrove Point, lava rock near beach and also inland, *Snodgrass & Heller*, nos. 310 (hb. Gr.), 311 (hb. Gr.); southern part, 615 m. altitude, abundant in heavy soil, *Snodgrass & Heller*, no. 334, a slender and doubtful form (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass & Heller*, no. 602 (hb. Gr.).

C. ROTUNDUS, Hook. f. (3), 177; Anderss. (1), 153, & (2), 54; not L ? — ALBEMARLE ISL.: *Macrae*, ex Hook. f., l. c. According to Hook. f. & Jacks. Ind. Kew. i. 698, the plant of Macrae was the Texan *C. lutescens*, Torr. & Hook., but acc. to Dr. N. L. Britton, who has examined the type of the latter, the Texan plant was only a form or variety of *C. esculentus*, L.

C. RUBIGINOSUS, Hook. f. (3), 178; Anderss. (1), 157, & (2), 56; Rob. & Greenm. (1), 148, as to pl. from Duncan Isl. — Spiculis oblongis, 6–10 mm. longis; squamulis 10–19. — GALAPAGOS IDS.: *Goodridge*, in part (hb. Gr.). CHARLES ISL.: *Darwin.* CHATHAM ISL.: *Snodgrass & Heller*, no. 501 (hb. Gr.). DUNCAN ISL.: *Baur*, no. 318 (hb. Gr.). Endemic.

Var. **cornutus**, spiculis perbrevibus valde congestis; squamulis 4–8. — *Mariscus cornutus*, Anderss. (1), 151. *Cyperus cornutus*, Anderss. (2), 53, t. 13, f. 1. *C. rubiginosus*, Rob. & Greenm. (1), 148, as to pl. from Chatham and Hood Ids. — GALAPAGOS IDS.: *Goodridge*, in part (hb. Gr.). CHARLES ISL.: *Andersson* (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.); southwest end, middle region, *Baur*, no. 315 (hb. Gr.). DUNCAN ISL.: *Snodgrass & Heller*, no. 683 (hb. Gr.). HOOD ISL.: *Baur*, no. 314 (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass & Heller*, no. 595 (hb. Gr.). Endemic.

C. STRIGOSUS, L. Sp. 47 (1753); Hook. f. (3), 177; Anderss. (1), 153, & (2), 54 ? — CHARLES ISL.: *Darwin.* CHATHAM ISL.: *Andersson.* Further distrib. N. Am. I have seen no Galapageian specimens of this species, and am inclined to believe that the plants so referred by Hooker and Andersson will prove to be forms rather of *C. esculentus*.

C. SURANIMENSIS, Rottb. Descr. Nov. Pl. 35, t. 6, f. 5 (1786); Kunth, Enum. ii. 43; Hook. f. (3), 177; Anderss. (1), 153, & (2), 54. — JAMES ISL.: *Darwin*, acc. to Hook. f., l. c. Further distrib. trop. S. Am., Mex., W. Ind., S. United States.

C. TRISTACHYUS, Boeck. Linnaea, xxxv. 454 (1867–1868); Rob. & Greenm. (1), 148. — CHATHAM ISL.: southwest end, middle region, *Baur*, no. 316 (hb. Gr.). Further distrib. Venezuela, Colombia, southern central Mexico.

C. sp. — NARBOROUGH ISL.: southern part, altitude 615 m., *Snodgrass & Heller*, no. 337 (hb. Gr.).

C. sp. WENMAN ISL.: dried stalks of a *Cyperus* are mentioned in Mr. Heller's field notes, as observed on a small islet near Wenman.

DICHRONEMA, Michx.

D. LEUCOCEPHALA, Michx. Fl. i. 37 (1803); Rob. & Greenm. (1), 148. — CHATHAM ISL.: southwest end, middle region, *Baur*, no. 353 (hb. Gr.). Further distrib., S. United States, Mex., W. Ind.

ELEOCHARIS, R. Br.

E. FISTULOSA, Schult. Mant. ii. 89 (1824); Caruel (1), 622. — CHATHAM ISL.: *Chierchia*, acc. to Caruel. Further distrib. trop. of the Old World. I cannot help a strong suspicion that this when reëxamined will prove to be the following species.

E. MUTATA, R. Br. Prodr. 224 (1810). *Scirpus mutatus*, L. Syst. Nat. ed. 10, 867 (1760). — ALBEMARLE ISL.: Elizabeth Bay, *Snodgrass & Heller*, no. 261 (hb. Gr.). Further distrib. S. Am., W. Ind.

FIMBRISTYLIS, Vahl.

F. CAPILLARIS, Gray, Man. Bot. ed. 5, 567 (1869). *Scirpus capillaris*, L. Sp. 49 (1753). — BINDLOE ISL.: *Snodgrass & Heller*, no. 790 a (hb. Gr.). NARBOROUGH ISL.: Mangrove Point, near beach in crevices of lava, abundant, *Snodgrass & Heller*, no. 304 (hb. Gr.). Further distrib. N. Am., Mex., W. Ind., southward to Uruguay.

F. DIPHYLLA, Vahl, Enum. ii. 289 (1805). — HOOD ISL.: *Snodgrass & Heller*, no. 730 (hb. Gr.). Widely distrib. in trop. regions.

HEMICARPHA, Nees.

H. SUBSQUARROSA, Nees in Mart. Fl. Bras. ii. pt. 1, 61 (1842); Rob. & Greenm. (1), 148. *H. micrantha*, Britton, Bull. Torr. Club, xv. 104 (1888); Coville, ibid. xxi. 34. *Scirpus micranthus*, Vahl, Enum. ii. 254 (1805). — CHATHAM ISL.: southwest end, middle region, *Baur*, no. 317 (hb. Gr.). Further distrib. U. S., Mex., W. Ind., S. Am.

KYLLINGA, Rottb.

K. PUMILA, Michx. Fl. i. 28 (1803), as *Kyllingia*. *K. caespitosa*, Nees in Mart. Fl. Bras. ii. pt. 1, 12 (excl. syn. Kunth); Rob. & Greenm. (1), 148, as *Kyllingia*. — CHARLES ISL.: *Baur*, no. 319 (hb. Gr.). Further distrib. U. S., W. Ind., Mex., S. Am.

SCLERIA, Berg.

S. PRATENSIS, Lindl. ex Nees, Nov. Act. Nat. Cur. xix. Suppl. i. 121 (1843); Rob. & Greenm. (1), 148. — CHATHAM ISL.: southwest end, upper region, *Baur*, no. 320 (hb. Gr.). Further distrib. Mex., W. Ind., S. Am.

LEMNACEAE.

LEMNA, L.

L. sp., Wolf (1), 284. — CHARLES ISL.: in brook near hacienda, acc. to Wolf, l. c.

BROMELIACEAE.

TILLANDSIA, L.

T. INSULARIS, Mez in DC. Monogr. ix. 756 (1896). *T.* sp. Rob. & Greenm. (1), 148. — GALAPAGOS IDS.: *Steindachner*, no. 28 (in hb. Naturh. Mus. Vienna). DUNCAN ISL.: *Baur*, no. 237 (hb. Gr.). Endemic. A *Tillandsia* sp. coll. on the Galapagos Ids. by Habel, may prove the same.

COMMELINACEAE.

COMMELINA, Plum.

C. NUDIFLORA, L. Sp. 41 (1753). *C. agraria*, Kunth, Enum. iv. 38 (1843); Hook. f. (3), 179; Anderss. (1), 158, & (2), 56; Rob. & Greenm. (1), 148. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass & Heller*, nos. 56 (hb. Gr.), 103 (hb. Gr.). CHARLES ISL.: *Darwin*. DUNCAN ISL.: *Baur*, no. 238 (hb. Gr.). JAMES ISL.: *Darwin*. Further distrib. general in the tropics.

Commelinacea? Caruel (1), 621. — Charles Isl.: *Chierchia*. Same as the preceding?

AMARYLLIDACEAE.

Hypoxis, L.

H. decumbens, L. Amoen. Acad. v. 396 (1759), & Syst. Nat. ed. 10, 986; Baker, Jour. Linn. Soc. xvii. 107. *H. erecta*, Hook. f. (3), 180; Anderss. (1), 158, & (2), 57; Rob. & Greenm. (1), 148, not L. — Charles Isl.: *Darwin*. Chatham Isl.: southwest end, upper region, *Baur*, no. 239 (hb. Gr.). Further distrib. Mex., W. Ind., trop. of S. Am.

ORCHIDACEAE.

Epidendrum, L.

E. spicatum, Hook. f. (3), 180; Anderss. (1), 158, & (2), 57. — Charles Isl.: *Lee* (hb. U. S. Nat. Mus.), fruiting specimen, the identity not quite certain. James Isl.: *Darwin*. Endemic.

PIPERACEAE.

Peperomia, R. & P.

P. flagelliformis, Hook. f. ex. Miq. in Hook. Lond. Jour. Bot. iv. 423 (1845), & (3), 181; Anderss. (1), 158, & (2), 57. — James Isl.: *Darwin*. Endemic.

P. galapagensis, Hook. f. ex. Miq. in Hook. Lond. Jour. Bot. iv. 426 (1845), & (3), 180; Anderss. (1), 158, & (2), 57. — James Isl.: *Darwin*. Endemic.

P. galioides, HBK. Nov. Gen. & Sp. i. 71, t. 17 (1815). — Abingdon Isl.: alt. 510 m., *Snodgrass* & *Heller*, no. 837 (hb. Gr.). Determined by Mr. Casimir de Candolle. Further distrib. Mex., trop. S. Am.

P. petiolata, Hook. f. (3), 181; Anderss. (1), 158, & (2), 57. — James Isl.: *Darwin*. Endemic.

P. ramulosa, Anderss. (1), 158, & (2), 57. — Charles Isl.: on tree trunks at the summit of the mountain, *Andersson*. Endemic.

P. **Snodgrassii**, C. DC. nov. sp., " foliis ternis-quaternis brevissime petiolatis a basi acuta ellipticis apice obtusis vel oblongo-obovatis utrinque dense pilosulis, amentis apice ramulorum verticillatis pedunculatis ipsis folia fere aequantibus, bractea orbiculari centro subsessili, ovario emerso ovato paullo sub apice stigmatifero, stigmate minuto et glabro. — Herbula ramulis ad 3 cm. longis et circa 1 mm. crassis, dense pilosulis. Foliorum limbi in sicco membranacei ad 7 mm. longi et ad 3½ mm. lati. Amenta confertiflora in sicco rufescentia pedunculis 3 mm. longis, rhachis glabra. Ovarium sessile." — ALBEMARLE ISL.: Iguana Cove, on trees and bushes, *Snodgrass* & *Heller*, no. 130 (hb. Gr. and hb. C. DC.). Endemic.

P. nov. sp. — ALBEMARLE ISL.: mountain east of Tagus Cove, alt. 925 m., *Snodgrass* & *Heller*, no. 232 (hb. Gr.). Determined by Mr. Casimir de Candolle. Endemic.

P. sp. Rob. & Greenm. (1), 148. — CHATHAM ISL.: *Baur*. (Specimen not in hb. Gr. but sent as unicate to hb. Clark Univ.).

URTICACEAE.

FLEURYA, Gaud.

F. AESTUANS, Gaud. in Freyc. Voy. Bot. 497 (1826): Anderss. (2), 58. *F. aestuans*, var. *tuberculata* (poorly marked), Wedd. in DC. Prodr. xvi. pt. 1, 72 (1869); Rob. & Greenm. (1), 148. *F. cordata*, Gaud. l. c. 498, & Bot. Bonite, t. 83; Wedd. Arch. Mus. ix. 110; Anderss. (2), 57. ? *Urtica canadensis*, Hook. f. (3), 182; Anderss. (1), 160; not L. *U. divaricata*, Hook. f. (3), 181; Anderss. (1), 159; not L. *U. latifolia*, & *tuberculata*, Anderss. (1), 159, & (2), 57, 58. *U. racemosa*, Anderss. (2), 57. ? *Laportea canadensis*, Anderss. (2), 58, not Gaud. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 55 (hb. Gr.), 100 (hb. Gr.); southern portion, *Baur*, no. 240 (hb. Gr.); Tagus Cove, tolerably common in shady places from the beach to 300 m. alt., *Snodgrass* & *Heller*, no. 186 (hb. Gr.). CHARLES ISL.: *Darwin; Andersson* (hb. Gr.). CHATHAM ISL.: *Snodgrass* & *Heller*, no. 525 (hb. Gr.). JAMES ISL.: *Darwin*, acc. to Anderss. l. c. under *Laportea canadensis*, but not mentioned by Hooker f.; James Bay, abundant in lava soil under bushes, *Snodgrass* & *Heller*, no. 375 (hb. Gr.). NARBOROUGH ISL.: southern portion, *Snodgrass* & *Heller*, no. 339 (hb. Gr.). Further distrib. trop. S. Am.

PARIETARIA, L.

P. DEBILIS, G. Forst. Fl. Ins. Aust. Prodr. 73 (1786). *P. floridana*, Hook. f. (3), 182 (see Wedd. Arch. Mus. ix. 516); Anderss. (1), 160, & (2), 58. — CHARLES ISL.: *Darwin*. JAMES ISL.: *Darwin*. Further distrib. trop. Am., Asia, Pacific Ids.

PILEA, Lindl.

P. **Baurii**, nov. sp., annua erecta debilis; caule glaberrimo rufo-brunneo parce ramoso; foliis oppositis graciliter petiolatis ovatis 3-nervis acutis basi obtusis vel rotundatis obtuse serratis tenuibus supra viridibus obscure adpresso-pubescentibus nervis leviter impressis subtus paullo pallidioribus pubescentibus nervis plus minusve prominentibus; stipulis late ovatis apice rotundatis strigosis; cymis masculis oppositis axillaribus compositis quam petiolum multo brevioribus leviter recurvatis, foeminis ignotis. — *P.* sp. Rob. & Greenm. (1), 148. — CHARLES ISL.: June, 1891, *Baur*, no. 241 (hb. Gr.). CHATHAM ISL.: southwest end, middle region, June, 1891, *Baur*, no. 242 (hb. Gr.). Endemic. Leaves 1.7 to 4 cm. long, three-fourths as broad; petioles 1.3 to 1.8 cm. long; staminate cymes 8 to 11 mm. long. A species with the habit and foliage much as in *P. pubescens*, Poir., but the inflorescence of *P. peploides*, HBK. PLATE 2, FIG. 1.

P. MUSCOSA, Lindl. Coll. Bot. t. 4; Wedd. Arch. Mus. ix. 174. *P. succulenta*, Hook. f. (3), 182; Anderss. (1), 160, & (2), 58; not Wedd. (see Hook. f. & Jacks. Ind. Kew. i. 524). *Parietaria microphylla*, L. Syst. Nat. ed. 10, 1308 (1760). — JAMES ISL.: *Darwin*, acc. to Hook. f., l. c. Further distrib. trop. S. Am., Mex., W. Ind.

P. PEPLOIDES, Hook. & Arn. Bot. Beech. 96 (1832); Hook. f. (3), 182; Wedd. Arch. Mus. ix. 179; Anderss. (1), 160, & (2), 58. *Dubreulia peploides*, Gaud. in Freyc. Voy. Bot. 495 (1826). — JAMES ISL.: *Darwin*. Further distrib. Pacific Ids., Asia.

LORANTHACEAE.

PHORADENDRON, Nutt.

P. **florianum**. *Viscum florianum*, Anderss. (1), 219, & (2), 92. — CHARLES ISL.: in middle region, *Andersson*. Endemic.

P. **galapageium**. *Viscum galapageium*, Hook. f. (3), 216; *V. galapagejum*, Anderss. (1), 220, & (2), 92. — GALAPAGOS IDS.: *Habel*. CHATHAM ISL.: *Darwin*; *Andersson*. Endemic.

P. **Henslovii**. *Viscum Henslovii*, Hook. f. (3), 216; Anderss. (1), 219, & (2), 92; Rob. & Greenm. (1), 148. — ABINGDON ISL.: *Snodgrass & Heller*, no. 825 (hb. Gr.). ALBEMARLE ISL.: Iguana Cove, *Snodgrass & Heller*, no. 40 (hb. Gr.); mountain east of Tagus Cove, *Snodgrass & Heller*, no. 228 (hb. Gr.). CHARLES ISL.: *Darwin*.

CHATHAM ISL.: southwest end, upper region, *Baur*, no. 286 (hb. Gr.). Endemic.

P. **uncinatum**, nov. sp., glabrum lignoso-carnosum; caulibus teretibus olivaceo-viridibus dichotomo ramosis; ramis arcuatis etiam teretibus gracilibus; internodiis elongatis; foliis oppositis breviter petiolatis lanceolatis vel ovati-lanceolatis integris 5-nervatis margine ad apicem falcato-uncinatam revoluta basi cuneata; spicis in dichotomis terminalibus elongatis saepius 7-nodosis; floribus 4-seriatim immersis; calycis segmentis late deltoideis obtusis. — NARBOROUGH ISL.: southern part, common on "palo santo" and coffee trees, at 300 to 600 m. alt., April, 1899, *Snodgrass* & *Heller*, no. 325 (hb. Gr.). Endemic. Leaves 7 cm. long, 1.6 to 2.3 cm. broad. Spikes 7 cm. long, segments 1.5 cm. long; sheath short, sharply 2-pointed. PLATE 1, FIG. 1.

POLYGONACEAE.

POLYGONUM, L.

P. ACUMINATUM, HBK. Nov. Gen. & Spec. ii. 178 (1817); Meisn. in Mart. Fl. Bras. v. pt. 1, 14, t. 4, & in DC. Prodr. xiv. 114; Griseb. Fl. Brit. W. Ind. 161; Small, Monog. Gen. Polyg. 52, t. 13. — GALAPAGOS IDS.: acc. to Griseb. l. c. Further distrib. trop. S. Am., Centr. Am., W. Ind.

P. GALAPAGENSE, Caruel (1), 624. — CHATHAM ISL.: *Chierchia* (hb. Kew.). Endemic.

CHENOPODIACEAE.

ATRIPLEX, L.

A. sp. — INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 676 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 571 (hb. Gr.).

A. sp. — WENMAN ISL.: *Snodgrass* & *Heller*, no. 9 (hb. Gr.).

AMARANTACEAE.

ALTERNANTHERA, Forsk.

A. RADICATA, Hook. f. (4), 261, 262; Anderss. (1), 164, & (2), 60, t. 3, f. 2. — *A. acaulis*, Anderss. (1), 164. — CHATHAM ISL.: *Darwin*; in upper very dry region, *Andersson* (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 753 (hb. Gr.). Endemic.

A. RIGIDA, Rob. & Greenm. (1), 143, 148. — JAMES ISL.: Orchilla Bay, *Baur*, no. 387 (hb. Gr.). Endemic.

A. SUBSCAPOSA, Hook. f. (3), 189; Moq.-Tand. in DC. Prodr. xiii. pt. 2, 353; Anderss. (1), 164 & (2), 60. — CHARLES ISL.: *Darwin.* Endemic.

AMARANTHUS, L.

A. CARACASANUS, HBK. Nov. Gen. & Sp. ii. 195 (1817); Hook. f. (3), 189; Anderss. (1), 161, & (2), 58; Rob. & Greenm. (1), 147. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 77 (hb. Gr.); on beach, not common, *Snodgrass* & *Heller*, no. 108 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 219 (hb. Gr.); southern portion, *Baur*, no. 287 (hb. Gr.). CHARLES ISL.: *Darwin;* in cultivated ground, *Andersson; Snodgrass* & *Heller*, no. 421 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.), but not recorded in his works; *Snodgrass* & *Heller*, no. 516 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson.* Further distrib. S. Am.

A. CELOSIOIDES, HBK. Nov. Gen. & Sp. ii. 194 (1817); Moq.-Tand. in DC. Prodr. xiii. pt. 2, 257; Hook. f. (3), 189; Anderss. (1), 161, & (2), 59. — CHARLES ISL.: *Darwin; Andersson.* CHATHAM ISL.: *Andersson.* — Further distrib. northern S. Am. A very doubtful species in need of further study. I have seen none of the specimens upon which the records of its occurrence in the Galapagos Islands rests.

A. SCLERANTOIDES, Anderss. (2), 59, t. 2, f. 1; Rob. & Greenm. (1), 139, 147. *Euxolus sclerantoides*, Anderss. (1), 163. — GALAPAGOS IDS.: *Habel.* BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 469 (hb. Gr.). CHARLES ISL.: in lower dry region, *Andersson.* NARBOROUGH ISL.: eastern part, *Snodgrass* & *Heller*, no. 354 (hb. Gr.). Endemic.

Forma CHATHAMENSIS, Rob. & Greenm. (1), 140. — CHATHAM ISL.: southwest end, lower region, *Baur*, no. 289 (hb. Gr.). Endemic.

Forma HOODENSIS, Rob. & Greenm. l. c. — GARDNER ISL.: *Snodgrass* & *Heller*, no. 635 (hb. Gr.). HOOD ISL.: *Baur*, no. 288 (hb. Gr.), *Snodgrass* & *Heller*, no. 731 (hb. Gr.). Endemic.

A. SPINOSUS, L. Sp. 991 (1753); Anderss. (1), 161, & (2), 59. — CHARLES ISL.: cultivated ground of lower region, *Andersson.* Doubtless an introduction, not seen since on the islands. Further distrib. general in warm and temperate countries.

A. SQUARRULOSUS, Uline & Bray, Bot. Gaz. xix. 270 (1894); Rob. & Greenm. (1), 147. *Amblogyne squarrulosa*, Gray, Proc. Am. Acad. v. 169 (1861). *Scleropus squarrulosus*, Anderss. ined. ex Gray, l. c.,

and Uline & Bray, l. c. *S. squamulatus*, Anderss. (1), 162, & (2), 60. — ALBEMARLE ISL.: quite common near beach, Tagus Cove, *Snodgrass & Heller*, no. 183 (hb. Gr.). CHARLES ISL.: *Snodgrass & Heller*, no. 422 (hb. Gr.). CHATHAM ISL.: in grassy places at low altitude, *Andersson* (hb. Gr.). DUNCAN ISL.: *Snodgrass & Heller*, no. 691 (hb. Gr.). INDEFATIGABLE ISL.: northern part, *Snodgrass & Heller*, no. 663 (hb. Gr.). JERVIS ISL.: *Baur*, no. 390 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass & Heller*, no. 570 (hb. Gr.). Endemic. Unfortunately Dr. Gray misread Andersson's label and altered the specific name.

A. URCEOLATUS, Benth. Bot. Sulph. 158 (1844); Uline & Bray, Bot. Gaz. xix. 269. *Sarratia urceolata*, Moq.-Tand. in DC. Prodr. xiii. pt. 2, 269 (1849). *Scleropus urceolatus*, Anderss. (1), 162. *Amblogyna urceolata*, Moq.-Tand. in DC. Prodr. xiii. pt. 2, 270; Anderss. (2), 59, t. 2, f. 2; Gray, Proc. Am. Acad. v. 168 (1861), as *Amblogyne*. — INDEFATIGABLE ISL.: grassy places at low altitude, *Andersson*. Further distrib. adj. S. Am., from Peru northward. Lower Calif.

A. VIRIDIS, L. Sp. ed. 2, 1405 (1763); Uline & Bray, Bot. Gaz. xix. 319. — BARRINGTON ISL.: *Snodgrass & Heller*, 481 (hb. Gr.). CHATHAM ISL.: *Snodgrass & Heller*, no. 529 (hb. Gr.). Probably of recent introduction. Further distrib. general in warm countries.

FROELICHIA, Moench.

F. JUNCEA, Rob. & Greenm. (1), 143, 148. — ALBEMARLE ISL.: mountain north of Elizabeth Bay, *Snodgrass & Heller*, no. 290 (hb. Gr.); southern portion, *Baur*, no. 388 (hb. Gr.); mountain east of Tagus Cove, alt. 925 m., *Snodgrass & Heller*, no. 243 (hb. Gr.). Endemic.

F. LANIGERA, Anderss. (2), 63. *F. lanata*, Anderss. (2), t. 3, f. 1. — ALBEMARLE ISL.: *Andersson*; Tagus Cove, *Snodgrass & Heller*, no. 895 (hb. Gr.). Endemic.

F. NUDICAULIS, Hook. f. (3), 192; Moq.-Tand. in DC. Prodr. xiii. pt. 2, 423; Anderss. (1), 170, & (2), 63, t. 4, f. 3. — CHARLES ISL.: *Darwin*; *Andersson* (hb. Gr.). CHATHAM ISL.: *Andersson*. Endemic.

F. **scoparia**, nov. sp. *F. lanigerae* valde affinis; caulibus decumbentibus basi plus minusve lignosis ramosissimis subteretibus glabratis cortice lutescente tectis; ramis ramulisque erectis teretibus gracilibus sparse adpresso-lanatis, internodiis elongatis quam folii lineares acuti patentes sparse adpresso-lanati multo longioribus; pedunculis terminalibus nudis elongatis; capitulis terminalibus paucifloris subglobosis verticillastro

uno paullo inferiori vel evoluto vel nullo; bracteis ovatis glabriusculis brevibus obtusis; perigonio ovoideo-conico; laciniis ovatis obtusis stramineis basi lanatis supra glabratis induratis. — NARBOROUGH ISL.: southern part, abundant at 600 m. alt., *Snodgrass* & *Heller*, March, 1899, no. 346 (hb. Gr.), and April, 1899, no. 322 (hb. Gr.). This species may prove a variety of *F. lanigera*, Anderss., with which it shares many characteristics. It is, however, taller, considerably more slender and less lanate. The leaves are longer, narrower, and more acute, and the heads smaller and in general fewer-flowered.

IRESINE, L.

I. EDMONSTONEI, Hook. f. (3), 190; Moq.-Tand. in DC. Prodr. xiii. pt. 2, 343; Anderss. (1), 164, & (2), 60. — CHARLES ISL.: *Darwin.* Endemic? A plant in herb. Gray, labelled "*Iresine Edmonstonei*, Hook. f. Guayaquil? Mr. Edmonston," answers well to the description of this species.

PLEUROPETALUM, Hook. f.

P. DARWINII, Hook. f. (1), t. 2, (3), 221, & Bot. Mag. cix. under t. 6674, where second species is figured and genus is referred as by Endlicher and by Moquin to *Amarantaceae;* Anderss. (1), 225, & (2), 96. *Allochlamys Darwinii*, Moq.-Tand. in DC. Prodr. xiii. pt. 2, 463. — ALBEMARLE ISL.: Iguana Cove near beach, *Snodgrass* & *Heller*, no. 646 (hb. Gr.). JAMES ISL.: *Darwin.* Endemic. The single specimen of this problematic plant secured by Messrs. Snodgrass & Heller is rather fragmentary but corresponds closely with the characterization and figure (Journ. of Bot.) published by Hooker. In its dense (although fruiting) inflorescence, relatively small leaves, 4 to 5 carpels and 6 to 8 stamens, it would seem sufficiently distinct from *P. costaricense* [H. Wendl.] Hemsl. Biol. Cent.-Am. Bot. iii. 12, figured by Hook. f. Bot. Mag. 6674. The congeneric *Melanocarpum Sprucei*, Hook. f. Gen. Pl. iii. 24 of Ecuador is said to have 2 to 3 carpels and 5 stamens.

TELANTHERA, R. Br.

T. ECHINOCEPHALA, Moq.-Tand. in DC. Prodr. xiii. pt. 2, 373 (1849); Anderss. (1), 167, & (2), 62; Rose (1), 137; Rob. & Greenm. (1), 148. *T. argentea*, Anderss. (1), 168. *T.* sp. Rose (1), 137. *Brandesia echinocephala*, Hook. f. (3), 189. — GALAPAGOS IDS.: *Habel.* ABINGDON ISL.: *Baur*, no. 299 (hb. Gr.); *Snodgrass* & *Heller*, no. 850 (hb. Gr.). ALBEMARLE ISL.: southern portion, *Baur*, no. 300 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 31 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 467 (hb. Gr.). CHARLES ISL.: *Darwin; Andersson; A. Agassiz; Snodgrass* & *Heller*, no. 414 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.); *A. Agassiz* (hb. U. S. Nat. Mus.); southwest end, lower region, *Baur*, no. 301 (hb. Gr.); *Snodgrass* & *Heller*, no. 497 (hb. Gr.). DUNCAN ISL.: *A. Agassiz* (hb. Gr. & hb. U. S. Nat. Mus.); *Baur*, no. 298 (hb. Gr.); *Snodgrass* & *Heller*, no. 711 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 624 (hb. Gr.). HOOD ISL.: *Baur*, no. 297 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson;* northern part, *Snodgrass* & *Heller*, no. 680 (hb. Gr.). JAMES ISL.: *Andersson:* James Bay, *Baur*, no. 296 (hb. Gr.); abundant, shrubby, spreading, 1 to 1.3 m. high, *Snodgrass* & *Heller*, no. 360 (hb. Gr.). Endemic. With the far more copious material now at hand I have been unable to follow the formal subdivision of this species suggested by Andersson or to correlate the trifling variations with occurrence upon the different islands.

T. FILIFOLIA, Moq.-Tand. in DC. Prodr. xiii. pt. 2, 368 (1849); Anderss. (1), 165, & (2), 61. *Bucholtzia filifolia*, Hook. f. (3), 192. — JAMES ISL.: *Scouler*. Endemic.

T. FLAVICOMA, Anderss. (1), 166, & (2), 61, t. 5, f. 2; Rob. & Greenm. (1), 148, in part (as to pl. Hood Isl.). — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 826 (hb. Gr.), identity doubtful. CHARLES ISL.: in dry grassy places at middle altitudes, *Andersson*. GARDNER ISL.: *Snodgrass* & *Heller*, no. 617 (hb. Gr.). HOOD ISL.: *Baur*, no. 291 (hb. Gr.); *Snodgrass* & *Heller*, no. 733 (hb. Gr.). Endemic.

T. FRUTESCENS, Moq.-Tand. in DC. Prodr. xiii. pt. 2, 265 (1849) q. v. for synon.; Caruel (1), 625. *Illicebrum frutescens*, L'Her. Stirp. Nov. iv. 75, t. 37 (1785). *Alternanthera frutescens*, R. Br. acc. to Spreng. Syst. i. 819 (1825), not Rob. & Greenm. (1), 148. — CHATHAM ISL.: *Chierchia*, acc. to Caruel, l. c. Further distrib. W. S. Am., Panama to Caracas and Chili.

T. GLAUCESCENS, Moq.-Tand. in DC. Prodr. xiii. pt. 2, 369 (1849); Anderss. (1), 167, & (2), 62, t. 5, f. 3. *Bucholtzia glaucescens*, Hook. f. (3), 191. — CHARLES ISL.: in the middle region, *Andersson*. CHATHAM ISL.: *Darwin:* in grassy places and dry parts of the lower region, *Andersson* (hb. Gr.). Endemic.

T. **Helleri**, nov. sp., stellato-tomentella grisea novellis subochraceis; ramis dichotomis teretibus gracilibus; internodiis quam folii multo longioribus; foliis late ovalibus vel orbicularibus apice rotundatis vel obtusis vel

in superioribus acutis, basi subtruncato, lamina 2–2.6 cm. longa 2.4–3 cm. lata penninervia, subtus haud vel paullo pallidiore ; petiolo 3–4 mm. longo recurvato : capitulis in dichotomis sessilibus ovoideis 4–6 mm. longis 4 mm. crassis apice rotundatis ochraceo-pubescentibus ; floribus arcte imbricatis obcompressis triquetris ; squamis exterioribus tribus, quam interiores multo brevioribus lanceolatis vel ovato-lanceolatis acutis, in parte superiore ochraceo-pubescentibus, carina subrigida, marginibus tenue membranaceis, squamis interioribus (sepalis) 3 ovato-lanceolatis acutis rigidioribus opacis 3.5 mm. longis ; staminibus 5 glabris, filamentis gracillimis 1.7 mm. longis, antheris 1.3 mm. longis ; staminodeis 5 tenuibus filamento subaequantibus, apice argute 2–3-partitis ; ovario obovoideo, stylo breve crasso cum stigmate depresse-globoso. — Culpepper Isl. : 10 December, 1898, *Snodgrass* & *Heller*, no. 1 (hb. Gr.). A species to be readily distinguished from its Galapageian congeners by its broadly ovate or suborbicular stem-leaves. Endemic. Plate 1, figs. 5, 6, and 7.

Var. **obtusior**, nov. var. Conspicue ochracea ; foliis omnibus latissime oblongis vel ovato-oblongis obtusissimis. — Wenman Isl. : 14 December, 1898, *Snodgrass* & *Heller*, no. 7 (hb. Gr.). Endemic. Plate 1, fig. 8.

T. nudicaulis, Moq.-Tand. in DC. Prodr. xiii. pt. 2, 369 (1849); Anderss. (1), 167, & (2), 62, t. 5, f. 1 ; Rob. & Greenm. (1), 148. *T. flavicoma*, Rob. & Greenm. (1), 148, in part (as to pl. Chatham Isl.). *Bucholtzia nudicaulis*, Hook. f. (3), 191. — Galapagos Ids. : without further data (hb. Gr.). Albemarle Isl. : eastern portion, Cowley Bay, *Baur*, no. 294 (hb. Gr.); southern part, *Baur*, no. 293 (hb. Gr.) ; on mountain east of Tagus Cove, alt. 925 m., *Snodgrass* & *Heller*, no. 249 (hb. Gr.). Charles Isl. : *Darwin ;* in very dry places, *Andersson* (hb. Gr.). Chatham Isl. : northern part, *Baur*, no. 292 (hb. Gr.), leafless and doubtful. Duncan Isl. : *Snodgrass* & *Heller*, no. 710 (hb. Gr.). James Isl. : Orchilla Bay, *Baur*, no. 295 (hb. Gr.). Further distrib. S. Chili, *J. G. Reynolds* (hb. Gr.) !

T. **rugulosa**, nov. sp., frutex arborescens ; ramulis teretibus dichotomis griseis et breviter stellato-tomentellis mox glabrescentibus ; foliis oppositis graciliter petiolatis oblongis penninerviis supra rugulosis tenuissime stellato-pubescentibus subtus multo pallidioribus canescentibus tomentellis basi cuneatis apice rotundatis vel obtusis apiculatisque ; spicis in dichotomis sessilibus ovoideis vel breviter teretibus albidis ; bracteis lanceolatis acutissimis apice plus minusve reflexis vel uncinatis ; sepalis exterioribus lanceolatis hyalinis acutissimis glabris carina rigidiuscula ; interioribus ovati-lanceolatis basi atrobrunneis valde nervatis et inter nervis albo-pubescentibus apice recto vel leviter recurvato; antheris linearibus staminodiis linearibus paullo indivisis superantibus. — *Alternanthera frutescens*, Rob. & Greenm. (1), 148, not R. Br. — CHATHAM ISL.: southwest end, middle region, June, 1891, *Baur*, no. 290 (hb. Gr.). Endemic. Leaf-blade 3 to 5 cm. long, 1.2 to 2 cm. broad; spikes 8 to 11 mm. long, 6 to 9 mm. in thickness. Nearly related to *T. frutescens*, Moq. in DC. Prodr. xiii. pt. 2, 365, but differing in the contour and rugosity of its leaves.

T. **Snodgrassii**, nov. sp., suffrutescens a basi valde ramosa; ramis oppositis vel alternis teretibus leviter striatis pallidis breviter adpresse et sordide pubescentibus vel novellis canescentibus tomentulosis ad nodos paullo incrassatis supra nodis non constrictis; foliis spatulatis adpresse pubescentibus erectis apice obtusis basi cuneatis sessilibus; spicis sessilibus ovoideis parvis albidis; bracteis late ovatis carinatis hyalinis acutis cuspidatis glabris adpressis apice rectis sepalis exterioribus bracteis similibus sub apice pubescentibus interioribus longioribus ovatis acutis basi atro-brunneis valde nervatis in parte superiore patente flavescentipilosis; filamentis filiformibus staminodiis oblongis apice laceratis. — SEYMOUR ISL.: north, May, 1899, *Snodgrass* & *Heller*, no. 578 (hb. Gr.). Endemic. Leaves 2.4 to 5 cm. long, 7 to 10 mm. broad; spikes 4 to 5 mm. in diameter. PLATE 2, FIG. 2.

T. STRICTIUSCULA, Anderss. (1), 166. *T. angustata*, Anderss. (2), 61, t. 4, f. 2; Rob. & Greenm. (1), 148. *T.* sp. Rose, (1), 137. — GALAPAGOS IDS.: *Habel.* ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 52 (hb. Gr.), 88 (hb. Gr.), 863 (hb. Gr.); southern portion, *Baur*, no. 302 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.); *A. Agassiz* (hb. Gr.); southwest end, lower region, *Baur*, no. 303 (hb. Gr.); *Snodgrass* & *Heller*, no. 502 (hb. Gr.). NARBOROUGH ISL.: southern part, not common, *Snodgrass* & *Heller*, no. 347 (hb. Gr.). Endemic. In his second publication Andersson credits this species to James Island, instead of Chatham Island as in his original description. A portion of the type material in hb. Gray is, however, labelled Chatham Island, where furthermore the species has been rediscovered by several subsequent collectors.

T. VESTITA, Anderss. (1), 169, & (2), 63, t. 4, f. 1; Rob. & Greenm. (1), 148. — INDEFATIGABLE ISL.: in dry grassy places of the middle region, *Andersson*; south of Conway Bay, *Baur*, no. 304 (hb. Gr.). Endemic.

BATIDACEAE.

BATIS, L.

B. MARITIMA, L. Syst. Nat. ed. 10, 1376 (1760). — JAMES ISL.: James Bay, near salt ponds in sand, *Snodgrass* & *Heller*, no. 355 (hb. Gr.). Widely distrib. on trop. shores.

BASELLACEAE.

BOUSSINGAULTIA, HBK.

B. BASELLOIDES, HBK. Nov. Gen. & Sp. vii. 196, t. 645 (1825); Hook. f. (3), 193; Anderss. (1), 227, & (2), 97. — CHARLES ISL.: *Darwin*. Further distrib. Mex., W. Ind., trop. S. Am.

PHYTOLACCACEAE.

PHYTOLACCA, L.

P. DECANDRA, L. Sp. ed. 2, 631 (1763); Hook. f. (3), 193; Anderss. (1), 227, & (2), 97. — JAMES ISL.: *Darwin*, acc. to Hook. f. Further distrib. U. S. and Canada. It is probable that the Galapageian plant, which curiously enough has not been secured by any subsequent collector, will prove to be *P. octandra* L., or a related trop. American species.

NYCTAGINACEAE.

BOERHAAVIA, L.

B. ERECTA, L. Sp. 3 (1753); Hook. f. (3), 193; Anderss. (1), 170, & (2), 64. — ALBEMARLE ISL.: *Macrae*. CHATHAM ISL.: *Andersson* (hb. Gr.). INDEFATIGABLE ISL.: *Andersson*.

B. PANICULATA, Rich. Act. Soc. Hist. Nat. Par. i. 105 (1792); Anderss. (1), 171, & (2), 64. *B. decumbens*, Vahl, Enum. i. 284 (1804); Hook. f. (3), 193. — JAMES ISL.: *Darwin*, and acc. to Andersson, l. c. (but probably by error) *Scouler*. Further distrib. S. United States, Mex., trop. S. Am. The identity of the Galapageian plant is doubtful.

B. SCANDENS, L. Sp. 3 (1753); Hook. f. (3), 193; Anderss. (1), 170, & (2), 64. — CHARLES ISL.: *Darwin;* stony, grassy ground in the middle and lower region, *Andersson* (hb. Gr.); *Snodgrass* & *Heller*, no. 427 (hb. Gr.). CHATHAM ISL.: *Andersson; Snodgrass* & *Heller*, no. 527 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson*. JAMES ISL.: *Darwin*. Further distrib. S. United States, Mex., W. Ind., S. Am.

B. viscosa, Lag. & Rod. Anal. Cienc. Nat. iv. 256 (1801); *B. hirsuta*, Hook. f. (3), 193; Choisy in DC. Prodr. xiii. pt. 2, 451, in part; Anderss. (1), 170, & (2), 64; Rob. & Greenm. (1), 147; not Jacq., nor L., nor Willd. *B. diffusa*, Anderss. (1), 171, & (2), 64, not L. *B. glandulosa*, Anderss. (1), 171, & (2), 64. — Albemarle Isl.: *Macrae; Andersson;* Tagus Cove, common in tufa soil up to 300 m., *Snodgrass* & *Heller*, no. 170 (hb. Gr.); mountain east of Tagus Cove, *Snodgrass* & *Heller*, no. 252 (hb. Gr.). Charles Isl.: in grassy, stony places, *Andersson* (hb. Gr.); *Lee* (hb. U. S. Nat. Mus.); southwest end lower region, *Baur*, no. 309 (hb. Gr.); *Snodgrass* & *Heller*, no. 428 (hb. Gr.). Chatham Isl.: *Andersson* (hb. Gr.); *Baur; Snodgrass* & *Heller*, no. 543 (hb. Gr.). Gardner Isl.: *Snodgrass* & *Heller*, no. 622 (hb. Gr.). Indefatigable Isl.: northern part, *Snodgrass* & *Heller*, no. 670 (hb. Gr.). James Isl.: James Bay, on bluffs near the beach, *Snodgrass* & *Heller*, no. 377 (hb. Gr.). Although I have not seen original material of Andersson's *B. glandulosa* I find plants from Albemarle collected by Messrs. Snodgrass & Heller, which correspond so closely with Andersson's description that I cannot avoid the conclusion that they represent just what Andersson had. Nevertheless the plants seem to be only starved and dwarfed *B. viscosa*, growing as Andersson says *in locis sterilissimis.* Further distrib. S. W. United States, Mex., W. S. Am. southward to Chili.

Cryptocarpus, HBK.

C. pyriformis, HBK. Nov. Gen. & Sp. ii. 188, t. 124 (1817); Hook. f. (3), 193; Anderss. (1), 161, & (2), 58; Rob. & Greenm. (1), 147. — Galapagos Ids.: *Edmonston* (hb. Gr.); *Habel.* Albemarle Isl.: Elizabeth Bay, on lava fields near beach, *Snodgrass* & *Heller*, no. 941 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 912 (hb. Gr.). Barrington Isl.: *Baur*, acc. to Rob. & Greenm. l. c. (specimen not in hb. Gr., but sent to hb. Clark Univ.). Bindloe Isl.: *Snodgrass* & *Heller*, no. 786 (hb. Gr.). Charles Isl.: *Baur*, acc. to Rob. & Greenm. l. c. (specimen not in hb. Gr., but sent to hb. Clark Univ.). Chatham Isl.: *Darwin.* Hood Isl.: *Baur*, no. 306 (hb. Gr.). James Isl.: James Bay, *Baur*, no. 305 (hb. Gr.); forming dense thickets 1 to 1.3 m. high on lava coast, *Snodgrass* & *Heller*, no. 372 (hb. Gr.). Narborough Isl.: east side, *Snodgrass* & *Heller*, no. 922 (hb. Gr.). Further distrib. Ecuador, Bolivia.

PISONIA, L.

P FLORIBUNDA, Hook. f. (3), 193; Choisy in DC. Prodr. xiii. pt. 2, 447; Anderss. (1), 171, & (2), 65. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 65 (hb. Gr.). JAMES ISL.: *Darwin*. Endemic.

NYCTAGINACEA? — SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 574 (hb. Gr.). A weak, armed shrub with orbicular petiolate leaves recalling those of a *Boerhaavia*. Sterile and indeterminate.

AIZOACEAE.

MOLLUGO, L.

M. FLAVESCENS, Anderss. (1), 226, & (2), 96, t. 15, f. 2; Rob. & Greenm. (1), 146 as to pl. Chatham and pl. Albemarle. ? *M. verticillata*, Hook. f. (3), 232; Anderss. (1), 225, & (2), 96; not L. — ALBEMARLE ISL.: *Darwin*, identity doubtful; *Macrae*, identity doubtful; *Baur*, identity doubtful. CHARLES ISL.: *Snodgrass* & *Heller*, no. 441 (hb. Gr.). CHATHAM ISL.: *Darwin*, identity doubtful; *Andersson* (hb. Gr.); northern part, *Baur*, no. 155 (hb. Gr.); southwestern part, lower region, *Baur*, no. 154 (hb. Gr.). INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 667 (hb. Gr.). Endemic.

In all these specimens so far as seen the pedicels are rather short, some ascending, others deflexed. Distinguishable is

Var. **floriana**. *M. flavescens*, Rob. & Greenm. l. c. as to pl. Charles Isl. Pedicellis 4–9 mm. longis, omnibus erectis vel adscendentibus; floribus paulo quam illi formæ typicæ majoribus. — CHARLES ISL.: Cormorant Bay, *Baur*, no. 137. Endemic. In this variety as in the typical form the seeds are black, dull, and minutely tessellated.

M. GRACILLIMA, Anderss. (1), 226, & (2), 96; Rob. & Greenm. (1), 146. *M. gracilis*, Anderss. (2), t. 15, f. 3. — GALAPAGOS IDS.: *Habel*. ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 79 (hb. Gr.), 122 (hb. Gr.); Tagus Cove, abundant everywhere up to 300 m. alt., *Snodgrass* & *Heller*, no. 192 (hb. Gr.). CHARLES ISL.: in the driest parts of the lower region, *Andersson* (hb. Gr.). JAMES ISL.: Orchilla Bay, *Baur*, no. 153 (hb. Gr.). Endemic. By Hook. f. & Jacks. Ind. Kew. ii. 253, this species is reduced to *M. verticillata*, L. It differs, however, in several ways, namely by having smaller flowers, a shorter capsule relatively to the blunter sepals and in its minute black tessellated, not ribbed seeds.

M. **Snodgrassii,** nov. sp., perennans; radice crassa; caudice ramosa crassa nodosa plus minusve suberifera; caulibus pluribus erectis ramosis nodosis nunc geniculatis subscopariis teretibus striatulis glaberrimis 3 dm. altis; internodiis perlongis (saepe 7 cm.); foliis crassiusculis, radicalibus spatulatis 1.6 cm. longis, 6 mm. latis apice rotundatis caulinibus anguste linearibus obtusis vel acutis glaberrimis vel minutissime papillosis 1–2 cm. longis 1.2 mm. latis, umbellis 2–6 floribus sessilibus; pedicellis 4–15 mm. longis capillaribus; sepalis ellipticis obtusis 3(–5)-nerviis 4 mm. longis staminibus 7 sepalis paulo superantibus; filamentis gracilibus glaberrimis; ovario ovoideo obtusissimo; stylis 3 filiformibus; stigmatibus plus minusve elongatis subrecurvatis; capsula 3 mm. longa; seminibus subglobosis nigrescentibus glaberrimis lucidis sub lenti lineas concentrice curvatas exhibentibus. — ALBEMARLE ISL.: Elizabeth Cove, 15 February, 1899, *Snodgrass* & *Heller*, no. 268 (hb. Gr.). Tagus Cove, 20 January, 1899, *Snodgrass* & *Heller*, no. 914 (hb. Gr.). NARBOROUGH ISL.: Mangrove Point, common, scattered everywhere in crevices of lava from near the beach to 600 m. alt., April, 1899, *Snodgrass* & *Heller*, no. 309 (hb. Gr., type); east side, *Snodgrass* & *Heller*, no. 921 (hb. Gr.). Endemic. Well marked among its Galapageian congeners by its perennial habit, suberiferous base, long pedicels, large flowers, etc.

SESUVIUM, L.

S. EDMONSTONEI, Hook. f. (3), 221; Anderss. (1), 225, & (2), 95; Rob. & Greenm. (1), 146. — GALAPAGOS IDS.: *Edmonston* (hb. Gr.); *Habel*. BARRINGTON ISL.: *Baur*, no. 151 (hb. Gr.). CHARLES ISL.: *Goodridge*. GARDNER ISL.: *Snodgrass* & *Heller*, no. 638 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 561 (hb. Gr.). Endemic.

S. PORTULACASTRUM, L. Syst. Nat. ed. 10, 1058 (1760). *S. Portulacastrum*, var. *revolutum*, Sims, Bot. Mag. xli. t. 1701. *Portulaca Portulacastrum*, L. Sp. 446 (1753). — ALBEMARLE ISL.: Elizabeth Cove, *Snodgrass* & *Heller*, no. 265 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 561 a (hb. Gr.). Further distrib. S. United States, Mex., W. Ind., S. Am., China, etc.

TRIANTHEMA, L.

T. PORTULACASTRUM, L. Sp. 223 (1753); Moench, Meth. 700. *T. monogyna*, L. Mant. i. 69 (1767); DC. Pl. Gras. t. 109; Anderss. (1), 225, & (2), 95; Rob. & Greenm. (1), 146. — BARRINGTON ISL.:

Snodgrass & *Heller*, no. 480 (hb. Gr.). CHARLES ISL.: *Andersson* (hb. Gr.); Cuevas Bay, *Baur*, no. 152 (hb. Gr.); *Snodgrass* & *Heller*, no. 409 (hb. Gr.). CHATHAM ISL.: *Andersson*. DUNCAN ISL.: *Snodgrass* & *Heller*, nos. 713 (hb. Gr.), 714 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 630 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 737 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson*. JAMES ISL.: *Andersson*. SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 575 (hb. Gr.); south, *Snodgrass* & *Heller*, no. 588 (hb. Gr.). Further distrib. S. United States, Mex., W. Ind., S. Am., E. Ind.

PORTULACACEAE.

PORTULACA, L.

P. OLERACEA, L. Sp. 445 (1753). *P. parvifolia*, Haw. Syn. Pl. Succ. 122 (1812); Anderss. (1), 225, & (2), 95. — GALAPAGOS IDS.: *Habel*. ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 30 (hb. Gr.). CHARLES ISL.: in cultivated ground of lower region, *Andersson*; *A. Agassiz* (hb. U. S. Nat. Mus.); *Snodgrass* & *Heller*, no. 409a (hb. Gr.). CHATHAM ISL.: *Snodgrass* & *Heller*, no. 528 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, nos. 619 (hb. Gr.), 620 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 735 (hb. Gr.). Further distrib. cosmop. None of the earlier explorers in the Galapagos Ids. mention this weed of cultivated ground. It was first noted in the group by *Andersson* on Charles Isl. in 1852, and its present much more general distribution among the islands well shows the fact that intercommunication between them must now be such as to carry seeds frequently from one to the other.

P. sp.? — WENMAN ISL.: *Snodgrass* & *Heller*, no. 916 (hb. Gr.), sterile and indeterminate.

CARYOPHYLLACEAE.

DRYMARIA, Willd.

D. CORDATA, Willd. ex Roem. & Schultes, Syst. v. 406 (1819); Rob. & Greenm. (1), 145. *D. glaberrima*, Bart. in J. & C. Presl, Rel. Haenk. 7 (1835), so far as char. goes; Hook. f. (3), 232; Anderss. (1), 227, & (2), 97; Caruel (1), 623. — CHATHAM ISL.: *Chierchia*; southwest end, upper region, *Baur*, no. 3 (hb. Gr.). JAMES ISL.: *Darwin*. From Bartling's detailed description of his *D. glaberrima*, I cannot avoid the conclusion that it is *D. cordata*. Further distrib. Mex., W. Ind., S. Am.

MENISPERMACEAE.

CISSAMPELOS, L.

C. PAREIRA, L. Sp. 1031 (1753); Lam. Ill. t. 830; Hook. f. (3), 233; Anderss. (1), 220, & (2), 92; Caruel (1), 624; Rob. & Greenm. (1), 145. *C. testudinum*, Miers, Contrib. iii. 143 (1864–1871). — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 849 (hb. Gr.). ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 67 (hb. Gr.), 91 (hb. Gr.), 871 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, nos. 221 (hb. Gr.), 902 (hb. Gr.). CHARLES ISL.: *Darwin;* in the upper region, *Andersson; Chierchia; Baur*, no. 2 (hb. Gr.). CHATHAM ISL.: southwest end, upper region, *Baur*, no. 1 (hb. Gr.). JAMES ISL.: *Darwin.* Further distrib. general in trop. regions. Miers, l. c., separates the Galapageian plant as *C. testudinum*, but good distinctions are not evident.

CRUCIFERAE.

BRASSICA, L.

B. CAMPESTRIS, L. Sp. 666 (1753). — CHARLES ISL.: *Snodgrass* & *Heller*, no. 406 (hb. Gr.). Further distrib. cosmop.

B. SINAPISTRUM, Boiss. Voy. Esp. ii. 39 (1839–1845). *Sinapis arvensis*, L. Sp. 668 (1753); Anderss. (1), 220, & (2), 93. — CHARLES ISL.: cultivated ground in upper and middle region, *Andersson.* Further distrib. cosmop.

RAPHANUS, L.

R. SATIVUS, L. Sp. ed. 2, 935 (1763); Anderss. (1), 220, & (2), 93. — CHARLES ISL.: cultivated ground in middle and upper regions, *Andersson.* Further distrib. cosmop. through cult.

SENEBIERA, Poir.

S. PINNATIFIDA, DC. Mém. Soc. Hist. Nat. Par. vii. 144, t. 9 (1799); Hook. f. (3), 233; Anderss. (1), 221, & (2), 93. *S. didyma*, Pers. Syn. ii. 185 (1807). *Lepidium didymum*, L. Mant. 92 (1767). *Coronopus didymus*, Smith, Fl. Brit. ii. 691. — JAMES ISL.: *Darwin.* Further distrib. cosmop.

ROSACEAE?

SANGUISORBEA? Wolf (1), 283, & (2), 259. — CHARLES ISL.: acc. to Wolf. Probably an error. Plant described as thorny and with pinnate leaves.

LEGUMINOSAE.

ACACIA, Willd.

A. FARNESIANA, Willd. Sp. iv. 1083 (1806). *A. Cavenia*, Bert. ex Bull. Férussac, xx. 108 (1830), acc. to Hook. f. & Jacks. Ind. Kew. i. 7; Hook. & Arn. Bot. Beech. 21 (1830); Hook. f. (3), 228; Anderss. (1), 255, & (2), 113. — ALBEMARLE ISL.: *Darwin; Macrae.* Further distrib. extra trop. S. Am., also the Texano-Mexican region?

A. MACRACANTHA, H. & B. in Willd. Sp. iv. 1080 (1806); Kunth, Mimos. t. 28; Benth. Trans. Linn. Soc. xxx. 500; Rob. & Greenm. (1), 146. *A. flexuosa*, H. & B. in Willd. l. c. 1082 (1806); Hook. f. (3), 229; Anderss. (1), 256, & (2), 113. — GALAPAGOS IDS.: *Habel.* CHARLES ISL.: *Baur; Snodgrass & Heller*, no. 415 (hb. Gr.). CHATHAM ISL.: *Andersson.* INDEFATIGABLE ISL.: south of Conway Bay, *Baur*, no. 94 (hb. Gr.). JAMES ISL.: *Scouler.* Further distrib. S. Am., W. Ind. An *Acacia* mentioned by Caruel (1), 623, as collected on Charles Isl., by *Chierchia* may well have been this species.

A. TORTUOSA, Willd. Sp. iv. 1083 (1806); Benth. Trans. Linn. Soc. xxx. 501; Rob. & Greenm. (1), 146. *A. tortuosa*, var. *glabrior*, Hook. f. (3), 229; Anderss. (1), 256, & (2), 113. *A. albida*, Lindl. Bot. Reg. t. 1317 (1830). — CHARLES ISL.: grassy places, middle region, *Andersson.* CHATHAM ISL.: similar situations, *Andersson.* JAMES ISL.: *Scouler; Darwin;* Orchilla Bay, *Baur*, no. 65 (hb. Gr.); James Bay, *Baur*, no. 64 (hb. Gr.). Further distrib. Mex., W. Ind., northern S. Am.

A. sp. affin. *A. macracantha*, H. & B. — ALBEMARLE ISL.: mountain north of Elizabeth Cove, *Snodgrass & Heller*, no. 283 (hb. Gr.); Tagus Cove, *Snodgrass & Heller*, no. 161 (hb. Gr.). In its minute leaflets, velvety branchlets, and small spines this is very different from the preceding or indeed from any specimen of *A. macracantha* at hand. The species, however, has been so widely drawn by Bentham, l. c., and contains such an aggregation of old species, that it is impossible to characterize with confidence any new species of this affinity until the whole group can be worked over again.

A. sp.? Hook. f. (4) 261. — CHARLES ISL.: *Edmonston*, acc. to Hook. f., l. c.

A. sp. — JAMES ISL.: *Snodgrass & Heller*, no. 398 (hb. Gr.). Near *A. tortuosa*, but with larger leaflets.

ASTRAGALUS, L.

A. **Edmonstonei.** *Phaca Edmonstonei,* Hook. f. (3), 227; Anderss. (1), 249, & (2), 110. — GALAPAGOS IDS.: *Du Petit-Thouars.* CHARLES ISL.: *Edmonston.* Endemic. Not secured by Andersson or any subsequent collector.

CAESALPINIA, L.

C. BONDUCELLA, Fleming in As. Res. xi. 159 (1810); Rob. & Greenm. (1), 146. — ALBEMARLE ISL.: *Baur,* no. 79 (hb. Gr.). Further distrib. general in warm reg.

C. PULCHERRIMA; Sw. Obs. 166 (1791); Maund, Bot. iv. t. 151. *Poinciana pulcherrima,* L. Sp. 380 (1753); Hook. f. (4), 261; Anderss. (1), 253, & (2), 112; Caruel (1), 624. — CHARLES ISL.: *Edmonston,* as a relic of former cultivation acc. to Andersson. CHATHAM ISL.: *Chierchia.* Further distrib. cosmop. trop.

CANAVALIA, Adans.

C. OBTUSIFOLIA, DC. Prodr. ii. 404 (1825); Mart. Fl. Bras. xv. pt. 1, 178, t. 48. *Dolichos obtusifolius,* Lam. Dict. ii. 295 (1786). — GALAPAGOS IDS.: *Habel.* BINDLOE ISL.: *Snodgrass & Heller,* no. 770 (hb. Gr.). Further distrib. general in tropics.

CASSIA, L.

C. HIRSUTA, L. Sp. i. 378; Benth. in Mart. Fl. Bras. xv. pt. 2, 114, to 34, f. 1, & Trans. Linn. Soc. xxx. 534. — CHARLES ISL.: *Lee* (hb. U. S. Nat. Mus.). Further distrib. Peru to Mex. and Brazil. Professor Lee's plant is only in flower and the identification is subject to some doubt.

C. OCCIDENTALIS, L. Sp. 377 (1753); DC. Prodr. ii. 497; Benth. Trans. Linn. Soc. xxvii. 532; Lindl. Bot. Reg. t. 83; Anderss. (1), 254, & (2), 112; Caruel, (1), 624; Rose (1), 137; Rob. & Greenm. (1), 146. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass & Heller,* nos. 68 (hb. Gr.), 95 (hb. Gr.). CHARLES ISL.: in grassy places of the upper region, *Andersson; Snodgrass & Heller,* no. 425 (hb. Gr.), a form with unusually obtuse leaflets. CHATHAM ISL.: in grassy places of the middle region, *Andersson; Chierchia; A. Agassiz* (hb. Gr. & hb. U. S. Nat. Mus.); southwest end, middle region, *Baur,* no. 85 (hb. Gr.); *Snodgrass & Heller,* no. 556 (hb. Gr.). Widely distrib.

C. PICTA, Don, Syst. ii. 444 (1832); Hook. f. (3), 229; Anderss. (1), 255, & (2), 112; Benth. Trans. Linn. Soc. xxvii. 552; Rose (1), 137. *C. applanata*, Anderss. (1), 254, acc. to Benth. l. c. — ALBEMARLE ISL.: *Macrae;* Iguana Cove, *Snodgrass* & *Heller*, no. 48 (hb. Gr.); Tagus Cove, not common near the shore but abundant inland at 150 m. alt., flowers Feb. to the middle of Mar., *Snodgrass* & *Heller*, no 169. CHATHAM ISL.: *Darwin; Andersson* (hb. Gr.); *A. Agassiz.* Further distrib. Ecuador.

C. SERICEA, Sw. Prodr. 66 (1788), & Fl. Ind. Occ. 724; Benth. in Mart. Fl. Bras. xv. pt. 2, 116, t. 35, f. 1; Anderss. (1), 254, & (2), 112. — CHATHAM ISL.: *Andersson.* INDEFATIGABLE ISL.: lower region, *Andersson.* SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 607 (hb. Gr.). Further distrib. Mex., W. Ind., trop. S. Am.

CROTALARIA, L.

C. GLABRESCENS, Anderss. (1), 248, & (2), 109; Rose (1), 137. — GALAPAGOS IDS.: *Habel.* ALBEMARLE ISL.: Tagus Cove, *Snodgrass* & *Heller*, no. 176 (hb. Gr.). CHATHAM ISL.: *Andersson; ?A. Agassiz* acc. to Rose, l. c. Endemic.

C. PUMILA, Ort. Dec. ii. 23 (1797); DC. Prodr. ii. 132; Anderss. (2), 109. *C. lupulina*, HBK. Nov. Gen. & Sp. vi. 402, t. 590 (1823); DC. Prodr. ii. 133; Hook. f. (3), 225; Anderss. (1), 248, & (2), 109. *C. puberula*, Hook. f. (3), 225; Anderss. (1), 248, & (2), 109; Rob. & Greenm. (1), 146. — GALAPAGOS IDS.: *Habel.* ALBEMARLE ISL.: *Darwin;* Iguana Cove, *Snodgrass* & *Heller*, no. 98 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, nos. 174 (hb. Gr.), 878 (hb. Gr.). CHARLES ISL.: *Darwin;* in the upper region, *Andersson; Snodgrass* & *Heller*, no. 435 (hb. Gr.). CHATHAM ISL.: in grassy places of the middle region, *Andersson;* southwest end, middle region, *Baur*, no. 99 (hb. Gr.); southwest end, upper region, *Baur*, no. 98 (hb. Gr.); *Snodgrass* & *Heller*, no. 554 (hb. Gr.). JAMES ISL.: James Bay, common everywhere, *Snodgrass* & *Heller*, no. 391 (hb. Gr.). NARBOROUGH ISL.: southern part, *Snodgrass* & *Heller*, no. 323 (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 610 (hb. Gr.). Further distrib. W. Ind., Mex.

C. SETIFERA, DC. Prodr. ii. 131 (1825); A. DC. Calques des Dess. t. 226. — ALBEMARLE ISL.: Tagus Cove, alt. 1230 m., *Snodgrass* & *Heller*, nos. 879 (hb. Gr.), 882 (hb. Gr.). Further distrib. Mex., Centr. Am.

DALEA, L.

D. PARVIFOLIA, Hook. f. (3), 225; Anderss. (1), 249, & (2), 109; Rob. & Greenm. (1), 146. — ALBEMARLE ISL.: Tagus Cove, on mountain, alt. 215 to 460 m., bush 1 to 2 m. high, *Snodgrass & Heller*, no. 222 (hb. Gr.). CHARLES ISL.: *Baur*, no. 102 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.). INDEFATIGABLE ISL.: south of Conway Bay, *Baur*, no. 104 (hb. Gr.). JAMES ISL.: *Darwin; Baur*, no. 103 (hb. Gr.). Endemic.

D. TENUICAULIS, Hook. f. (3), 226; Anderss. (1), 249, & (2), 110; Rob. & Greenm. (1), 146. — ALBEMARLE ISL.: *Darwin;* southern part, *Baur*, no. 105 (hb. Gr.). CHATHAM ISL.: *Andersson.* Endemic.

DESMANTHUS, Willd.

D. DEPRESSUS, H. & B. ex Willd. Sp. iv. 1046 (1806); Kunth, Mimos. 115, t. 35; Hook. f. (3), 228; Anderss. (1), 255, & (2), 113; Benth. Trans. Linn. Soc. xxx. 386; Rose (1), 137; Rob. & Greenm. (1), 146. — CHARLES ISL.: *Darwin.* CHATHAM ISL.: *A. Agassiz*, acc. to Rose, l. c.; southwestern end, middle region, *Baur*, no. 77 (hb. Gr.). DUNCAN ISL.: *Snodgrass & Heller*, no. 704 (hb. Gr.). GARDNER ISL.: *Snodgrass & Heller*, no. 627 (hb. Gr.). HOOD ISL.: *Baur*, no. 78 (hb. Gr.); *Snodgrass & Heller*, no. 749 (hb. Gr.), no. 750 (hb. Gr.). Further distrib. S. United States, Mex., W. Ind., S. Am.

DESMODIUM, Desv.

D. **galapagense**. *D. filiforme*, Hook. f. (3), 227; Anderss. (1), 249, & (2), 110; not Zoll. & Mor. — JAMES ISL.: *Darwin.* Endemic.

D. INCANUM, DC. Prodr. ii. 332 (1825); Caruel, (1), 624; Rob. & Greenm. (1), 146. — CHATHAM ISL.: *Chierchia;* southwest end, middle region, *Baur*, no. 71 (hb. Gr.); also a questioned variety of this species, *Chierchia*, acc. to Caruel, (1), 624. Further distrib. Mex., W. Ind., S. Am.

D. MOLLE, DC. Prodr. ii. 332; Anderss. (1), 250, & (2), 110; Rob. & Greenm. (1), 146. — ABINGDON ISL.: *Snodgrass & Heller*, no. 824 (hb. Gr.). ALBEMARLE ISL.: rather common in tufa soil up to 185 m. alt., *Snodgrass & Heller*, no. 203 (hb. Gr.). BINDLOE ISL.: *Baur*, no. 83 (hb. Gr.); *Snodgrass & Heller*, no. 765 (hb. Gr.). CHARLES ISL. *Andersson; Baur.* GARDNER ISL.: *Snodgrass & Heller*, no. 618 (hb. Gr.). HOOD ISL.: *Baur*, no. 84 (hb. Gr.); *Snodgrass & Heller*, no. 727 (hb. Gr.). Indefatigable Isl.: *Baur;* northern part, *Snodgrass* & *Heller*, no. 652 (hb. Gr.). Jervis Isl.: *Baur*, no. 81 (hb. Gr.). Not noticed by Darwin and the earlier collectors, and found only on one island (Charles) by Andersson; hence probably a recently introduced although now generally distributed plant in the islands. Further distrib. general in trop. and sub-trop. Am.

D. spirale, DC. Prodr. ii. 332 (1825); Benth. in Mart. Fl. Bras. xv. pt. 1, 105; Rob. & Greenm. (1), 146. *D. tenuiculum*, DC. Prodr. ii. 333 (1825); Anderss. (1), 249, & (2), 110. — Albemarle Isl.: Tagus Cove, tolerably common in shady places, from the beach to 300 m. alt., *Snodgrass* & *Heller*, no. 193 (hb. Gr.). Bindloe Isl.: *Baur* (hb. Gr.). Charles Isl.: on dry places of middle region, *Andersson; Snodgrass* & *Heller*, no. 444 (hb. Gr.). Gardner Isl.: *Snodgrass* & *Heller*, no. 644 (hb. Gr.). Hood Isl.: *Snodgrass* & *Heller*, no. 734 (hb. Gr.). James Isl.: on rocks near beach, *Snodgrass* & *Heller*, no. 379 (hb. Gr.). Seymour Isl.: south, *Snodgrass* & *Heller*, no. 609 (hb. Gr.). Further distrib. Mex., W. Ind., trop. S. Am.

D. uncinatum, DC. Prodr. ii. 331 (1825); Rob. & Greenm. (1), 146. — Chatham Isl.: southwest end, middle region, *Baur*, no. 93 (hb. Gr.). Further distrib. N. Am., S. Am.

Erythrina, L.

E. velutina, Willd. Ges. Naturf. Fr. Neue Schr. iii. 426 (1801); Hook. Bot. Mag. lx. t. 3227; Rob. & Greenm. (1), 146. — James Isl.: James Bay, *Baur*, no. 72 (hb. Gr.). Further distrib. S. Am., W. Ind. This is doubtless the red-seeded tree mentioned in Nature, vi. 353 as observed on James Island by the members of the Hassler expedition.

Galactea, P. Br.

G. Jussiaeana, Kunth, var. volubilis, Benth. in Mart. Fl. Bras. xv. pt. 1, 143 (1859). *G. velutina*, Benth. in Hook. Jour. Bot. ii. 59 (1840); Anderss. (2), 111. *Phaseolus tomentosus*, Anderss. (1), 250. — Albemarle Isl.: Iguana Cove, *Snodgrass* & *Heller*, no. 63 (hb. Gr.); Tagus Cove, not common, *Snodgrass* & *Heller*, no. 208 (hb. Gr.). Charles Isl.: *Andersson.* Chatham Isl.: *Andersson* (hb. Gr.). Indefatigable Isl.: northern part, *Snodgrass* & *Heller*, no. 666 (hb. Gr.). Narborough Isl.: northern part, *Snodgrass* & *Heller*, no.

300 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 576 (hb. Gr.). Further distrib. S. Am.

Var. GLABRESCENS, Benth. in Mart. Fl. Bras. xv. pt. 1, 143 (1859). CHATHAM ISL.: *Snodgrass* & *Heller*, no. 544 (hb. Gr.). Further distrib. Brazil.

G. n. sp., Hook. f. (4), 261. — CHARLES ISL.: *Edmonston*, acc. to Hook. f.

GEOFFRAEA, L.

G. SUPERBA, H. & B. Pl. Aequin. ii. 69, t. 100 (1809) as *Geoffroya*. — HOOD ISL.: *Snodgrass* & *Heller*, no. 738 (hb. Gr.). Further distrib. trop. S. Am. Not before reported from the Galapagos Ids.

MIMOSA, L.

M. ASPERATA, L. Syst. Nat. ed. 10, 1312 (1760); Benth. Trans. Linn. Soc. xxx. 437; Hook. f. (4), 262; Anderss. (1), 256, & (2), 113. *M. pellita*, Kunth, Mimos. 27, t. 9 (1819). — CHARLES ISL.: *Edmonston*. Further distrib. S. United States, Mex., W. Ind., S. Am., Afr., and Madagascar.

NEPTUNIA, Lour.

N. PLENA, Benth. in Hook. Jour. Bot. iv. 355 (1842), & Trans. Linn. Soc. xxx. 383. *N. surinamensis*, Steud. Flora, xxvi. 759 (1843); Anderss. (1), 255, & (2), 113; Rob. & Greenm. (1), 146. Perhaps the *Piptadenia ?* sp. of Hook. f. (4), 262. — CHARLES ISL.: *Edmonston ? Andersson ;* Cuevas Bay, *Baur*, no. 69 (hb. Gr.). CHATHAM ISL.: in grassy places of the middle region, *Andersson; Baur; Snodgrass* & *Heller*, no. 495 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 626 (hb. Gr.). INDEFATIGABLE ISL.: south of Conway Bay, *Baur*, no. 66 (hb. Gr.); northern part, *Snodgrass* & *Heller*, no. 658 (hb. Gr.). JERVIS ISL.: *Baur*, no. 68 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 577 (hb. Gr.); south, *Snodgrass* & *Heller*, no. 603 (hb. Gr.). Further distrib. Mex., W. Ind., S. Am.

PARKINSONIA, L.

P. ACULEATA, L. Sp. 375 (1753); Benth. in Mart. Fl. Bras. xv. pt. 2, 76, t. 26; Hook. f. (4), 262; Anderss. (1), 253, & (2), 112; Rose (1), 137; Rob. & Greenm. (1), 146. — GALAPAGOS IDS.: *Habel*. ALBEMARLE ISL.: southern part, *Baur*, no. 75 (hb. Gr.). CHARLES ISL.: *Edmonston ;* Cuevas Bay, *Baur*, no. 74 (hb. Gr.); *Snodgrass* & *Heller*, no. 405 (hb. Gr.). CHATHAM ISL.: *Andersson ; A. Agassiz ;*

southwestern part, lower region, *Baur*, no. 73 (hb. Gr.); *Snodgrass* & *Heller*, no. 498 (hb. Gr.). DUNCAN ISL.: *Baur*, no. 76 (hb. Gr.); *Snodgrass* & *Heller*, no. 693 (hb. Gr.). HOOD ISL.: *Baur*, no. 80 (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 606 (hb. Gr.). Further distrib. S. United States, Mex., W. Ind., S. Am.

PHASEOLUS, L.

P. ADENANTHUS, G. F. W. Mey. Prim. Fl. Esseq. 239 (1818). *P. truxillensis*, HBK. Nov. Gen. & Sp. vi. 451 (1823). — GALAPAGOS IDS.: *Habel.* HOOD ISL.: *Snodgrass* & *Heller*, no. 754 (hb. Gr.). Further distrib. tropics of the New World, also introd. (?) in E. Ind.

P. MOLLIS, Hook. f. (3), 228; Anderss. (1), 250, & (2), 110; ? Rob. & Greenm. (1), 146. — JAMES ISL.: *Darwin.* JERVIS ISL.: *Baur*, no. 86. (hb. Gr.), identity somewhat doubtful. Endemic.

P. SEMIERECTUS, L. Mant. i. 100 (1767); Jacq. Ic. t. 558; Bot. Reg. ix. t. 743; Rob. & Greenm. (1), 146. *P. cytisoides*, Anderss. (1), 251, & (2), 110, not Zoll. & Mor. — CHARLES ISL.: *Andersson* (hb. Gr.); *Lee* (hb. U. S. Nat. Mus.); *Baur*, no. 88 (hb. Gr.); *Snodgrass* & *Heller*, no. 424 (hb. Gr.). CHATHAM ISL.: *Baur*; *Snodgrass* & *Heller*, no. 555 (hb. Gr.). Further distrib. general in the tropics.

PISCIDIA, L.

P. ERYTHRINA, L. Sp. ed. 2, 993 (1763); Hook. f. (3), 228; Anderss. (1), 253, & (2), 112. — CHATHAM ISL.: *Darwin*; *Snodgrass* & *Heller*, no. 503 (hb. Gr.). JAMES ISL.: ? Further distrib. trop. Am. There can be little doubt that this is the species with winged pods, mentioned in Nature, vi. 353, as observed by the members of the Hassler expedition.

PROSOPIS, L.

P. DULCIS, Kunth, Mimos. 110, t. 34 (1819); Hook. f. (3), 229; Anderss. (1), 255, & (2), 112; Rob. & Greenm. (1), 146. — GALAPAGOS IDS.: *Habel.* ABINGDON ISL.: *Snodgrass* & *Heller*, no. 852 (hb. Gr.). CHARLES ISL.: *Darwin*; *Snodgrass* & *Heller*, no. 416 (hb. Gr.). CHATHAM ISL.: *Baur* (hb. Gr.); *Snodgrass* & *Heller*, no. 531 (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller*, nos. 694 (hb. Gr.), 695, (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 623 (hb. Gr.).

HOOD ISL.: *Snodgrass* & *Heller*, no. 739 (hb. Gr.). INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 677 (hb. Gr.). JAMES ISL.: James Bay, abundant near beach in sand and lava soil, *Snodgrass* & *Heller*, no. 366 (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 586 (hb. Gr.). Further distrib. Mex., S. Am.

RHYNCHOSIA, Lour.

R. MINIMA, DC. Mém. Leg. ix. 363 (1825), & Prodr. ii. 385; Hook. f. (3), 228; Anderss. (1), 251, & (2), 111; Rob. & Greenm. (1), 146. *R. punctata*, DC. Mém. Leg. ix. t. 56, & Prodr. ii. 385; Anderss. (1), 252, & (2), 111. *R. aureo-guttata*, Anderss. (1), 252, & (2), 111. *R. exigua*, Anderss. (1), 252, & (2), 111. — GALAPAGOS IDS.: *Habel*. ABINGDON ISL.: *Snodgrass* & *Heller*, no. 833 (hb. Gr.). ALBEMARLE ISL.: *Darwin*; Iguana Cove, *Snodgrass* & *Heller*, nos. 62 (hb. Gr.), 121 (hb. Gr.); Elizabeth Bay, north mountain, *Snodgrass* & *Heller*, no. 286 (hb. Gr.); Tagus Cove, abundant from the beach to 300 m. alt., *Snodgrass* & *Heller*, nos. 207 (hb. Gr.), 901 (hb. Gr.), 154 (hb. Gr.), the last number representing the small-leaved form (*R. exigua* of Andersson). BARRINGTON ISL.: *Baur*, no. 100 (hb. Gr.); *Snodgrass* & *Heller*, no. 477 (hb. Gr.). BINDLOE ISL.: *Baur*, no. 101 (hb. Gr.). *Snodgrass* & *Heller*, no. 760 (hb. Gr.). CHARLES ISL.: *Andersson* (hb. Gr.). CHATHAM ISL.: *Darwin*; *Andersson* (hb. Gr.); *Snodgrass* & *Heller*, no. 546 (hb. Gr.). INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 651 (hb. Gr.). NARBOROUGH ISL.: southern part, common up to 615 m. alt., *Snodgrass* & *Heller*, no. 313 (hb. Gr.). Further distrib. cosmop. trop. and subtrop.

R. RETICULATA, DC. Prodr. ii. 385 (1825); Hook. f. (3), 227; Anderss. (1), 251, & (2), 111. — CHATHAM ISL.: *Darwin*. Further distrib. trop. Am.

R. sp. — ALBEMARLE ISL.: mountain east of Tagus Cove, *Snodgrass* & *Heller*, no. 250 (hb. Gr.), also below 650 m., *Snodgrass* & *Heller*, no. 905 (hb. Gr.)

R. sp. — BINDLOE ISL.: *Snodgrass* & *Heller*, no. 773 (hb. Gr.). Sterile and doubtful.

STYLOSANTHES, Sw.

S. SCABRA, Vog. Linnaea, xii. 69 (1838); Taubert, Abh. Bot. Ver. Prov. Brandenb. xxxii. 24; Rob. & Greenm. (1), 146. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 838 (hb. Gr.). ALBEMARLE ISL.:

Tagus Cove, common up to 185 m. alt., *Snodgrass* & *Heller*, no. 171 (hb. Gr.). BINDLOE ISL.: *Baur*, no. 90 (hb. Gr.); *Snodgrass* & *Heller*; no. 767 (hb. Gr.). CHARLES ISL.: *Baur*, no. 92 (hb. Gr.); *Snodgrass* & *Heller*, no. 450 (hb. Gr.). INDEFATIGABLE ISL.: *Baur* (no specimen in hb. Gr.); northern part, *Snodgrass* & *Heller*, no. 673 (hb. Gr.). JERVIS ISL.: *Baur*, no. 89 (hb. Gr.). Further distrib. Centr. & S. Am. Taubert, l. c., distinguishes the Galapageian plant seen in the Vienna Herbarium (coll. *Steindachner?*) as being villous-pubescent rather than scabrous. The material of the species at hand shows variation in its pubescence even upon the islands.

TEPHROSIA, Pers.

T. CINEREA, Pers. Syn. ii. 328 (1807). *T. litoralis*, Pers. l. c. 329; Hook. f. (3), 226 (*littoralis*); Anderss. (1), 249, & (2), 110; Rob. & Greenm. (1), 146. — GALAPAGOS IDS.: *Du Petit-Thouars*, acc. to Hook. f.; *Habel*. ALBEMARLE ISL.: *Macrae*; Tagus Cove, not common, alt. 30 to 185 m., *Snodgrass* & *Heller*, no. 188 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 483 (hb. Gr.). BINDLOE ISL.: *Baur*, no. 97 (hb. Gr.); *Snodgrass* & *Heller*, no. 774 (hb. Gr.). CHARLES ISL.: in stony places of lower and middle region, *Andersson*; *Snodgrass* & *Heller*, no. 440 (hb. Gr.). CHATHAM ISL.: *Andersson*; *Baur* (not in hb. Gr.); *Snodgrass* & *Heller*, no. 545 (hb. Gr.). INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 682 (hb. Gr.). NARBOROUGH ISL.: abundant acc. to Mr. Heller's field notes. SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 601 (hb. Gr.). Further distrib. throughout trop. Am. The Galapageian plant often has a velvety spreading pubescence on the stem, but is apparently only a form of this common and widely distributed species.

VIGNA, Savi.

V. OWAHUENSIS, Vog. Linnaea, x. 585 (1836). *V. oahuensis*, A. Gray, Bot. U. S. Expl. Exped. i. 450. *V. owyhensis*, var., Hook. f. (3), 228. *V. owaihensis*, Anderss. (1), 251, & (2), 111. — JAMES ISL.: *Darwin*, a variety acc. to Hook. f., l. c., who also credits this Hawaiian species to Chili. The identity of the Galapageian plant with the typical Hawaiian needs further confirmation. Further distrib. Hawaiian Ids., "Chili." Acc. to Hemsl. in litt., a doubtful *Vigna* was secured on the Galapagos Islands by Dr. Habel.

OXALIDACEAE.

Oxalis, L.

O. carnosa, Molina, Sagg. Chile, ed. 2, 288 (1810); Lindl. Bot. Reg. t. 1063; Hook. Bot. Mag. t. 2866; Walp. Rep. i. 487; Anderss. (1), 247, & (2), 108; Rob. & Greenm. (1), 145. — Charles Isl.: at the summit, *Andersson.* Duncan Isl.: *Baur*, without number (hb. Gr.). Further distrib. Chili.

O. Cornelli, Anderss. (1), 246, & (2), 108. *O. Barrelieri*, Anderss. ll. cc., not L. *O. Agassizi*, Rose (1), 136; Rob. & Greenm. (1), 145. — Galapagos Ids.: *Habel.* Albemarle Isl.: Iguana Cove, *Snodgrass & Heller*, no. 78 (hb. Gr.), and abundant near beach, *Snodgrass & Heller*, no. 118 (hb. Gr.); Tagus Cove, *Snodgrass & Heller*, no. 197 (hb. Gr.). Barrington Isl.: *Snodgrass & Heller*, no. 471 (hb. Gr.). Charles Isl.: in sandy places, middle region, *Andersson* (hb. Gr.); *Baur* (no specimen in hb. Gr.). Chatham Isl.: in sterile parts of the upper region, *Andersson* (hb. Gr.); *Snodgrass & Heller*, no. 504 (hb. Gr.). Duncan Isl.: *A. Agassiz* (hb. Gr.); *Snodgrass & Heller*, no. 703 (hb. Gr.). Gardner Isl.: *Snodgrass & Heller*, no 621 (hb. Gr.). Hood Isl.: *Baur*, no. 35 (hb. Gr.); *Snodgrass & Heller*, no. 748 (hb. Gr.). Indefatigable Isl.: *Andersson;* south of Conway Bay, *Baur*, no. 36 (hb. Gr.); northern part, *Snodgrass & Heller*, no. 672 (hb. Gr.). James Isl.: James Bay, common on sea bluff, *Snodgrass & Heller*, no. 390 (hb. Gr.). Endemic.

O. corniculata, L. Sp. 435 (1753); Rob. & Greenm. (1), 145. — Chatham Isl.: southwest end, upper region, *Baur* (hb. Gr.). Further distrib. cosmop.

LINACEAE.

Linum, L.

L. oligophyllum, Willd. ex Schult. Syst. vi. 758 (1820); Urban, Linnaea, xli. 634. — Albemarle Isl.: Tagus Cove Mountain, 215 to 925 m. alt., *Snodgrass & Heller*, nos. 225 (hb. Gr.), 898 (hb. Gr.). Further distrib. Andes of Ecuador and Peru.

ZYGOPHYLLACEAE.

Kallstroemia, Scop.

K. adscendens. *Tribulus adscendens*, Anderss. (1), 245. *T. maximus*, var. *adscendens*, Anderss. (2), 107. *T. maximus*, Rose (1), 136. —

CHARLES ISL.: *Andersson.* CHATHAM ISL.: *Andersson* (hb. Gr.). DUNCAN ISL.: *A. Agassiz* (hb. U. S. Nat. Mus.). GARDNER ISL.: *Snodgrass & Heller*, no. 615 (hb. Gr.). HOOD ISL.: *Snodgrass & Heller*, no. 756 (hb. Gr.). Endemic. This species is by its much shorter style, etc., clearly distinct from *K. maximus*, Wight & Arn. (*Tribulus maximus*, L). It is, however, exceedingly close to the plant recently characterized from the southwestern United States as *Kallstroemia brachystylis*, Vail, Bull. Torr. Club, xxiv. 206 (*Tribulus brachystylis*, Robinson, Syn. Fl. i. pt. 1, 354). The Galapageian plant, however, differs from the continental in slightly more slender habit, smaller and more unequal leaflets (usually 2 or 3 rather than 4 pairs), and more enduring sepals which persist even at the maturity of the fruit. *K. brachystylis* has been found in Mexico, but apparently not in S. America.

TRIBULUS, L.

T. CISTOIDES, L. Sp. 387 (1753); Hook. f. (3), 231; Anderss. (1), 244, & (2), 107. — ALBEMARLE ISL.: *Macrae.* CHARLES ISL.: *Snodgrass & Heller*, no. 400 (hb. Gr.). HOOD ISL.: *Snodgrass & Heller*, no. 743 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson* (hb. Gr.). JAMES ISL.: *Darwin.* SEYMOUR ISL.: south, *Snodgrass & Heller*, no. 598 (hb. Gr.). Further distrib. general in warm and temperate countries.

Var. **anacanthus**, nov. var. Formae typicae habitu foliisque similis; floribus plus minusve minoribus; petalibus 1–1.2 cm. longis; fructus carpellis vel tribus vel quatuor vel saepius omnibus ab aculeis omnino destitutis. — ALBEMARLE ISL.: *Lee* (hb. Gr. & hb. U. S. Nat. Mus.). Tagus Cove, March, 1899, common near beach and on sandy hillsides up to 200 m. alt., *Snodgrass & Heller*, nos. 163 (hb. Gr.), 165 (hb. Gr.), & 915 (hb. Gr.). Endemic.

T. SERICEUS, Anderss. (1), 245, & (2), 107; Rob. & Greenm. (1), 145. *T. servicens*, var. *humifusus*, Rose, (1), 136. — CHARLES ISL.: in lower arid region, *Andersson* (var. *humifusus*); *A. Agassiz*; *Baur*, no. 39 (hb. Gr.). CHATHAM ISL.; in lower arid region, *Andersson* (var. *erectus*), (hb. Gr.). Endemic but too near the cosmopolitan *T. terrestris*, L.

T. sp. — CULPEPPER ISL.; *Snodgrass & Heller*, no. 3 (hb. Gr.). A velvety-tomentose species with short internodes and geniculate stem. The specimens are sterile and indeterminate.

RUTACEAE.

Zanthoxylum, L.

Z. Pterota, HBK. Nov. Gen. & Sp. vi. 3 (1823); Hook. f. (3), 231; Anderss. (1), 244, & (2), 106; Rob. & Greenm. (1), 145 (*Xanthoxylon*). *Z. lentiscifolium*, Anderss. ll. cc. *Fagara Pterota*, L. Syst. Nat. ed. 10, 897 (1760); Engl. & Prantl, Nat. Pflanzenf. iii. Ab. 4, 117. *F. lentiscifolia*, H. & B. acc. to Willd. Enum. Hort. Berol. 165. — Galapagos Ids.: *Habel.* Abingdon Isl.: alt. 525 m., *Snodgrass* & *Heller*, no. 836 (hb. Gr.). Albemarle Isl.: Iguana Cove, *Snodgrass* & *Heller*, nos. 53 (hb. Gr.), 903 (hb. Gr.); mountain east of Tagus Cove, alt. 925 m., *Snodgrass* & *Heller*, no. 231 (hb. Gr.). Charles Isl.: *Andersson*, identity doubtful; *Lee* (hb. U. S. Nat. Mus.). Chatham Isl.: southwest end, middle region, *Baur*, no. 40 (hb. Gr.). James Isl.: *Darwin.* — Narborough Isl.: southern part, small tree, abundant at 615 m. alt., *Snodgrass* & *Heller*, no. 333 (hb. Gr.). Further distrib. S. United States, Mex., W. Ind., S. Am.

SIMARUBACEAE.

Castela, Turp.

C. galapageia, Hook. f. (3), 229, & (4), 262; Rob. & Greenm. (1), 145. *C. galapageja*, Anderss. (1), 243, & (2), 106. *C. Galapagei*, Engl. & Prantl, Nat. Pflanzenf. iii. Ab. 4, 219.

This species, like *Euphorbia viminea* and several other Galapageian plants, shows pronounced formal differences on the different islands, thus:

Forma typica, inarmata; foliis lineari-lanceolatis acutis. — Chatham Isl.: *Darwin; Baur?*

Forma **albemarlensis**, inarmata vel aculeis parvis gracilibusque armatis; foliis oblongis obtusis mucronatis, 1.5–3.4 cm. longis, saepissime dentibus 1–2 parvis lateralibus instructis, supra haud lucidis. — Albemarle Isl.; southern part, *Baur*, no. 45 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, nos. 156 (hb. Gr.), 924 (hb. Gr.), the latter showing the hitherto undescribed fruit to be obovoid, 12 mm. long, 9 mm. thick, very smooth and shining.

Forma **bindloensis**, inarmata; foliis integerrimis, anguste obovatis, rotundatis vel mucronulatis, 1–1.6 cm. longis, supra haud lucidis. — Bindloe Isl.: *Baur*, no. 43 (hb. Gr.).

Forma **carolensis**, spinis brevibus robustioribus armata; foliis spatulatis basi manifeste attenuatis, demum supra valde lucidis 2–2.5 cm.

longis. — CHARLES ISL.: *Snodgrass* & *Heller*, no. 452 (hb. Gr.); *Baur*, no. 41 (hb. Gr.), and (?) *Baur*, no. 42 (hb. Gr.).

Forma **duncanensis**, inarmata; foliis anguste lanceolatis, base apiceque acutis. — DUNCAN ISL.: *Baur*, no. 47 (hb. Gr.). Perhaps only a young form.

Forma **jacobensis**, spinis perbrevibus armata; foliis late oblongis, integerrimis, basi apiceque rotundatis mucronatis supra lucidis et transverse rugulosis 1–1.3 cm. longis. — JAMES ISL.: a shrub 3 to 6 dm. high, in sand near beach, James Bay, *Snodgrass* & *Heller*, no. 364 (hb. Gr.).

Forma **jervensis**, spinis brevibus gracilibusque armata; foliis late oblongis integerrimis 1.7 cm. longis apice rotundatis margine valde revolutis supra haud lucidis. — JERVIS ISL.: *Baur*, no. 46 (hb. Gr.).

These forms possess, as will be seen from the above brief characters, rather striking differences. It is, however, very difficult to tell how much of this variation is due to permanent divergence of character and how much to individual environment and age. The species as a whole is very near *C. Nicholsonii*, Hook., but the branches do not exhibit the tendency to induration at the tip so conspicuous in the species just mentioned. *C. galapageia* was also collected by Habel acc. to Hemsl. in litt.

BURSERACEAE.

BURSERA, L.

B. GRAVEOLENS, Trian. & Planch. Ann. Sci. Nat. ser. 5, xiv. 303 (1872); Engl. in DC. Monogr. iv. 49. *Elaphrium graveolens*, HBK. Nov. Gen. & Sp. vii. 31 (1825). *E. Tacamaco*, Tul. Ann. Sci. Nat. ser. 3, vi. 368 (1846). *Spondias Edmonstonei*, Hook. f. (3), 230; Anderss. (1), 243, & (2), 106; Rob. & Greenm. (1), 145. *Guaiacum*, Baur, Am. Nat. xxv. 220. — GALAPAGOS IDS.: *Goodridge* (hb. Gr.); *Habel*. ABINGDON ISL.: common, large, extending from the beach to 350 m. alt., *Snodgrass* & *Heller*, no. 809a (hb. Gr.). ALBEMARLE ISL.: *Macrae;* southern part, *Baur*, no. 58 (hb. Gr.); Elizabeth Bay, abundant on the north shore, *Snodgrass* & *Heller*, no. 259 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 21 (hb. Gr.); mountain east of Tagus Cove, *Snodgrass* & *Heller*, no. 227 (hb. Gr.). BARRINGTON ISL.: small and leafless, 28 May, acc. to Mr. Heller. CHARLES ISL.: *Edmonston*, acc. to Hook. f., l. c. CHATHAM ISL.: rather common, acc. to Mr. Heller. HOOD ISL.: scattered and rather small, acc. to Mr. Heller. INDEFATIGABLE ISL.: *Steindachner* in hb. Vienna Mus., acc. to Engl. l. c.; common on the north coast, acc. to Mr. Heller. JAMES

Isl.: James Bay, abundant, acc. to Mr. Heller. Narborough Isl.: southern part, common, *Snodgrass & Heller*, no. 340 (hb. Gr.). Tower Isl.: common, 3 to 5 m. high, acc. to Mr. Heller. Further distrib. Peru to U. S. Colombia, Mex., W. Ind. This is the Palo Santo of the Galapagos Islands.

B. **malacophylla**, nov. sp. *B. graveolenti* affinis; ramis crassis lignosis fistulosis a cortice griseo-brunneo tectis apice foliosis; foliis 7–9-foliolatis ubique molliter tomentosis, petiolo gracili 2–3 cm. longo, rachide alato 4–5 cm. longo, aliis inter jugis inferioribus paulo dilatatis sub jugo supremo in auriculis rotundatis ampliatis, foliolis ovato-oblongis crenatis 2–3 cm. longis basi obliquis apice rotundatis; pedunculis fructiferis 3 vel pluribus 9 cm. longis simplicibus vel paulo ramosis; capsulis ovoideis compressis glabris 1.2 cm. longis 8 mm. latis. — Seymour Isl.: south, abundant, May, 1899, *Snodgrass & Heller*, no. 596 (hb. Gr.). Endemic. Plate 1, fig. 2. A Palo Santo of North Seymour Isl., entered in Mr. Heller's field notes as differing in foliage from the Palo Santo of the other islands, was probably this, although the only specimens received are labelled South Seymour.

POLYGALACEAE.

Polygala, L.

P. **Anderssonii**. *P. puberula*, Anderss. (1), 232, & (2), 100; Rob. & Greenm. (1), 145; not Gray. — Indefatigable Isl.: in dry places on the sides of the mountain, *Andersson;* south of Conway Bay, *Baur*, no. 13 (hb. Gr.). Endemic. This species is omitted from Professor Chodat's monograph. It is reduced by Mr. Bennett, Jour. Bot. xvii. 204, but as I think erroneously, to *P. galapageia*. It is noteworthy that both species occur upon the same island but no intergradation has been observed in the matter of pubescence.

P. galapageia, Hook. f. (3), 233; Bennett, Jour. Bot. xvii. 204; Rob. & Greenm. (1), 145. *P. galapageja*, Anderss. (1), 232, & (2), 100, t. 10, f. 1. *P. galopagensis*, Chodat, Monogr. Polyg. 230 (1893). *P. chathamensis*, Anderss. (1), 232, & (2), 99, t. 10, f. 3. *P. insularis*, Rob. & Greenm. (1), 145, in part. — Galapagos Ids.: *Habel.* Abingdon Isl.: *Snodgrass & Heller*, no. 844 (hb. Gr.). Albemarle Isl.: *Macrae;* eastern part, Cowley Bay, *Baur*, no. 12 (hb. Gr.); Tagus Cove, *Snodgrass & Heller*, nos. 142, 177 (both in hb. Gr.). Bindloe Isl.: *Baur*, no. 10 (hb. Gr.); *Snodgrass & Heller*, no. 763 (hb. Gr.). Charles Isl.: *Darwin; Andersson* (hb. Gr.); *Baur*, no. 11 (hb. Gr.).

CHATHAM ISL.: *Andersson;* southwest end, lower region, *Baur*, no. 9 (hb. Gr.); northern part, *Baur*, no. 7 (hb. Gr.); *Snodgrass* & *Heller*, no. 494 (hb. Gr.). INDEFATIGABLE ISL.: Conway Bay, *Baur*, no. 4 (hb. Gr.); northern part, *Snodgrass* & *Heller*, no. 668 (hb. Gr.). JERVIS ISL.: *Baur*, no. 6 (hb. Gr.). Endemic.

Var. **insularis**. P. OBOVATA, Hook. f. (3), 233; Anderss. (1), 231, & (2), 99, t. 10, f. 2; Rob. & Greenm. (1), 146; not St. Hil. *P. insularis*, Bennett, Jour. Bot. xvii. 204 (1879); Rob. & Greenm. (1), 146, in part. — Foliis latioribus, obovato-oblanceolatis vel obovatis, crassioribus; floribus minoribus, racemis brevibus. — GALAPAGOS IDS.: *Cuming*, no. 103, acc. to Bennett, l. c.; *Mr. Brace* (hb. Gr.). ALBEMARLE ISL.: *Macrae*, acc. to Bennett, l. c. CHARLES ISL.: *Edmonston; Andersson; Baur*. CHATHAM ISL.: *Darwin; Andersson* (hb. Gr.). JERVIS ISL.: *Baur*, no. 8 (hb. Gr.). Endemic. None of the distinctions between this and *P. galapageia*, given by Bennett, hold in the series of specimens now at hand. Andersson names two foliar forms (*latifolia* and *angustifolia*).

EUPHORBIACEAE.

ACALYPHA, L.[1]

A. **Adamsii**, nov. sp.; perennis; caule tenui plus minusve repente copiose ramoso fusco griseo-tomentuloso; indumento e pilis brevibus albidis recurvatis sed in apice caulis plus minusve longioribus et flavescentibus composito; foliis late ovatis vel suborbicularibus crenatis

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[1] With the material now at hand it is impossible to accept the treatment of *Acalypha* given by Mueller Argovensis, who reduces all Galapageian forms to varieties of one polymorphous species. We have here to do not only with annuals and perennials, with erect and prostrate habit, and with leaves of very different size, covered with pubescence varying from velvety and non-glandular to hirsute or glandular, but accompanying these considerable differences are others in the inflorescence sufficiently striking and constant to be classed as specific distinctions. The following key will suggest the leading diagnostic features.

SYNOPSIS OF THE GALAPAGEIAN ACALYPHAS.

* Involucre of the ♀ flowers 3-parted.

1. A. FLACCIDA, Hook. f.

* * Involucre irregularly 9-toothed: leaves large, 4 to 6 cm. long: floriferous part of the ♂ inflorescence 5 cm. in length, the peduncles very short.

2. A. BAURII, Rob. & Greenm.

* * * Involucre irregularly 7–9-toothed: leaves rarely over 3 cm. in length: floriferous part of ♂ inflorescence 1 mm. to 2 cm. long, commonly equalled

obtusis vel rotundatis 6–20 mm. longis 5–14 mm. latis supra adpresse pilosis subtus sordide pubescentibus et minute albo-punctatis a basi rotundato nec cordato 5-nerviis petiolo 6–19 mm. longo *tomentuloso*; spicis brevibus axillaribus parte mascula perbrevi 1–3 mm. longa in pedunculo filiforme 6–9 mm. longo terminali; involucris foemineis 1–3 in basi pedunculi sessilibus 3-floris ad mediam partem 7-partitis; dentibus

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and often much exceeded by the peduncle which often bears below 1 to 3 or more ♀ involucres.

+– Leaves tomentose.

++ Indumentum of the stem non-glandular.

= Terminal (chiefly ♀) spike well developed.

3. A. SERICEA, Anderss.

= = Pistillate involucres scattered chiefly at or near the base of lateral spikes: no well developed terminal spike.

4. A VELUTINA, Hook. f.

++ ++ Indumentum of the stem glandular.

5. A. STROBILIFERA, Hook. f.

+– +– Leaves puberulent or pubescent.

++ Terminal spike developed.

= Involucres glandular-pubescent.

6. A. RENIFORMIS, Hook. f.

= = Involucres hirsute with long non-glandular as well as shorter glandular hairs.

7. A. SPICATA, Anderss.

++ ++ No terminal spike.

= Prostrate perennials branching from the base.

a. Pistillate involucres 1 to 3, sessile at the very base of the rachis.

1. Leaves about equalling the petioles, cordate, reniform.

8. A. DIFFUSA, Anderss.

2. Leaves about equalling the petioles, suborbicular, rounded at the base, obtuse or rounded at the apex.

9. A. ADAMSII, Robinson.

3. Leaves considerably shorter than the petioles, acutish.

10. A. ALBEMARLENSIS, Robinson.

b. Pistillate involucres borne somewhat above the base of the rachis.

11. A. PARVULA, Hook. f.

= = Erect annuals.

a. Leaves cordate, 8 to 12 mm. long, obtuse or rounded: stem hirsute and glandular.

12. A. CORDIFOLIA, Hook. f.

b. Leaves 2 to 4 cm. long, rhombic-ovate, subacute, somewhat narrowed to a scarcely or not at all cordate base; stem glandular-puberulent.

13. A. CHATHAMENSIS, Robinson.

oblongis obtusiusculis pubescentibus. — *A. parvula*, var. *procumbens*, Rob. & Greenm. (1), 148, in part. — CHATHAM ISL.: southwest end, middle region, June, 1891, *Dr. George Baur*, no. 282 (hb. Gr.). Dedicated to the late C. F. Adams of the University of Illinois, the companion and efficient assistant of Dr. Baur upon his voyage to the Galapagos Islands. Endemic.

A. **albemarlensis**, nov. sp., gracilis sed basi ramoso lignescenti; ramis longis teretibus subsimplicibus purpurascentibus uno solo latere pubescentibus; indumento e pilis albidis brevissimis recurvatis et aliis longioribus rectis divergentibus composito; foliis parvis ovatis acutiusculis crenatis cordatis 7–10 mm. longis supra pilosis subtus pallidioribus creberrime punctatis et in nervis hirsutulis; petiolo filiforme flexuoso hirsutulo 9–17 mm. longo; spicis axillaribus parte mascula tenui gracili 6 mm. longa in pedunculo filiforme flexuoso 10–14 mm. longo; involucris foemineis in basi pedunculi saepius solitariis sessilibus glanduloso-pubescentibus 2–3-floris fere ad mediam partem 7-partitis; segmentis ovatis obtusiusculis. — ALBEMARLE ISL.: Tagus Cove, alt. 1220 m., 15 June, 1899, *Snodgrass & Heller*, no. 885 (hb. Gr.). Endemic.

A. BAURII, Rob. & Greenm. (1), 144, 148. — CHATHAM ISL.: southwest end, middle region, *Baur*, no. 285 (hb. Gr.). Endemic.

A. **chathamensis**, nov. sp., annua erecta in parte superiori ramosa; caule tereti fuscescente dense glanduloso-tomentello; foliis ovatis tenuibus obtusiusculis basi rotundatis vel leviter angustatis grosse supra basi crenatis in pagina superiori sparse pubescentibus subtus pallidioribus sub lente squamosis in nervis pubescentibus 2.5–4 cm. longis 2–3 cm. latis; petiolo 2.5–3.5 cm. longo sordide glanduloso-piloso; spicis axillaribus ad 4–5 cm. longis; parte mascula terminali 2 mm. crassa 10–15 mm. longa supra densa infra laxiori; rachide filiforme glanduloso-puberulo involucros foemineos 1–5 glanduloso-pubescentes 2–3-floros ad mediam partem in dentes 6–7 inaequales deltoideos acutiusculos glanduloso-ciliatos sectos plus minusve supra basi gerente. — CHATHAM ISL.: May, 1899, *Snodgrass & Heller*, no. 541 (hb. Gr.), and (?) no. 540 (hb. Gr.). Endemic.

A. CORDIFOLIA, Hook. f. (3), 186; Anderss. (1), 238, & (2), 103. *A. parvula*, var. *cordifolia*, Muell. Arg. in DC. Prodr. xv. pt. 2, 877 (1862), but surely not Rob. & Greenm. (1), 148. — CHARLES ISL.: *Darwin*. CHATHAM ISL.: *Andersson* fide ipsi l. c., but identity doubted. Endemic.

A. DIFFUSA, Anderss. (1), 240, & (2), 104, t. 14, f. 4; Rose (1),

137. — ALBEMARLE ISL.: in very sterile places, *Andersson* (hb. Gr.). CHATHAM ISL.: *A. Agassiz*, acc. to Rose, l. c. Endemic.

A. FLACCIDA, Hook. f. (3), 186; Anderss. (1), 238, & (2), 103. *A. parvula*, var. *flaccida*, Muell. Arg. in DC. Prodr. xv. pt. 2, 878 (1862). — JAMES ISL.: *Darwin*. Endemic.

A. PARVULA, Hook. f. (3), 185; Anderss. (1), 240, & (2), 104. *A. parvula*, var. *genuina*, Muell. Arg. l. c. 878 (1862); Rob. & Greenm. (1), 148. — ALBEMARLE ISL.: *Macrae; Capt. King*, acc. to Muell. Arg. l. c.; Iguana Cove, *Snodgrass* & *Heller*, nos. 83 (hb. Gr.), 97 (hb. Gr.); Tagus Cove, not common, from beach to 1230 m. alt., *Snodgrass* & *Heller*, nos. 202 (hb. Gr.), 245? (hb. Gr.), 886 (hb. Gr.); southern portion, *Baur*, no. 281 (hb. Gr.). Endemic.

A. RENIFORMIS, Hook. f. (3), 187; Anderss. (1), 240, & (2), 104. *A. parvula*, var. *reniformis*, Muell. Arg. l. c. — CHARLES ISL.: *Darwin*. Endemic.

A. SERICEA, Anderss. (1), 238, & (2), 103, t. 14, f. 1. *A. parvula*, var. *pubescens*, Muell. Arg. l. c. 877, in part; Rob. & Greenm. (1), 148, in part. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 842 (hb. Gr.). ALBEMARLE ISL.: in stony places of lower region, *Andersson*. BINDLOE ISL.: *Baur*, no. 277 (hb. Gr.); *Snodgrass* & *Heller*, no. 762 (hb. Gr.). Endemic. Ascribed to Chatham Isl. by Anderss. (1), 238, but apparently by clerical error, as Albemarle Isl. is substituted in his later work.

A. SPICATA, Anderss. (1), 239, (2), 104, t. 14, f. 3. *A. parvula*, var. *procumbens*, Muell. Arg. l. c. 878, in part. — CHARLES ISL.: *Baur*, no. 283 (hb. Gr.), identity doubtful. CHATHAM ISL.: in dry places of the lower region, *Andersson* (hb. Gr.). DUNCAN ISL.: *Baur*, no. 276 (hb. Gr.), identity doubtful. GARDNER ISL.: *Snodgrass* & *Heller*, no. 637 (hb. Gr.), identity doubtful. HOOD ISL.: *Snodgrass* & *Heller*, no. 723 (hb. Gr.), identity doubtful. JERVIS ISL.: *Baur*, no. 279 (hb. Gr.). Endemic.

A. STROBILIFERA, Hook. f. (3), 187; Anderss. (1), 238, & (2), 103. *A. parvula*, var. *strobilifera*, Muell. Arg. l. c. 877; Rob. & Greenm. (1), 148. — ALBEMARLE ISL.: eastern portion, Cowley Bay, *Baur*, no. 273 (hb. Gr.). CHATHAM ISL.: *Darwin*; in dry places of middle region, *Andersson*; northern part, *Baur*, no. 272 (hb. Gr.). Endemic.

A. VELUTINA, Hook. f. (3), 186; Anderss. (1), 237, & (2), 103, t. 14, f. 2. *A. parvula*, var. *pubescens*, Muell. Arg. l. c. 877, in part. —

CHARLES ISL.: *Darwin*; in arid places of the middle region, *Andersson* (hb. Gr.). CHATHAM ISL.: *Wood*, acc. to Muell. Arg. Endemic.

Var. MINOR, Hook. f. (3), 187; Anderss. (1), 238, & (2), 103. *A. parvula*, var. *pubescens*, Muell. Arg. l. c. 877, in part; Rob. & Greenm. (1), 148, in part. — CHARLES ISL.: *Darwin*; Cuevas Bay, *Baur*, no. 275 (hb. Gr.). Endemic.

A. sp. — *A. parvula*, var. *cordifolia?* Rob. & Greenm. (1), 148 (where by error referred to Jervis Isl.), not Muell. Arg. — BARRINGTON ISL.: *Baur*, no. 274 (hb. Gr.). Endemic.

A. sp. — BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 472 (hb. Gr.). Endemic.

A. sp. — BARRINGTON ISL.: *Baur*, no. 280 (hb. Gr.). Endemic.

A. sp. — INDEFATIGABLE ISL.: south of Conway Bay, *Baur*, no. 278 (hb. Gr.). Endemic.

A. sp. — *A. parvula*, var. *flaccida*, Rob. & Greenm. (1), 148, not Muell. Arg. — DUNCAN ISL.: *Baur*, no. 284 (hb. Gr.).

CROTON, L.

C. SCOULERI, Hook. f. (3), 188; Anderss. (1), 242, & (2), 105; Rob. & Greenm. (1), 148. *C. Scouleri*, var. *genuinus*, Muell. Arg. in DC. Prodr. xv. pt. 2, 605 (1862) ? *C. flavens*, var., Caruel (1), 625. — GALAPAGOS IDS.: *Goodridge* (hb. Gr.); *Habel*. BARRINGTON ISL.: *Baur*, no. 265 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 785 (hb. Gr.), large-leaved form. CHARLES ISL.: *Snodgrass* & *Heller*, no. 453 (hb. Gr.), approaching var. *incanus*. CHATHAM ISL.: *Darwin*; northern part, *Baur*, without number (hb. Gr.); *Chierchia?* HOOD ISL.: *Baur*, no. 264 (hb. Gr.). JAMES ISL.: *Scouler*; *Douglas*; *Andersson*; James Bay, abundant on lava soil, small trees 2.5 to 4 m. high, *Snodgrass* & *Heller*, no. 386 (hb. Gr.). NARBOROUGH ISL.: southern part, *Snodgrass* & *Heller*, nos. 330 (hb. Gr.), 352 (hb. Gr.). TOWER ISL.: *Baur*, without number (hb. Gr.), broad-leaved form.

Var. ALBESCENS, Muell. Arg. l. c. *C. albescens*, Anderss. (1), 242, & (2), 105. *C. incanus*, Anderss. (1), 243, & (2), 106; Rob. & Greenm. (1), 148; not Kunth. — ALBEMARLE ISL.: *Andersson*; mountain north of Elizabeth Bay, *Snodgrass* & *Heller*, no. 292 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 226 (hb. Gr.). BINDLOE ISL.: *Baur*, no. 266 (hb. Gr.). CHARLES ISL.: *Andersson*; *A. Agassiz* (hb. U. S. Nat. Mus.); *Baur*, no. 267 (hb. Gr.). JAMES ISL.: *Andersson* (hb. Gr.). Endemic.

Forma MICROPHYLLUS, Muell. Arg. l. c. *C. incanus*, var. *microphylla*, Anderss. (1), 243, & (2), 106. — ALBEMARLE ISL.: *Andersson.* Endemic.

Var. BREVIFOLIUS, Muell. Arg. l. c. *C. brevifolius*, Anderss. (1), 241, & (2), 105. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 47 (hb. Gr.). CHARLES ISL.: *Andersson.* CULPEPPER ISL.: *Snodgrass* & *Heller*, no. 4 (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller*, no. 686 (hb. Gr.), passing to typical form. GARDNER ISL.: *Snodgrass* & *Heller*, no. 641 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 572 (hb. Gr.). WENMAN ISL.: *Snodgrass* & *Heller*, no. 6 (hb. Gr.). Endemic.

Var. GRANDIFOLIUS, Muell. Arg. l. c. *C. xalapensis*, Hook. f. (3), 188; Anderss. (1), 241, & (2), 105; ? Rob. & Greenm. (1), 148; not HBK. — ABINGDON ISL.: *Snodgrass* & *Heller* no. 822 (hb. Gr.). CHARLES ISL.: *Lee* (hb. U. S. Nat. Mus., & hb. Gr.). CHATHAM ISL.: southwest end, upper region, *Baur*, no. 269 (hb. Gr.). JAMES ISL.: *Darwin.* TOWER ISL.: *Snodgrass* & *Heller*, no. 802 (hb. Gr.).

Var. MACRAEI, Muell. Arg. l. c. *C. Macraei*, Hook. f. (3), 188; Anderss. (1), 243, & (2), 106; Rob. & Greenm. (1), 148. — GALAPAGOS IDS.: *Habel.* ALBEMARLE ISL.: *Macrae; King*, acc. to Muell. Arg. l. c.; eastern part, Cowley Bay, *Baur*, no. 270 (hb. Gr.); Tagus Cove, most abundant bush from beach to 1230 m. alt., *Snodgrass* & *Heller*, nos. 153 (hb. Gr.), 189 (hb. Gr.). JAMES ISL.: in woods, *Andersson;* Orchilla Bay, *Baur*, no. 271 (hb. Gr.). Endemic.

EUPHORBIA, L.

E. AMPLEXICAULIS, Hook. f. (3), 183; Anderss. (1), 235, & (2), 101; Boiss. in DC. Prodr. xv. pt. 2, 13; Rob. & Greenm. (1), 149. — GALAPAGOS IDS.: *Habel.* BINDLOE ISL.: *Snodgrass* & *Heller*, no 771 (hb. Gr.). CHATHAM ISL.: *Darwin.* SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 582 (hb. Gr.). TOWER ISL.: *Baur*, no. 255 (hb. Gr.); *Snodgrass* & *Heller*, no. 795, (hb. Gr.). WENMAN ISL.: *Snodgrass* & *Heller*, no. 11 (hb. Gr.). Endemic.

E. APICULATA, Anderss. (1), 234, & (2), 101; Boiss. in DC. Prodr. xv. pt. 2, 17. — CHATHAM ISL.: near shore, *Andersson.* Endemic.

E. ARTICULATA, Anderss. (1), 236, & (2), 102, t. 12, f. 2; Boiss. l. c. 17; Rose (1), 137; Rob. & Greenm. (1), 148. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 834 (hb. Gr.). ALBEMARLE ISL.: eastern part, Cowley Bay, *Baur*, no. 258 (hb. Gr.). Elizabeth Bay, *Snodgrass* & *Heller*, no. 276 (hb. Gr.); Tagus Cove, common near beach and up to 185 m. alt., *Snodgrass* & *Heller*, nos. 149 (hb. Gr.), 181 (hb. Gr.). BINDLOE ISL.: *Baur*, no. 257 (hb. Gr.). CHARLES ISL.: Cormorant Bay, *Baur*, no. 259 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.); *A. Agassiz*, acc. to Rose, l. c. INDEFATIGABLE ISL.: *Andersson*. JAMES ISL.: Orchilla Bay, *Baur*, no. 260 (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 585 (hb. Gr.). Endemic.

E. DIFFUSA, Hook. f. (3), 184; Anderss. (1), 234, & (2), 101; Boiss. in DC. Prodr. xv. pt. 2, 17; Rob. & Greenm. (1), 148. — GALAPAGOS IDS.: *Habel*. ALBEMARLE ISL.: *Cuming; Macrae; Andersson*. INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 650 (hb. Gr.). JERVIS ISL.: *Baur*, no. 243 (hb. Gr.). Endemic.

E. FLABELLARIS, Anderss. acc. to Boiss. in DC. Prodr. xv. pt. 2, 17 (1862); Rob. & Greenm. (1), 148. *E. maculata*, Hook. f. (3), 182; Anderss. (1), 233, & (2), 101; not L. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 829 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 485 (hb. Gr.), identity doubtful. CHARLES ISL.: *Darwin*. GARDNER ISL.: *Snodgrass* & *Heller*, no. 631 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson;* south of Conway Bay, *Baur*, no. 256 (hb. Gr.). JAMES ISL.: James Bay, common on lava rocks near sand beach, *Snodgrass* & *Heller*, no. 383 (hb. Gr.), identity doubtful. SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 562 (hb. Gr.). TOWER ISL.: *Snodgrass* & *Heller*, no. 796 (hb. Gr.). Endemic.

E. GALAPAGEIA, Rob. & Greenm. (1), 144, 148. — CHARLES ISL.: *Baur*, no. 261 (hb. Gr.). Endemic.

E. **nesiotica**, nov. sp., verisimiliter annua gracilis glaberrima glaucescens; caulibus pluribus flexuosis dichotomis procumbentibus teretibus laevibus; foliis oppositis oblongis plus minusve denticulatis glabris obsolete venulosis apice rotundatis basi obliquis obtusis breviter petiolatis quam internodia valde brevioribus patentibus; involucris axillaribus breviter pedunculatis ad nodos superiores solitariis vel oppositis minimis turbinatis; lobis lanceolato-deltoideis plus minusve ciliatis; glandulis quatuor sessilibus transverse oblongis atroviolaceis; capsula nutante crassiore quam longa straminea-brunnea glabra laeve; coccis obtuse carinatis; stylis brevibus ad partem mediam bifidis. — SEYMOUR ISL.: south, May, 1899, *Snodgrass* & *Heller*, no. 589 (hb. Gr.). Endemic. Leaves 7 mm. long, half as broad; involucre 1 mm. long, capsule 1.4 mm. long, 2 mm. thick. PLATE 2, FIG. 5.

E. NUMMULARIA, Hook. f. (3), 183; Anderss. (1), 235, & (2), 101, t. 12, f. 1; Boiss. in DC. Prodr. xv. pt. 2, 16, & Icon. Euphorb. t. 5; Rose (1), 137; Rob. & Greenm. (1), 148. — CHATHAM ISL.: *Darwin;* covering large volcanic rocks near the shore, *Andersson* (hb. Gr.); *A. Agassiz;* southwest end, lower region, *Baur,* no. 253 (hb. Gr.); northern part, *Baur,* no. 254 (hb. Gr.); *Snodgrass* & *Heller,* no. 493 (hb. Gr.). Endemic.

Var. GLABRA, Rob. & Greenm. (1), 144, 148. — CHARLES ISL.: Cuevas Bay, *Baur,* no. 386 (hb. Gr.). Endemic.

E. PILULIFERA, L. Amoen. Acad. iii. 115 (1756); Hook. f. (3), 182; Anderss. (1), 233, & (2), 100; Boiss. in DC. Prodr. xv. pt. 2, 21. — CHARLES ISL.: cultivated ground, middle region, *Andersson; Lee* (hb. U. S. Nat. Mus.); *Snodgrass* & *Heller,* no. 439 (hb. Gr.). CHATHAM ISL.: *Snodgrass* & *Heller,* no. 519 (hb. Gr.). JAMES ISL.: *Darwin.* Distrib. general in warm countries.

E. PUNCTULATA, Anderss. (1), 235, & (2), 102; Boiss. l. c. 17; Rob. & Greenm. (1), 148. — ALBEMARLE ISL.: in very dry places, *Andersson.* DUNCAN ISL.: *Baur,* no. 262 (hb. Gr.). HOOD ISL.: *Baur,* no. 263 (hb. Gr.). Endemic.

E. RECURVA, Hook. f. (3), 182; Anderss. (1), 234, & (2), 101; Boiss. in DC. Prodr. xv. pt. 2, 16. — CHATHAM ISL.: *Darwin;* in lower stony region, *Andersson* (hb. Gr.). Endemic.

E. VIMINEA, Hook. f. (3), 184; Anderss. (1), 235, & (2), 101, t. 12, f. 3; Boiss. in DC. Prodr. xv. pt. 2, 17; Rob. & Greenm. (1), 136, 138, 148. *E. viminea,* forma *albemarlensis* (typica), Rob. & Greenm. (1), 138. — ALBEMARLE ISL.: *Macrae;* eastern part, Cowley Bay, *Baur,* no. 252 (hb. Gr.); southern portion, *Baur,* no. 251 (hb. Gr.); Elizabeth Bay, *Snodgrass* & *Heller,* no. 277 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller,* no. 164 (hb. Gr.). Endemic.

Forma BARRINGTONENSIS, Rob. & Greenm. (1), 139. — BARRINGTON ISL.: *Baur,* no. 244 (hb. Gr.). BINDLOE ISL.: *Baur,* no. 248 (hb. Gr.); *Snodgrass* & *Heller,* no. 761 (hb. Gr.). Endemic.

Forma CAROLENSIS, Rob. & Greenm. (1), 139. — CHARLES ISL.: *Andersson* (hb. Gr.). Endemic.

Forma CASTELLANA, Rob. & Greenm. (1), 138. — TOWER ISL.: *Baur,* no. 247 (hb. Gr.); *Snodgrass* & *Heller,* no. 805 (hb. Gr.). Endemic.

Forma CHATHAMENSIS, Rob. & Greenm. (1), 138. — CHATHAM ISL.: *Andersson; Baur,* no. 245 (hb. Gr.). Endemic.

Forma JACOBENSIS, Rob. & Greenm. (1), 138. — JAMES ISL.: Orchilla Bay, *Baur*, no. 249 (hb. Gr.). Endemic.

Forma JERVENSIS, Rob. & Greenm. (1), 139. — JERVIS ISL.: *Baur*, no. 250 (hb. Gr.). Endemic.

Var. ABINGDONENSIS, Rob. & Greenm. (1), 139. — ABINGDON ISL.: *Baur*, no. 246 (hb. Gr.). Endemic.

For a discussion of these forms see Rob. & Greenm. (1), 138, 139. *E. viminea* was collected on the Galapagos Ids. also by *Habel*.

E. sp. aff. *E. articulata*, Anderss. — BINDLOE ISL.: *Snodgrass & Heller*, no. 780 (hb. Gr.), sterile. Leaves narrowly ovate-oblong from a cordate base, otherwise much like *E. articulata*, which in its typical form grows upon the same island.

E. sp. Anderss. (1), 237, & (2), 102. — CHATHAM ISL.: *Andersson*.

E. sp. Hook. f. (3), 185; Anderss. (1), 237, & (2), 102. — CHARLES ISL.: *Darwin*.

HIPPOMANE, L.

H. MANCINELLA, L. Sp. 1191 (1753); Anderss. (1), 237, & (2), 103. — GALAPAGOS IDS.: *Habel*. ALBEMARLE ISL.: Elizabeth Bay, *Snodgrass & Heller*, no. 279 (hb. Gr.); Iguana Cove, *Snodgrass & Heller*, no. 49 (hb. Gr.). CHATHAM ISL.: *Andersson*. Further distrib. S. United States, Mex., W. Ind., Northwestern S. Am.

MANIHOT, Adans.

M. UTILISSIMA, Pohl. Pl. Bras. Ic. i. 32, t. 24 (1827); Muell. Arg. in DC. Prodr. xv. pt. 2, 1064; Caruel (1), 625. — CHATHAM ISL.: *Chierchia*. Widely distrib. in tropical countries.

PHYLLANTHUS, L.

P. CAROLINENSIS, Walt. Fl. Car. 228, as *caroliniensis* (1788); Caruel (1), 625; Rob. & Greenm. (1), 148. *P. obovatus*, Muhl. ex Willd. Sp. iv. 574 (1805); Hook. f. (3), 185; Anderss. (1), 237, & (2), 103. — ALBEMARLE ISL.: *Andersson*; Iguana Cove, *Snodgrass & Heller*, no. 106 (hb. Gr.); Tagus Cove, *Snodgrass & Heller*, no. 194 (hb. Gr.). CHARLES ISL.: *Darwin*; *Andersson* (hb. Gr.). CHATHAM ISL.: *Chierchia*; southwest end, middle region, *Baur*, no. 268 (hb. Gr.). JAMES ISL.: James Bay, common on lava rocks, *Snodgrass & Heller*, no. 385 (hb. Gr.). NARBOROUGH ISL.: northern part, *Snodgrass &*

Heller, no. 296 (hb. Gr.); southern part, *Snodgrass* & *Heller*, no. 332 (hb. Gr.). Further distrib. northern S. Am., Mex., W. Ind., southern and central U. S.

RICINUS, L.

R. COMMUNIS, L. Sp. 1007 (1753); Anderss. (1) 241, & (2), 105. — CHARLES ISL.: in cultivated ground, *Andersson*. Widely distrib. in warm countries.

CALLITRICHACEAE.

CALLITRICHE, L.

C. sp. Wolf, (1), 284. — CHARLES ISL.: in brook near hacienda acc. to Wolf. l. c.

CELASTRACEAE.

MAYTENUS, Feuill.

M. OBOVATA, Hook. f. (3), 230; Anderss. (1), 233, & (2), 100; Rob. & Greenm. (1), 145. — ALBEMARLE ISL.: southern part, *Baur*, no. 49 (hb. Gr.); Elizabeth Bay, on lava fields near beach, *Snodgrass* & *Heller*, no. 936 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 66 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, nos. 158 (hb. Gr.), 877 (hb. Gr.). BARRINGTON ISL.: *Baur*, no. 52 (hb. Gr.). CHARLES ISL.: *Andersson; Baur*. CHATHAM ISL.: *Darwin;* in woods of lower region, *Andersson;* southwest end, middle region, *Baur*, no. 48½ (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller*, no. 685 (hb. Gr.). HOOD ISL.: *Baur*, no. 51 (hb. Gr.). JAMES ISL.: James Bay, scattered along the sandy beach, *Snodgrass* & *Heller*, no. 371 (hb. Gr.). JERVIS ISL.: *Baur*. NARBOROUGH ISL.: common acc. to field notes of Mr. Heller. SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 597 (hb. Gr.). Endemic.

SAPINDACEAE.

CARDIOSPERMUM, L.

C. CORINDUM, L. Sp. ed. 2, 526 (1762); Radlk. Sitzungsb. Kgl. Bayer. Akad. 1878, p. 261; Rob. & Greenm. (1), 145, in part (as to pl. Chatham). *C. molle*, HBK. Nov. Gen. & Sp. v. 103 (1821); Hook. f. (3), 231; Anderss. (1), 231, & (2), 99; Caruel (1), 623. — ALBEMARLE ISL.: *Andersson;* mountain north of Elizabeth Bay, *Snod-*

grass & *Heller*, no. 281 (hb. Gr.). CHARLES ISL.: *Andersson.* CHATHAM ISL.: *Darwin; Andersson* (hb. Gr.); *Chierchia:* southwest end, lower region, *Baur*, no. 60 (hb. Gr.); southwest end, upper region, *Baur*, no. 59 (hb. Gr.); *Snodgrass* & *Heller*, no. 512 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson.* JAMES ISL.: *Andersson;* common about James Bay, *Snodgrass* & *Heller*, no. 359 (hb. Gr.). WENMAN ISL.: *Snodgrass* & *Heller*, no. 8 (hb. Gr.). Further distrib. trop. S. Am., Mex., southwestern U. S.

C. GALAPAGEIUM, Rob. & Greenm. Proc. Am. Acad. xxxii. 38 (1896). *C. Corindum*, Rob. & Greenm. (1), 145, in part (as to pl. Albemarle), not L. — ALBEMARLE ISL.: southern part, *Baur*, no. 61 (hb. Gr.). INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 675 (hb. Gr.). Endemic.

DODONAEA, L.

D. VISCOSA, Jacq. Enum. Pl. Carib. 19 (1762); L. Mant. ii. 228; Rob. & Greenm. (1), 145. Forma typica. — ALBEMARLE ISL.: eastern part, Cowley Bay, *Baur*, no. 62 (hb. Gr.); southern part, *Baur*, no. 63 (hb. Gr.). Further distrib. general in warm countries.

Var. SPATHULATA, Benth. Fl. Aust. i. 476 (1863). — ALBEMARLE ISL.: Tagus Cove, *Snodgrass* & *Heller*, nos. 876 (hb. Gr.), 904 (hb. Gr.); mountain east of Tagus Cove, *Snodgrass* & *Heller*, no. 244 (hb. Gr.). Specimen imperfect and doubtful, but clearly representing a very different form of the species from those collected on the same island by *Baur.* Further distrib. general in warm countries.

SAPINDUS, L.

S. SAPONARIA, L. Sp. 367 (1753); Sarg. Silv. ii. 69, t. 74, 75. "*S.* near *S. acuminatus*, Willd." Rob. & Greenm. (1), 145. — ALBEMARLE ISL.: southern part, *Baur*, no. 57 (hb. Gr.). Form with leaflets glabrous beneath. Further distrib. of species trop. Am., Mex., W. Ind., southeastern U. S.

RHAMNACEAE.

DISCARIA, Hook.

D. PAUCIFLORA, Hook. f. (3), 229; Anderss. (1), 233, & (2), 100; Rob. & Greenm. (1), 145. *D.* sp. Rob. & Greenm. (1), 145. — ALBEMARLE ISL.: *Darwin; Baur;* common about Elizabeth Bay, acc. to Mr. Heller; forming dense thickets, Tagus Cove, *Snodgrass* & *Heller*, no. 143 (hb. Gr.). BARRINGTON ISL.: *Baur*, no. 55 (hb. Gr.); abundant, *Snodgrass* & *Heller*, no. 478 (hb. Gr.). CHARLES ISL.: from this island Professor Caruel (1), 624, reports a spiny shrub collected by Chierchia and supposed to belong to the *Rhamnaceae*. It may well have been this species. CHATHAM ISL.: woods near shore, *Andersson;* southwest end, lower region, *Baur*. DUNCAN ISL.; not common, acc. to Mr. Heller. HOOD ISL.: *Baur*, (hb. Gr.), juvenile form with serrate leaves. INDEFATIGABLE ISL.: north coast, acc. to Mr. Heller. JAMES ISL.: James Bay, near beach in sand, acc. to Mr. Heller. SEYMOUR ISL.: north, uncommon, acc. to Mr. Heller; south, abundant, *Snodgrass* & *Heller*, no. 608 (hb. Gr.). Further distrib. collected by Dr. Baur at Posorja on the Gulf of Guayaquil, Ecuador, where perhaps introduced from the islands.

VITACEAE.

CISSUS, L.

C. SICYOIDES, L. Syst. Nat. ed. 10, 897 (1760). *Vitis sicyoides*, Miq. Ann. Mus. Bot. Lugd. Bat. i. 83 (1863–1864). — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 26, 45, 89, 101 (all in hb. Gr.). BINDLOE ISL.: common near the shore, acc. to field notes of Mr. Heller. CHARLES ISL.: *Lee* (hb. U. S. Nat. Mus.). NARBOROUGH ISL.: bordering beaches, acc. to Mr. Heller. Further distrib. Mex., W. Ind., S. Am., southward to Paraguay.

VITIS, L.

V. VINIFERA, L. Sp. 202 (1753); Caruel (1), 623. — CHARLES ISL.: in sterile state, *Chierchia* acc. to Caruel. Probably introduced through cultivation. Further distrib. Old World.

TILIACEAE.

CORCHORUS, L.

C. PILOBOLUS, Link, Enum. Hort. Berol. ii. 72 (1822); Jacq. Ecl. t. 163 as *C. bullatus*. — GALAPAGOS IDS.: *Habel?* ALBEMARLE ISL.: *Snodgrass* & *Heller*, no. 90. GARDNER ISL.: *Snodgrass* & *Heller*, no. 645 (hb. Gr.). Further distrib. trop. S. Am., W. Ind., Mex.

TRIUMFETTA, L.

T. SEMITRILOBA, Jacq. Enum. Pl. Carib. 22 (1762), Stirp. Am. 147, & Hort. Vindob. iii. t. 16. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 69, 70, 107, 858, 859 (all in hb. Gr.). Further distrib. general in trop. and subtrop. Am.

MALVACEAE.

ABUTILON, Gaertn.

A. ANDERSSONIANUM, Garcke in Anderss. (1), 230, & (2), 98, t. 15, f. 1. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 847 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 479 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 788 (hb. Gr.). CHARLES ISL.: dry places, lower region, *Andersson*. CHATHAM ISL.: dry places, lower region, *Andersson* (hb. Gr.); *Snodgrass* & *Heller*, no. 507 (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller*, no. 702 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 632 (hb. Gr.). INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 662 (hb. Gr.). TOWER ISL.: *Snodgrass* & *Heller*, no. 794 (hb. Gr.). Endemic. The forms on Duncan and Gardner Ids. are smoother and greener than the others.

A. DEPAUPERATUM, Anderss. (1), 230, & (2), 98. *A. Anderssonianum*, Rob. & Greenm. (1), 145, not Garcke. *Sida depauperata*, Hook. f. (3), 232. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 76, 82 (both in hb. Gr.). BARRINGTON ISL.: *Baur*, no. 16 (hb. Gr.), determined from character. CHARLES ISL.: *Darwin*. Endemic. Perhaps only a dry soil form of *A. Anderssonianum*.

ANODA, Cav.

A. HASTATA, Cav. Diss. i. 38, t. 11, f. 2 (1790). *A. acerifolia*, DC. Prodr. i. 459 (1824); Rob. & Greenm. (1), 145, where by error ascribed to Chatham Island. — CHARLES ISL.: *Lee* (hb. U. S. Nat. Mus.); *Baur*, no. 20 (hb. Gr.); *Snodgrass* & *Heller*, no. 449 (hb. Gr.). Further distrib. southern U. S., Mex. to Chili.

BASTARDIA, HBK.

B. VISCOSA, HBK. Nov. Gen. & Sp. v. 256 (1821); L'Her. Stirp. t. 53 bis; E. G. Baker, Jour. Bot. xxxi. 68. *B. guayaquilensis*, Turcz. Bull. Soc. Nat. Mosc. 1858, p. 201. *Sida viscosa*, L. Syst. ed. 10, 1145 (1760). — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 105 (hb. Gr.). CHATHAM ISL.: *Snodgrass* & *Heller*, no. 533 (hb. Gr.). Further distrib. western S. Am., Mex., W. Ind.

GOSSYPIUM, L.

G. BARBADENSE, L. Sp. 693 (1753). *G. purpurascens*, Hook. f. (3), 231; Anderss. (1), 228, & (2), 97; Rose (1), 136; not Poir.

G. Klotzschianum, Rob. & Greenm. (1), 145, not Anderss. — GALAPAGOS IDS.: *Habel.* ABINGDON ISL.: *Snodgrass* & *Heller*, no. 828 (hb. Gr.); ALBEMARLE ISL.: *Baur;* Iguana Cove, *Snodgrass* & *Heller*, nos. 46 (hb. Gr.), 854 (hb. Gr.). BARRINGTON ISL.: *Baur*, no. 24 (hb. Gr.). CHARLES ISL.: *Andersson* (hb. Gr.); *Snodgrass* & *Heller*, no. 402 (hb. Gr.). CHATHAM ISL.: *Darwin;* southwest end lower region, *Baur*, no. 22 (hb. Gr.); northern part, *Baur*, no. 25 (hb. Gr.). DUNCAN ISL.: *A. Agassiz.* GARDNER ISL.: *Snodgrass* & *Heller*, no. 633 (hb. Gr.). HOOD ISL.: *Baur*, no. 21 (hb. Gr.); *Snodgrass* & *Heller*, no. 752 (hb. Gr.). JAMES ISL.: *Darwin.* SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 599 (hb. Gr.). Further distrib. general in tropics.

G. KLOTZSCHIANUM, Anderss. (1), 228, & (2), 97. — GALAPAGOS IDS.: *Edmonston* (hb. Gr.). ALBEMARLE ISL.: *Andersson.* BINDLOE ISL.: *Snodgrass* & *Heller*, no. 772 (hb. Gr.). CHARLES ISL.: *Andersson* (l. c., but his spec. so labelled in hb. Gr. is clearly *G. barbadense*). CHATHAM ISL.: *Andersson.* INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 656 (hb. Gr.). Endemic. *Andersson* (2), 97, states that this species was collected upon Chatham and James Islands by Darwin, but this appears to have been a clerical error and to refer to the preceding species.

HIBISCUS, L.

I. TILIACEUS, L. Sp. 694 (1753). *Paritium tiliaceum*, Hook. f. (4), 262; Anderss. (1), 229, & (2), 98. — CHARLES ISL.: *Edmonston.* Further distrib. general in tropics.

MALACHRA, L.

M. CAPITATA, L. Syst. ed. 12, 458 (1767); Hook. f. (3), 231; Anderss. (1), 229, & (2), 98; Gürcke in Engl. Bot. Jahrb. xvi. 348. — JAMES ISL.: *Darwin* acc. to Hook. f., l. c. Further distrib. trop. S. Am., W. Ind., Mex., sparingly introd. in trop. of Old World.

SIDA, L.

S. ACUTA, Burm., var. CARPINIFOLIA, K. Schum. in Mart. Fl. Bras. xii. pt. 3, 326 (1891); E. G. Baker, Jour. Bot. xxx. 238. *S. carpinifolia*, L. f. Suppl. 307 (1781); Anderss. (1), 229, & (2), 98, excl. synon. — CHARLES ISL.: dry places of middle region, *Andersson.* Further distrib. general in trop. and subtrop. regions.

S. ANGUSTIFOLIA, Lam. Dict. i. 4 (1783). *S. tenuicaulis*, Hook. f. (3), 232, acc. to E. G. Baker. *S. spinosa* & *S. tenuicaulis*, Anderss. (1), 229, & (2), 98. *S. spinosa*, var. *angustifolia*, Griseb. Fl. Brit. W. Ind. 74 (1859); E. G. Baker, Jour. Bot. xxx. 237. *S. rhombifolia*, Rob. & Greenm. (1), 145, in part (as to pl. Charles and pl. Indefatigable). — ALBEMARLE ISL.: *Lee* (hb. U. S. Nat. Mus.); Iguana Cove, *Snodgrass* & *Heller*, nos. 84 (hb. Gr.), 104 (hb. Gr.). CHARLES ISL.: *Andersson; Baur*, no. 17 (hb. Gr.); *Snodgrass* & *Heller*, no. 438 (hb. Gr.). CHATHAM ISL.: lower region, *Andersson* (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller*, no. 701 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 629 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson;* south of Conway Bay, *Baur*, no. 18 (hb. Gr.). JAMES ISL: *Darwin ; Andersson.* NARBOROUGH ISL.: northern part, *Snodgrass* & *Heller*, no. 298 (hb. Gr.). Further distrib. general in warm countries.

S. CORDIFOLIA, L. Sp. 684 (1753); K. Schum. in Mart. Fl. Bras. xii. pt. 3, 331, t. 62. — ALBEMARLE ISL.: Iguana Cove, below 310 m. alt., *Snodgrass* & *Heller*, no. 860 (hb. Gr.). Sterile. Further distrib. general in trop. and subtrop. regions.

S. PANICULATA, L. Syst. ed. 10, 1145 (1760); Rob. & Greenm. (1), 145; E. G. Baker, Jour. Bot. xxx. 294. *S. atrosanguinea*, Jacq. Ic. Rar. t. 136. *S. floribunda*, HBK. Nov. Gen. & Sp. v. 258, t. 473 (1821). — GALAPAGOS IDS.: *Habel* acc. to Hemsl. in litt. ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 71 (hb. Gr.), 86 (hb. Gr.). CHARLES ISL.: *Baur*, no. 19 (hb. Gr.); *Snodgrass* & *Heller*, no. 401 (hb. Gr.). Further distrib. trop. S. Am., W. Ind., Mex.

S. RHOMBIFOLIA, L. Sp. 684 (1753); Hook. f. (4), 262; Anderss. (1), 229, & (2), 98; Rob. & Greenm. (1), 145, in part (as to pl. Chatham); E. G. Baker, Jour. Bot. xxx. 239, q. v. for synon. — CHARLES ISL.: *Edmonston ; Lee* (hb. U. S. Nat. Mus.); *Snodgrass* & *Heller*, no. 448 (hb. Gr.). CHATHAM ISL.: stony places, lower region, *Andersson;* southwest end, middle region, *Baur*, no. 15 (hb. Gr.); *Snodgrass* & *Heller*, no. 532 (hb. Gr.). Further distrib. general in warm countries.

S. SPINOSA, L. Sp. 683 (1753); Gray, Syn. Fl. i. pt. 1, 324. — GALAPAGOS IDS.: *Habel.* ALBEMARLE ISL.: common on tufa soil, Tagus Cove, *Snodgrass* & *Heller*, no. 206 (hb. Gr.). Further distrib. general in warm countries. In distinguishing this species and *S. angustifolia*, I have followed Dr. Gray's treatment and distinctions.

S. VERONICAEFOLIA, Lam., var. HUMILIS, K. Schum. in Mart. Fl. Bras. xii. pt. 3, 320 (1891), q. v. for extensive synon.; E. G. Baker, Jour. Bot. xxx. 293. *S. humilis*, Cav. Diss. v. 277, t. 134, f. 2 (1788). — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 85 (hb. Gr.). Further distrib. general in tropics.

STERCULIACEAE.

WALTHERIA, L.

W. RETICULATA, Hook. f. (3), 231; Anderss. (1), 231, & (2), 99; Rob. & Greenm. (1), 145. — This endemic species which has now been found upon no less than nine islands of the archipelago shows the same sort of formal variation that has been described in the case of *Euphorbia viminea*, Hook. f. (see Rob. & Greenm. (1), 138–139). The original characterization was unfortunately drawn from mixed material from Chatham, James, and Albemarle islands. We may, however, take as typical the form with a very close fine tomentum, small thickish leaves, 1.5 to 2.5 cm. long, with strongly crisped margins. The leaves are rounded rather than cordate at the base, and their indumentum becomes decidedly yellowish in a dried state. This form, assumed as typical, has been collected on GALAPAGOS IDS.: *Edmonston* (hb. Gr.). ALBEMARLE ISL.: *Macrae;* southern part, *Baur*, no. 32 (hb. Gr.); Elizabeth Bay, *Snodgrass* & *Heller*, nos. 269 (hb. Gr.), 289 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 73 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 162 (hb. Gr.). CHARLES ISL.: *Andersson; Baur*, no. 33 (hb. Gr.). JAMES ISL.: *Douglas;* "*Macrae;*"[1] James Bay, along edge of new lava, *Snodgrass* & *Heller*, no. 367 (hb. Gr.). JERVIS ISL.: *Baur* (this may belong to one of the following forms). Endemic. From this may be distinguished: —

Forma **acamata**, tomento imprimis ramulorum crassiore valde flavescente; foliis magnis crassiusculis cordatis. — INDEFATIGABLE ISL.: *Baur*, no. 27 (hb. Gr.); *Andersson?* Endemic.

Forma **Anderssonii**, indumento ac illud formae typicae tenui densoque sed griseo et nullo modo flavescente; foliis multo majoribus 4–5.5 cm. longis, 3.2–4.2 latis cordatis. — BARRINGTON ISL.: *Baur*, no. 26 (hb. Gr.); *Snodgrass* & *Heller*, no. 474 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.); *Darwin ?* NARBOROUGH ISL.: northern part, *Snodgrass* & *Heller*, no. 301 (hb. Gr.). TOWER ISL.: *Baur*, no. 30 (hb. Gr.); *Snodgrass* & *Heller*, no. 797 (hb. Gr.). Endemic.

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[1] Cited by Andersson, but probably a mistake for *Scouler*.

Forma **intermedia**, tomento typico sed minus flavescente: foliis quam illi formae typicae majoribus 3-4 cm. longis sed quam hi formae *Anderssonii* minoribus et minus cordatis. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 840 (hb. Gr.). BINDLOE ISL.: *Baur*, no. 28 (hb. Gr.); *Snodgrass* & *Heller*, no. 757 (hb. Gr.). CHARLES ISL.: *Lee* (hb. U. S. Nat. Mus.); Cuevas Bay, *Baur*, no. 34 (hb. Gr.); *Snodgrass* & *Heller*, no. 426 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 640 (hb. Gr.). NARBOROUGH ISL.: southern part, 300 to 615 m. alt., *Snodgrass* & *Heller*, no. 326 (hb. Gr.). Endemic.

TURNERACEAE.

TURNERA, L.

T. ULMIFOLIA, L. Sp. 271 (1753); Hook. f. (4), 262; Anderss. (1), 221, & (2), 93; Urb. Jahrb. Bot. Gard. Berlin, ii. 138 (where spec. is elaborately subdivided). — CHARLES ISL.: *Edmonston* (hb. Gr.). Further distrib. general in the tropics.

PASSIFLORACEAE.

PASSIFLORA, L.

P. FOETIDA, L. Sp. 959 (1753); Cav. Diss. 458, t. 289; Anderss. (1), 221, & (2), 93; Rose (2), 137; Rob. & Greenm. (1), 146. — CHARLES ISL.: upper region, *Andersson; A. Agassiz*. CHATHAM ISL: *A. Agassiz;* southwest end, middle region, *Baur*, no. 159 (hb. Gr.); *Snodgrass* & *Heller*, no. 496 (hb. Gr.). Further distrib. general in trop. and subtrop. Amer.

P. LINEARILOBA, Hook. f. (3), 222. *P. linearifolia*, Anderss. (1), 221, & (2), 93. *? P. tridactylites*, Hook. f. (3), 222; Anderss. (1), 221, & (2), 93. *P. suberosa*, var. *lineariloba*, Masters in Mart. Fl. Bras. xiii. pt. 1, 579 (1872). *P. suberosa*, var., Rob. & Greenm. (1), 146. — GALAPAGOS IDS.: *Habel*. CHARLES ISL.: *Darwin ;? Andersson*. GARDNER ISL.: *Snodgrass* & *Heller*, no. 625 (hb. Gr.). HOOD ISL.: *Baur*, no. 160 (hb. Gr.). JAMES ISL.: *Scouler; Douglas*. NARBOROUGH ISL.: southern part, trailing on bushes, rare, 615 m. alt., *Snodgrass* & *Heller*, no. 321 (hb. Gr.). Endemic. This species seems very different from continental specimens of *P. suberosa*, L., at hand.

P. PUBERULA, Hook. f. (3), 223; Anderss. (1), 221, & (2), 93. — JAMES ISL.: *Darwin*. Endemic. Perhaps a form of *P. suberosa*, L., as considered by several writers.

CARICACEAE.

CARICA, L.

C. PAPAYA, L. Hort. Cliff. 461, & Sp. 1036 (1753); Anderss. (1), 223, & (2), 94. — CHARLES ISL.: about habitations, *Andersson.* Further distrib. general in tropics.

LOASACEAE.

MENTZELIA, L.

M. ASPERA, L. Sp. 516 (1753); Anderss. (1), 222, & (2), 94; Rob. & Greenm. (1), 146. ?*Acrolasia squalida*, Hook. f. (3), 222; Anderss. (1), 222, & (2), 93. — ALBEMARLE ISL.: *Andersson;* Iguana Cove, *Snodgrass* & *Heller*, no. 102 (hb. Gr.); Tagus Cove, common from beach to 310 m. alt., in shady places, *Snodgrass* & *Heller*, no. 175 (hb. Gr.). CHARLES ISL.: *Darwin; Andersson;* Cuevas Bay, *Baur*, no. 162 (hb. Gr.); *Snodgrass* & *Heller*, no. 429 (hb. Gr.). CHATHAM ISL.: *Andersson;* southwest end, lower region, *Baur*, no. 161 (hb. Gr.); *Snodgrass* & *Heller*, no. 530 (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller*, no. 689 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 614 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 719 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson;* northern part, *Snodgrass* & *Heller*, no. 664 (hb. Gr.). JAMES ISL.: *Andersson;* James Bay, abundant on lava soil in shade, *Snodgrass* & *Heller*, no. 376 (hb. Gr.). TOWER ISL.: *Snodgrass* & *Heller*, no. 803 (hb. Gr.). Further distrib. N. Am.

SCLEROTHRIX, Presl.

S. FASCICULATA, Presl, Symb. Bot. ii. 3, t. 53 (1858). *Ancyrostemma micranthum*, Poepp. & Endl. Nov. Gen. & Sp. iii. 65 (1845). — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 123, 128 (both in hb. Gr.). JAMES ISL.: James Bay, common on lava rocks near beach, *Snodgrass* & *Heller*, no. 373 (hb. Gr.). NARBOROUGH ISL.: southern part, tolerably common, alt. 615 m., *Snodgrass* & *Heller*, no. 315a. Further distrib. Mex. to Peru and Brazil.

CACTACEAE.

[The plants of this family secured by Messrs. Snodgrass & Heller have been kindly identified by Prof. Karl Schumann of the Royal Botanical Museum, Berlin.]

CEREUS, Mill.

C. GALAPAGENSIS, Weber, Bull. du Mus. d'hist. nat. Paris, 1899, p. 312 (1899). — CHARLES ISL.: *Du Petit-Thouars.* Endemic. This can scarcely be regarded as a described species.

C. **nesioticus**, K. Sch. nov. sp. in litt., "humilis 30 cm. longitudinem non attingens; caulibus caespitosis et e basi communi in omnes partes more spinarum Echini radiantibus costatis ubique spinulis numerosis atrocastaneis tectis; costis humilibus 3 mm. vix superantibus alte crenatis et in tubercula fere perfecte dissolutis; areolis orbicularibus, 2.5 mm. diametro lano sparso exigue tectis; spinis quam 40 pluribus inaequalibus saepe (praesertim infimis) apice fractis in unam centralem et alias exteriores non distributis, maximis 3 cm. longis omnibus quam setae equinae vix rigidioribus erectis strictis divaricatis non pungentibus; floribus 7 cm. longis; ovario subgloboso subobliquo leviter tuberculato et spinulis ad 5 mm. longis flavo-fuscis radiantibus armato; tubo perigonii angusto item in areolis spinoso, lobis exterioribus lanceolatis 1.2 cm. longis vix 2 mm. latis, interioribus etiam angustioribus et magis linearibus verosimiliter albis; staminibus prope faucem, filamentis perbrevibus non 1 mm. longis, antheris bis vel paulo ultra longioribus; fructu ellipsoideo in summo spinoso-areolato basi acuto 2.5 cm. longo et 1.3 cm. crasso; seminibus numerosis, 1.2 mm. longis ellipsoideis rufis leviter punctatis." — ALBEMARLE ISL.: Black Bight, *Snodgrass* & *Heller*, no. 923 (hb. Berl. & hb. Gr.); lava fields, Elizabeth Bay, *Snodgrass* & *Heller*, no. 939 (hb. Berl. & hb. Gr.); Point Christopher, *Snodgrass* & *Heller*, no. 932 (hb. Berl. & hb. Gr.). NARBOROUGH ISL.: eastern side, on barren black lava, *Snodgrass* & *Heller*, nos. 919 (hb. Berl. & hb. Gr.), 927 (hb. Berl. & hb. Gr.). "Note. — This species is a very peculiar one from its long brown non-pungent spines, which clothe the stem so densely that its surface is invisible. I have never before seen a species of the genus with such short filaments as in this. The petals are also uncommonly narrow. Probably the flower is white. I think it can hardly be compared with either of the two species of *Cereus* thus far known from the Galapagos Islands. From *C. Thouarsii*, Weber, which is said to be similar to *C. multangularis*, it is perfectly distinguished by the mode of growth and by the much smaller fruit which does not at all resemble a large prune" [K. Sch. in litt.]. Endemic.

C. **sclerocarpus**, K. Sch. nov. sp. in litt., "arborescens; caule principali 3–6 m. alto 15–22 cm. diametro ramos paucos sibi saepius parallelos gerente; articulis costatis, costis in sectione transversa triangularibus

obtusis 1 cm. altis ope sulcorum acutorum separatis; areolis orbicularibus 3.5 mm. diametro tomento brevi tectis; spinis 20–25 in radiales et centrales non distributis rectis pungentibus maximo 4.5 cm. longo; floribus circa 10–11 cm. longis; ovario tereti squamato; squamis sparsis 1–1.5 mm. longis ovatis acutis; perigonii tubo gracili paucis bracteolis brevibus ornato; lobis exterioribus subspathulatis 3 cm. longis apice eroso-denticulatis, interioribus brevioribus angustioribus acuminatisque; staminibus prope faucem perigonii affixis 2 cm. longis; stilo florem fere aequante; stigmatibus 11 fere 2 cm. longis filiformibus; fructu duro strato exteriori perfirmo ovoideo acuto flore marcido coronato 7 cm. longo 3 cm. diametro; seminibus haud numerosis (ovulis plurimis abortivis) disciformibus 1 mm. diametro nigris lucidis leviter foveolatis." — ALBEMARLE ISL.: Pt. Christopher, *Snodgrass & Heller*, nos. 933 (hb. Berl. & hb. Gr.), 934 (hb. Berl. & hb. Gr.); Black Bight, *Snodgrass & Heller*, no. 935 (hb. Berl. & hb. Gr.). "Note. — There is only one species in the Galapagos Islands which may be compared with this, namely *C. galapagensis*, Weber. The characterization of the latter is, however, so imperfect that I cannot identify my species with it." [K. Sch. in litt.]. Endemic. In his field notes Mr. Heller reports a giant *Cereus* on Bindloe (where rare), Chatham, James, and Narborough Islands which was probably *C. sclerocarpus*. It grew upon barren lava.

C. THOUARSII, Weber, Bull. du. Mus. d'hist. nat. Paris, 1899, p. 312 (1899). — CHARLES ISL.: *Du Petit-Thouars*. Endemic. Not satisfactorily described.

OPUNTIA, Raf.

O. GALAPAGEIA, Hensl. Mag. Zoöl. & Bot. i. 467, t. 14, f. 2 (1837); Hook. f. (3), 223; Anderss. (1), 224, & (2), 95; Hemsl. Gard. Chron. ser. 3, xxiv. (1898), p. 265, f. 75; Lindberg, Monatschr. Kakteenk. iv. 120–122, 134–135, & v. 10; K. Schumann, ibid. ix. 19, 20, x. 173, & Gesammtbeschr. Kakteen, 747. — CHARLES ISL.: *Hassler Exp.*, acc. to ms. note of Engelm. (hb. Mo. Bot. Gard.). JAMES ISL.: *Darwin; Hassler Exp.*, acc. to ms. note of Engelm. (hb. Mo. Bot. Gard.). JERVIS ISL.: *Hassler Exp.*, acc. to ms. note of Engelm. (hb. Mo. Bot. Gard.). Endemic.

O. **Helleri**, K. Sch. nov. sp. in litt., "humilis plus minusve prostrata 30–60 cm. alta dense caespitosa ramosa; articulis lineari-oblongis apice basique acutis planis 11 cm. longis 4 cm. latis tenuibus apice setis fuscis munitis; areolis orbicularibus 2 mm. diametro non manifeste in tuberculis editis capillis paucis albis tectis etiam tomento flavido in parte

superiori instructis; spinis circa 20 inaequalibus 1.5 cm. non superantibus non pungentibus plus minusve arcuatis flavo-fuscis; glochidiis paucis (circa 50) in summa parte areolae sitis et non arcte pungentibus apice obtusis; spinis lateralibus glochidiarum non eminentibus; ovario paulo quam 3 cm. longiore turbinato tuberculato; areolis quam illae articularum non minoribus et a lano modice copioso tectis et a spinis capillaceis modice laxis ad 2 cm. longis flavo-fuscis munitis; perigonio 3–3.5 cm. maximo diametro; lobis exterioribus subulatis, 3 mm. longis, sequentibus late ovatis acutis mucronatisve verosimiliter flavo-viridibus, intimis subobovatis fere 2 cm. longis; staminibus fere in fundo infimo floris crateriformis insertis permultis; stilo 2 cm. longo crasso; stigmatibus 6 erectis incurvatis carnosis percrassis." — WENMAN ISL.: *Snodgrass & Heller*, no. 917 (hb. Berl. & hb. Gr.). "Note. — The relatively large flower brings this species near *O. myriacantha*, Web., from which it differs in the mode of growth, smaller articles, and non-pungent spines." [K. Sch. in litt.]. Endemic.

O. MYRIACANTHA, Weber in Bois, Dictionn. d'horticult. 894 (1898), & Bull. du Mus. d'hist. nat. Paris, 1899, p. 313 (1899). — ALBEMARLE ISL.: *Hassler Exp.*, acc. to ms. note of Engelm. (hb. Mo. Bot. Gard.). CHARLES ISL.: *Du Petit-Thouars* [*Dr. Néboux*] (hb. Mus. d'hist. nat. Paris, & hb. Mo. Bot. Gard.). INDEFATIGABLE ISL.: *Hassler Exp.*, acc. to ms. note of Engelm. (hb. Mo. Bot. Gard.). Endemic.

Opuntias also occur (acc. to field notes of several collectors) upon Abingdon, Barrington, Bindloe, Chatham, Culpepper, Duncan, Hood, Narborough, North and South Seymour, and Tower Islands, but as no specimens of them have been secured it is impossible to refer them with definiteness to any of the foregoing species. Dr. Baur[1] says of the genus: "Die grosse Opuntia hat einen verschiedenen Charakter beinahe auf jeder Insel. Die Opuntia von Barrington, Indefatigable und süd-Albemarle z. B. entwickelt einen sehr hohen Stamm; die von Hood und Charles besitzt einen verhältnissmässig niederen und dickeren Stamm; die Opuntia von Jervis wiederum einen sehr niederen; die Verzweigung beginnt schon kurz über dem Boden; die Opuntia von Tower hat gar keinen Stamm, die Verzweigung beginnt sofort am Boden, es ist ein niederer Busch, aber kein Baum. Die Form von Bindloe zeigt Charaktere, die zwischen den Individuen von Tower und Jervis liegen." It seems not unlikely that the low plant on Tower lacking the main trunk may be the same as Professor Schumann's *O. Helleri* from the not very distant island of Wenman.

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[1] Biol. Centralbl. xii. 247 (1892).

LYTHRACEAE.

CUPHEA, P. Br.

C. PATULA, St. Hil. Fl. Bras. Merid. iii. 101; Caruel (1), 624; Koehne in Engl. Jahrb. ii. 165 (1882). — CHATHAM ISL.: *Chierchia*, acc. to Caruel, l. c. Further distrib. Brazil.

RHIZOPHORACEAE.

RHIZOPHORA, L.

R. MANGLE, L. Sp. 443 (1753); Hook. f. (3), 225; Anderss. (1), 247, & (2), 108. — ALBEMARLE ISL.: in swamps north of Tagus Cove, acc. to ms. notes of Mr. Heller; also fringing lagoons on the south and west shores of Elizabeth Bay, acc. to Mr. Heller. CHATHAM ISL.: *Darwin*. DUNCAN ISL.: in a swamp on the west coast, acc. to Mr. Heller. INDEFATIGABLE ISL.: about lagoons, acc. to Mr. Heller. NARBOROUGH ISL.: forming large swamps fringing lagoons, on the east and southeast sides of the island, *Snodgrass* & *Heller*, no 918 (hb. Gr.). Further distrib. general on trop. shores.

MYRTACEAE.

PSIDIUM, L.

P. GALAPAGEIUM, Hook. f. (3), 224. *P. galapagejum*, Anderss. (1), 247, & (2), 109. — ALBEMARLE ISL.: Iguana Cove, above 125 m. alt., bushes 2.4 to 3.6 m. high, *Snodgrass* & *Heller*, no. 126 (hb. Gr.). JAMES ISL.: *Scouler; Darwin*. Endemic.

COMBRETACEAE.

CONOCARPUS, Gaertn.

C. ERECTUS, L. Sp. 176 (1753); Jacq. Stirp. Am. 78, t. 52; Anderss. (1), 247, & (2), 108. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 44 (hb. Gr.). CHATHAM ISL.: somewhat marshy woodland in the littoral region, *Andersson*. JAMES ISL.: *Andersson* (hb. Gr.); abundant along sandy beach, James Bay, *Snodgrass* & *Heller*, no. 382 (hb. Gr.). Further distrib. W. Ind., Mex., southern U. S., trop. S. Am., introd. in Africa.

LAGUNCULARIA, Gaertn.

L. RACEMOSA, Gaertn. Fruct. iii. 209, t. 217, f. 2 (1805); Brandis in Engl. & Prantl, Nat. Pflanzenf. iii. Ab. 7, 127, f. 64. — ALBEMARLE ISL.: Elizabeth Bay, lava fields near beach, *Snodgrass & Heller*, nos. 270 (hb. Gr.), 937 (hb. Gr.); Point Christopher, *Snodgrass & Heller*, no. 931 (hb. Gr.). NARBOROUGH ISL.: east side, *Snodgrass & Heller*, no. 920 (hb. Gr.). Further distrib. shores, Mex., W. Ind., S. Am., trop. Afr. Not before recorded from the Galapagos Ids.

MELASTOMACEAE.

MICONIA, R. & P.

M. Robinsoniana, Cogniaux, nov. sp. (sect. *Tamonea*); *M.* sp. Rob. & Greenm. (1), 146; "glaberrima; ramis junioribus acute tetragonis non alatis; foliis breviuscule petiolatis, anguste ligulato-oblongis, leviter obtuseque acuminatis, basi breviter attenuatis vel subrotundatis, margine integerrimis, trinerviis vel obscure 5-nerviis, nervulis transversalibus numerosis tenuissimis; floribus sessilibus, secus ramulorum paniculae solitariis; calyce oblongo, limbo leviter dilatato, brevissime 5-lobato, lobis late rotundatis; staminum filamentis glaberrimis.

"Rami robustiusculi, laeves. Petiolus gracilis, leviter tortuosus, 1–1.5 cm. longus. Folia rigidiuscula, utrinque laevia, siccitate non nitida, 14–16 cm. longa, 3–3.5 cm. lata, nervis subtus satis prominentibus. Paniculae majusculae, multiflorae, trichotome ramosae, ramis gracilibus, erectis vel paulo patulis, leviter compressis, articulatis. Bracteae patentissimae, rigidae, triangulari-ovatae, acutae, 1–1.5 mm. longae. Calyx laevis, basi rotundatus, sub apice leviter constrictus, circiter 5 mm. longus. Petala erecta, anguste obovata, obtusa, 6 mm. longa. Staminum filamenta capillaria, 3–4 mm. longa; antherae valde arcuatae, apice longe attenuatae, 4–5 mm. longae. Stylus crassiusculus, glaber, apice arcuatus caeteris rectus, 5 mm. longus, stigmate paulo incrassato. — Affinis *M. nitidissimae*, Cogn. in DC. Monogr. Phan. vii. 748, sed bene distincta." — CHATHAM ISL.: southwest end, upper region, *Baur*, no. 163 (hb. Gr. & hb. Cogniaux). Endemic.

HALORRHAGIDACEAE.

MYRIOPHYLLUM, L.

M. sp. Wolf (1), 284. — CHARLES ISL.: in brook near the hacienda, acc. to Wolf, l. c.

UMBELLIFERAE.

APIUM, L.

A. LACINIATUM, Urb. in Mart. Fl. Bras. xi. pt. 1, 343 (1879). *Helosciadium laciniatum,* DC. Mém. Soc. Phys. Genèv. iv. 495 (1828); Hook. f. (3), 215; Anderss. (1), 219, & (2), 92. — CHARLES ISL.: *Darwin,* acc. to Hook. f., l. c. Further distrib. Peru to Chili.

A. LEPTOPHYLLUM, F. Muell. acc. to Benth. Fl. Austr. iii. 372 (1866). *A. Ammi,* Urb. in Mart. Fl. Bras. xi. pt. 1, 341, q. v. for extensive synon. *Helosciadium leptophyllum,* DC. Mém. Soc. Phys. Genèv. iv. 493 (1828); Hook. f. (3), 215; Anderss. (1), 219, & (2), 92. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller,* no. 41 (hb. Gr.). JAMES ISL.: *Darwin.* Further distrib. N. Am , Mex., S. Am., Austral., etc.

CENTELLA, L.

C. ASIATICA, Urb. in Mart. Fl. Bras. xi. pt. 1, 287 (1879). *Hydrocotyle asiatica,* L. Sp. 234 (1753). *H. repanda,* Pers. Syn. i. 302 (1805); Caruel (1), 623. — CHATHAM ISL.: *Chierchia,* acc. to Caruel. Further distrib. N. Am., S. Am., Asia, etc.

HYDROCOTYLE, L.

H. **galapagensis**, nov. sp., glaberrima, repens, radicibus ad nodos fibrosis petiolis erectis teretibus, foliis orbicularibus peltatis prope centrum insertis 12–13-radiati-nervatis, margine duplo dentata, nervis venulisque translucentibus, pedunculis erectis, umbellis simplicibus sub 16-floribus non proliferis: involucris bracteis ovatis brevibus pedicellis flores quadri-quintuple excedentibus: calycis limbo obsoleto, petalibus ovatis obtusis patentibus, albis, fructu latiori quam longo basi rotundato vel subcordato. *H. umbellata,* Rob. & Greenm. (1), 146, not L. — CHATHAM ISL.: southwest end, upper region, *Baur,* no. 150 (hb. Gr.). Near *H. umbellata,* L., which it resembles rather closely in habit, flowers, and fruit, the leaves, however, differ rather conspicuously by their doubly dentate not crenate margin from the Linnaean species. Transitional forms have been sought in vain in material of *H. umbellata* from many other parts of the world.

PETROSELINUM, Koch.

P. SATIVUM, Hoffm. Gen. Umb. 177 (1814); Anderss. (1), 219, & (2), 92. *Apium Petroselinum,* L. Sp. 264 (1753). *Carum Petrose-*

linum, Benth. & Hook. f. Gen. i. 891 (1867). — CHARLES ISL.: cultivated ground, middle and upper region, *Andersson* (hb. Gr.). Introd. from the Old World.

PLUMBAGINACEAE.

PLUMBAGO, L.

P. SCANDENS, L. Sp. ed. 2, 215 (1762); Hook. f. (3), 194; Anderss. (1), 172, & (2), 65; Caruel (1), 623; Rob. & Greenm. (1), 147. — ALBEMARLE ISL.: *Darwin; southern part, Baur*, no. 235 (hb. Gr.); mountain north of Elizabeth Bay, *Snodgrass & Heller*, no. 284 (hb. Gr.); Tagus Cove, *Snodgrass & Heller*, no. 220 (hb. Gr.). CHARLES ISL.: *Darwin; Andersson* (hb. Gr.); *Chierchia; Lee* (hb. U. S. Nat. Mus.); *Baur*, no. 234 (hb. Gr.): *Snodgrass & Heller*, no. 411 (hb. Gr.). CHATHAM ISL.: *Andersson; Snodgrass & Heller*, no. 506 (hb. Gr.). DUNCAN ISL.: *Baur*, no. 236 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson*. Further distrib. general in warm countries.

(*P. tomentosa*, Hook. f., l. c., is a typographical error for *Plantago tomentosa*.)

APOCYNACEAE.

VALLESIA, R. & P.

V. CYMBAEFOLIA, Ort. Hort. Matr. Dec. 58 (1798). *V. glabra*, Link, Enum. Hort. Berol. i. 207 (1821); Hook. f. (3), 205; Anderss. (1) 195, & (2), 78. *Rauwolfia glabra*, Cav. Ic. iii. 50, t. 297 (1795). *Psychotria angustata*, Rob. & Greenm. (1), 146, not Anderss. — GALAPAGOS IDS.: *Edmonston* (hb. Gr.). ALBEMARLE ISL.: Point Christopher, *Snodgrass & Heller*, no. 929 (hb. Gr.); Tagus Cove, *Snodgrass & Heller*, no. 926 (hb. Gr.). CHATHAM ISL.: *Darwin; Andersson*. HOOD ISL.: *Baur*, no. 149 (hb. Gr.). Further distrib. S. Am., Mex., W. Ind., Florida.

V. PUBESCENS, Anderss. (1), 195, & (2), 79. — CHARLES ISL.: *Snodgrass & Heller*, no. 451 (hb. Gr.). CHATHAM ISL.: woods in lower region, *Andersson; Snodgrass & Heller*, no. 518 (hb. Gr.). Endemic.

ASCLEPIADACEAE.

ASCLEPIAS, L.

A. ANGUSTISSIMA, Anderss. (1), 196, & (2), 79. *Vincetoxicum?* Rob. & Greenm. (1), 147, in part (as to pl. Albemarle). — ABINGDON

ISL.: *Snodgrass* & *Heller*, no. 845 (hb. Gr.). ALBEMARLE ISL.: in the driest part of the middle region, *Andersson;* southern part, *Baur*, without number (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 145 (hb. Gr.). Endemic.

VINCETOXICUM, Moench.

V.? Rob. & Greenm. (1), 147, in part (as to pl. Charles). — CHARLES ISL.: Cuevas Bay, *Baur*, without number (hb. Gr.). Sterile and doubtful.

CONVOLVULACEAE.

CALYSTEGIA, R. Br.

C. SOLDANELLA, R. Br. Prodr. 483 (1810); Hook. f. (4), 261; Anderss. (1), 212, & (2), 87. *Convolvulus Soldanella*, L. Sp. 159 (1753); Engl. Bot. v. t. 314; Gray, Syn. Fl. N. A. ii. pt. 1, 215. — CHARLES ISL.: *Edmonston*, acc. to Hook. f., l. c. Further distrib. general, Pacific shores of N. and S. Am., also in Eu., N. Zealand, etc.

CUSCUTA, L.

C. ACUTA, Engelm. Trans. Acad. Sci. St. Louis, i. 497 (1859); Anderss. (2), 89. — BINDLOE ISL.: *Snodgrass* & *Heller*, no. 769 (hb. Gr.), identity doubtful. CHARLES ISL.: *Andersson*. CHATHAM ISL.: *Andersson*. NARBOROUGH ISL.: southern part, not common, 600 m. alt., *Snodgrass* & *Heller*, no. 318 (hb. Gr.). Endemic.

C. GYMNOCARPA, Engelm. Trans. Acad. Sci. St. Louis, i. 496 (1859); Anderss. (2), 89; Rob. & Greenm. (1), 147. *C. sandvicensis*, var. *Mimosae*, Hook. f. (3), 205; Anderss. (2), 89 (*sandwicensis*). *C. sandwichiana*, var. *Mimosae*, Anderss. (1), 214. — GALAPAGOS IDS.: *Habel*. ALBEMARLE ISL.: eastern portion, Cowley Bay, *Baur*, no. 205 (hb. Gr.), parasitic on *Borreria suberecta*, Hook. f., and doubtfully identical with the plant of Darwin. JAMES ISL.: *Darwin*. Endemic.

EVOLVULUS, L.

E. GLABER, Spreng. Syst. i. 862 (1825); Hallier, Jahrb. Hamb. Wiss. Anst. xvi. 22. *E. hirsutus*, HBK. Nov. Gen. & Sp. iii. 117 (1818), not Lam. *E. mucronatus*, Sw. acc. to Wikstr. Vet. Acad. Handl. Stockh. 1827, p. 61. *E. glabriusculus*, Chois. Diss. Conv. 156 (1838); Hook. f. (3), 205; Anderss. (1), 211, & (2), 87; Rose (1), 137; Rob. & Greenm. (1), 147. — ALBEMARLE ISL.: *Andersson;* Iguana Cove, *Snodgrass* & *Heller,* no. 32 (hb. Gr.). CHARLES ISL.: *Snodgrass* & *Heller,* no. 432 (hb. Gr.); *A. Agassiz* (hb. Gr.); *Baur.* CHATHAM ISL.: *Andersson* (hb. Gr.); southwest end, middle region, *Baur,* no. 203 (hb. Gr.); *Snodgrass* & *Heller,* no. 517 (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller,* no. 687 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson.* JAMES ISL.: *Scouler.* SEYMOUR ISL.: north, *Snodgrass* & *Heller,* no. 564 (hb. Gr.). Further distrib. S. Am., W. Ind. Said by Hallier, l. c., to have been collected on Chatham and Hood Islands by Steindachner; but Dr. Steindachner visited neither of these islands.

E. SIMPLEX, Anderss. (1), 211, & (2), 87; Rob. & Greenm. (1), 147. — CHARLES ISL.: *Baur* (hb. Gr.); *Snodgrass* & *Heller,* no. 431 (hb. Gr.). CHATHAM ISL.: *Andersson; Baur,* no. 201 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson* (hb. Gr.); northern part, *Snodgrass* & *Heller,* no. 659 (hb. Gr.). JAMES ISL.: common on bluff near beach, James Bay, *Snodgrass* & *Heller,* no. 384 (hb. Gr.). Endemic.

IPOMOEA, L.

I. BILOBA, Forsk. Fl. Aegypt.-Arab. 44 (1775); Hook. f. & Jacks. Ind. Kew. i. 1223. *I. Pes-caprae,* Sweet, Hort. Suburb. Lond. 35 (1818); Roth, Nov. Pl. Sp. 109; Anderss. (1), 212, & (2), 87. *I. maritima,* R. Br. Prodr. 486 (1810); Hook. f. (3), 204. — ALBEMARLE ISL.: Black Bight, *Snodgrass* & *Heller,* no. 257 (hb. Gr.); pebbly beach, Iguana Cove, *Snodgrass* & *Heller,* no. 125 (hb. Gr.). CHATHAM ISL.: *Darwin; Andersson.* Widely distrib. in trop. reg.

I. BONA-NOX, L. Sp. ed. 2, 228 (1762); Sims, Bot. Mag. t. 752. — ALBEMARLE ISL.: *Snodgrass* & *Heller,* no. 872 (hb. Gr.). Widely distrib. and extensively cultivated.

I. CAMPANULATA, L. Sp. 160 (1753); Wight, Ic. Pl. Ind. iv. t. 1375. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller,* no. 43 (hb. Gr.). Further distrib. East India, Malayan Archipelago. This is, I believe, the only instance in which an Old World species occurs on the Galapagos Ids. which has not been reported from any part of the American continent.

I. GALAPAGENSIS, Anderss. (1), 213, & (2), 88. — ALBEMARLE ISL.: *Lee* (hb. U. S. Nat. Mus.); Iguana Cove, *Snodgrass* & *Heller,* nos. 33 (hb. Gr.), 94 (hb. Gr.). CHARLES ISL.: *Andersson; Snodgrass* & *Heller,* no. 464 (hb. Gr.). CHATHAM ISL.: *Andersson.* DUNCAN ISL.: *Snodgrass* & *Heller,* no. 690 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller,* no. 728 (hb. Gr.). JAMES ISL.: common in any soil, James

Bay, *Snodgrass* & *Heller*, no. 374 (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 583 (hb. Gr.). Endemic.

I. HABELIANA, Oliv. in Hook. Ic. t. 1099 (1871). *I.* sp. Rose (1), 137. — BINDLOE ISL.: *Snodgrass* & *Heller*, no. 759 (hb. Gr.). DUNCAN ISL.: *A. Agassiz* (hb. U. S. Nat. Mus.); *Snodgrass* & *Heller*, no. 712 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 616 (hb. Gr.). HOOD ISL.: *Habel* (hb. Kew); *Snodgrass* & *Heller*, no. 751 (hb. Gr.). TOWER ISL.: *Snodgrass* & *Heller*, no. 799 (hb. Gr.). Endemic. A peculiar species with entire ovate-lanceolate attenuate glabrous leaves 12 to 18 cm. long, oblong obtuse sepals and a tubular corolla 9 to 15 cm. long. I am indebted to Dr. H. Hallier of Hamburg for his examination of this species.

I. KINBERGI, Anderss. (1), 212, & (2), 88; Rob. & Greenm. (1), 147. — GALAPAGOS IDS.: *Habel.* ABINGDON ISL.: *Snodgrass* & *Heller*, no. 839 (hb. Gr.). CHATHAM ISL.: *Andersson.* INDEFATIGABLE ISL.: *Andersson;* south of Conway Bay, *Baur*, no. 195 (hb. Gr.); northern part, *Snodgrass* & *Heller*, no. 674 (hb. Gr.). JERVIS ISL.: *Baur*, no. 196 (hb. Gr.). TOWER ISL.: *Snodgrass* & *Heller*, no. 801 (hb. Gr.). WENMAN ISL.: *Snodgrass* & *Heller*, no. 5 (hb. Gr.). Endemic.

I. LINEARIFOLIA, Hook. f. (3), 204; Anderss. (1), 212, & (2), 88. — JAMES ISL.: *Darwin*, acc. to Hook. f. Endemic. Secured by no other collector. Ascribed in the Index Kewensis to the Cape Verde Ids.

I. NIL, Roth, Catalect. i. 36 (1797). *I. acuminata*, Morong & Britt. Ann. N. Y. Acad. Sci. vii. 169 (1893); Rob. & Greenm. (1), 147; not R. & S. *Pharbitis Nil*, Choisy, Mém. Soc. Phys. Genèv. vi. 441 (1833). — CHARLES ISL.: *Snodgrass* & *Heller*, no. 418 (hb. Gr.). CHATHAM ISL.: southwestern end, lower region, *Baur*, no. 200 (hb. Gr.); *Snodgrass* & *Heller*, no. 511 (hb. Gr.). Further distrib. general in warm regions.

An indeterminate *Ipomoea* from Indefatigable Isl., mentioned and briefly described by Andersson (1), 214, & (2), 88, may well have been this species.

I. PENTAPHYLLA, Jacq. Coll. ii. 297 (1788), & Ic. Pl. Rar. ii. 10, t. 319; Rob. & Greenm. (1), 147. *Batatas pentaphylla*, Choisy, Mém. Soc. Phys. Genèv. vi. 408 (1833); Anderss. (1), 214, & (2), 89. *Merremia pentaphylla*, Hallier in Engl. Jahrb. xvi. 552 (1893). — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 846 (hb. Gr.). CHARLES ISL.: *Andersson.* CHATHAM ISL.: *Andersson.* DUNCAN ISL.: *Snod-*

grass & *Heller*, no. 707 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 628 (hb. Gr.). HOOD ISL.: *Baur*, no. 198 (hb. Gr.); *Snodgrass* & *Heller*, no. 729 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson*; northern part, *Snodgrass* & *Heller*, no. 671 (hb. Gr.). JAMES ISL.: *Andersson*. JERVIS ISL.: *Baur*, no. 199 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 568 (hb. Gr.). TOWER ISL.: *Snodgrass* & *Heller*, no. 804 (hb. Gr.). Further distrib. general in trop.

I. TUBIFLORA, Hook. f. (3), 204; Anderss. (1), 213, & (2), 88.—JAMES ISL.: *Darwin*, acc. to Hook. f., l. c. Endemic. Secured by no other collector.

BORAGINACEAE.

COLDENIA, L.

C. DARWINI, Gürke in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 3a, 90 (1893). *C. Darwinii*, Rob. & Greenm. (1), 147 (excl. pl. Albemarle). *Galapagoa Darwini*, Hook. f. (3), 196; Anderss. (1), 210, & (2), 86, t. 16, f. 1.—GALAPAGOS IDS.: *Edmonston* (hb. Gr.); *Habel*. ALBEMARLE ISL.: *Macrae*. BINDLOE ISL.: *Baur*, no. 383 (hb. Gr.); *Snodgrass* & *Heller*, no. 764 (hb. Gr.). CHATHAM ISL.: *Darwin*; *Andersson* (hb. Gr.). INDEFATIGABLE ISL.: Conway Bay, *Baur*, no. 385 (hb. Gr.); northern part, *Snodgrass* & *Heller*, no. 679 (hb. Gr.). JAMES ISL.: Orchilla Bay, *Baur*, no. 384 (hb. Gr.). Endemic.

C. FUSCA, Gürke in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 3a, 90 (1893); Rob. & Greenm. (1), 147. *C. Darwinii*, Rob. & Greenm. (1), 147, as to pl. Albemarle. *Galapagoa fusca*, Hook. f. (3), 197; Anderss. (1), 210, & (2), 87, t. 16, f. 2.—ALBEMARLE ISL.: common on tufa soil about Tagus Cove, from beach to summit of hills, *Snodgrass* & *Heller*, no. 180 (hb. Gr.); southern portion, *Baur*, no. 382 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 468 (hb. Gr.). CHARLES ISL.: *Darwin*. CHATHAM ISL.: southwest end, lower region, *Baur*, no. 217 (hb. Gr.). HOOD ISL.: *Baur*, no. 218 (hb. Gr.). INDEFATIGABLE ISL.: in dry sandy places on the shore, *Andersson* (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 587 (hb. Gr.). Endemic.

CORDIA, L.

C. ANDERSSONI, Gürke in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 3a, 83 (1893). *Varronia canescens*, Anderss. (1), 203, & (2), 83, t. 11, f. 2. *V. leucophlyctis*, Anderss. (1), 203, & (2), 83, t. 11, f. 1, not

C. leucophlyctis, Hook. f. — CHARLES ISL.: in wooded places, lower region, *Andersson* (hb. Gr.); *Lee* (hb. U. S. Nat. Mus. & hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.). Endemic.

C. GALAPAGENSIS, Gürke, l. c. *C. scaberrima*, Rob. & Greenm. (1), 147, not HBK. *C.* sp., Rob. & Greenm. (1), 147. *Varronia scaberrima*, Anderss. (1), 202, & (2), 82, t. 11, f. 3. — ALBEMARLE ISL.: eastern part, *Baur*, no. 210 (hb. Gr.); Cowley Bay, *Baur*, no. 212 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, nos. 75 (hb. Gr.), 136 (hb. Gr.), 857 (hb. Gr.); mountain north of Elizabeth Bay, *Snodgrass* & *Heller*, no. 291 (hb. Gr.); Tagus Cove, from near beach to 1300 m. alt., *Snodgrass* & *Heller*, nos. 195 (hb. Gr.), 881 (hb. Gr.), 893 (hb. Gr.). DUNCAN ISL.: *Baur*, no. 215 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson;* south of Conway Bay, *Baur*, no. 211 (hb. Gr.). NARBOROUGH ISL.: southern part, growing 1.3 m. high, spreading 2 m. or more, altitude 650 m., *Snodgrass* & *Heller*, nos. 331 (hb. Gr.), 342 (hb. Gr.). Endemic.

C. HOOKERIANA, Gürke, l. c. *C. linearis*, Hook. f. (3), 199; Rob. & Greenm. (1), 147; not DC. *Varronia linearis*, Anderss. (1), 204, & (2), 84, t. 11, f. 4. *Lithocardium Hookerianum*, O. Kuntze, Rev. Gen. ii. 976 (1891). — ALBEMARLE ISL.: Elizabeth Cove, *Snodgrass* & *Heller*, no. 272 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 28 (hb. Gr.); southern portion, *Baur*, no. 213 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, nos. 155 (hb. Gr.), 196 (hb. Gr.), above 650 m., no. 897 (hb. Gr.). CHARLES ISL.: *Baur*, no. 214 (hb. Gr.). JAMES ISL.; *Darwin; Andersson*. NARBOROUGH ISL.: arborescent, 3 or 4 m. high, common in southern part at 600 m. alt., *Snodgrass* & *Heller*, no. 327 (hb. Gr.). Endemic.

C. LEUCOPHLYCTIS, Hook. f. (3), 199; Gürke, l. c. 83; Rob. & Greenm. (1), 147. *Lithocardium leucophlyctis*, O. Kuntze, Rev. Gen. ii. 977 (1891). — GALAPAGOS IDS.; *Habel*. ALBEMARLE ISL.: *Macrae; Darwin*. JAMES ISL.: *Scouler;* ? Orchilla Bay, *Baur*, no. 209 (hb. Gr.). Endemic.

C. LUTEA, Lam. Ill. i. 421 (1791); Hook. f. (3), 198; Rose (1), 137; Rob. & Greenm. (1), 147. *C. rotundifolia*, Ruiz & Pavon, Fl. Per. ii. 24, t. 148, fig. *a* (very bad, especially as to corolla), 1799; Hook. f. & Jacks. Ind. Kew. i. 614 (where ascribed to Panama instead of Peru). *Varronia rotundifolia*, DC. Prodr. ix. 469 (1845). *V. flava*, Anderss. (1), 201, & (2), 82. — GALAPAGOS IDS.: *Habel*. ABINGDON ISL.: *Snodgrass* & *Heller*, no. 821 (hb. Gr.). ALBEMARLE ISL.: *Lee* (hb. U. S. Nat. Mus.); *Macrae:* Iguana Cove, *Snodgrass* & *Heller*, no. 74 (hb. Gr.); mountain north of Elizabeth Bay, *Snodgrass* & *Heller*, no. 294 (hb. Gr.); also on lava fields near beach, Elizabeth Bay, *Snodgrass* & *Heller*, no. 940 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, nos. 159 (hb. Gr.), 925 (hb. Gr.). BARRINGTON ISL.: *Baur*, no. 231 (hb. Gr.); *Snodgrass* & *Heller*, no. 473 (hb. Gr.). BINDLOE ISL.: *Baur*, no. 232 (hb. Gr.); *Snodgrass* & *Heller*, no. 787 (hb. Gr.). CHARLES ISL.: *Andersson; A. Agassiz; Baur*, no. 228 (hb. Gr.); *Snodgrass* & *Heller*, no. 419 (hb. Gr.). CHATHAM ISL.: *Darwin: Andersson* (hb. Gr.): southwest end, lower region, *Baur*, no. 208 (hb. Gr.); *Snodgrass* & *Heller*, no. 524 (hb. Gr.). DUNCAN ISL.: *A. Agassiz* (hb. Gr.); *Baur*, no. 230 (hb. Gr.); very abundant on hills, *Snodgrass* & *Heller*, no. 708 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 642 (hb. Gr.). HOOD ISL.: *Baur*, no. 229 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson*. JAMES ISL.: *Andersson;* James Bay, 2 to 3 m. high, not common, *Snodgrass* & *Heller*, no. 365 (hb. Gr.). JERVIS ISL.: *Baur*, no. 233 (hb. Gr.). SEYMOUR ISL.: north, *Snodgrass* & *Heller*, no. 569 (hb. Gr.); south, *Snodgrass* & *Heller*, no. 604 (hb. Gr.). TOWER ISL.: *Snodgrass* & *Heller*, no. 793 (hb. Gr.). Further distrib. Ecuador, Peru, Bolivia. To our present knowledge this is the most widely distributed plant on the Archipelago, occurring as it does upon no less than fourteen islands. It is, however, remarkably constant in its characters.

C. REVOLUTA, Hook. f. (3), 199. *Varronia revoluta*, Anderss. (1), 204, & (2), 84. *Lithocardium revolutum*, O. Kuntze, Rev. Gen. ii. 977 (1891). — GALAPAGOS IDS.: *Habel*. CHARLES ISL.: *Darwin*. Endemic.

Var. NIGRICANS, Hook. f. (3), 199. *Varronia revoluta*, var. *nigricans*, Anderss. (1), 204, & (2), 84. — ALBEMARLE ISL.: *Macrae*. Endemic.

C. SCOULERI, Hook. f. (3), 200; Rob. & Greenm. (1), 147. *Varronia Scouleri*, Anderss. (1), 204, & (2), 83. *Lithocardium Scouleri*, O. Kuntze, Rev. Gen. ii. 977 (1891). — CHATHAM ISL.: *Andersson;* southwest end, middle region, *Baur*, no. 216 (hb. Gr.). JAMES ISL.: *Scouler*. Endemic.

C. nov. sp.? *C. dasycephala*, Anderss. (1), 204, & (2), 84, not HBK. *Varronia dasycephala*, Hook. f. (4), 261, not Desv. — CHARLES ISL.: *Edmonston* (hb. Gr.). Certainly different from any other species on the Islands, and clearly distinguished from *C. dasycephala* by the simple widely spreading tawny setae which cover the stem. Unfortunately the material is too poor for description. Endemic.

HELIOTROPIUM, L.

H. **Anderssonii.** *H. asperrimum*, Anderss. (2), 86, not R. Br. *Sarcanthus asperrimus*, Anderss. (1), 209. — INDEFATIGABLE ISL.: *Andersson* (hb. Gr.). Endemic.

H. CURASSAVICUM, L. Sp. 130 (1753); Hook. f. (3), 198; Anderss. (1), 208, & (2), 86; Rose (2), 137, in part; Rob. & Greenm. (1), 147. — GALAPAGOS IDS.: *Habel.* BINDLOE ISL.: *Snodgrass* & *Heller*, no. 766 (hb. Gr.). CHATHAM ISL.: *Darwin; Andersson* (hb. Gr.); *A. Agassiz* (hb. Gr. & hb. U. S. Nat. Mus.); *Baur*, no. 219 (hb. Gr.); *Snodgrass* & *Heller*, no. 514 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 725 (hb. Gr.). INDEFATIGABLE ISL.: northern part, *Snodgrass* & *Heller*, no. 660 (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 560 (hb. Gr.). Further distrib. wide, especially upon trop. shores.

H. INDICUM, L. Sp. 130 (1753); Hook. f. (4), 261. *Heliophytum indicum*, DC. Prodr. ix. 556; Anderss. (1), 208, & (2) 86. — CHARLES ISL.: *Edmonston; Lee* (hb. Gr. & hb. U. S. Nat. Mus.); *Snodgrass* & *Heller*, no. 447 (hb. Gr.). Widely distrib. weed of warm countries.

H. PARVIFLORUM, L. Mant. 201 (1771); Hook. f. (3), 198; Rob. & Greenm. (1), 147. *H. curassavicum*, Rose (1), 137, in part. *Heliophytum parviflorum*, DC. Prodr. ix. 553 (1845); Anderss. (1), 208, & (2), 86. — GALAPAGOS IDS.: *Habel.* ABINGDON ISL.: *Snodgrass* & *Heller*, no. 812 (hb. Gr.). ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 80 (hb. Gr.); Tagus Cove, most abundant near the coast, but also found inland to 300 m. alt., *Snodgrass* & *Heller*, nos. 160 (hb. Gr.), 200 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 486 (hb. Gr.). CHARLES ISL.: *Darwin; Andersson; Lee* (hb. Gr.); *Baur; Snodgrass* & *Heller*, no. 463 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.); *A. Agassiz* (hb. Gr. & hb. U. S. Nat. Mus.); southwest end, upper region, *Baur*, no. 220 (hb. Gr.); lower region, *Baur*, no. 221 (hb. Gr.); *Snodgrass* & *Heller*, no. 510 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 639 (hb. Gr.). HOOD ISL.: *Baur*, no. 222 (hb. Gr.), through typographical error ascribed to Duncan Isl. by Rob. & Greenm. l. c.; *Snodgrass* & *Heller*, no. 747 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson.* JAMES ISL.: *Douglas;* James Bay, scattered in lava soil, *Snodgrass* & *Heller*, no. 357 (hb. Gr.). NARBOROUGH ISL.: southern part, rather common at 650 m. alt., *Snodgrass* & *Heller*, no. 348 (hb. Gr.). TOWER ISL.: *Snodgrass* & *Heller*, no. 792 (hb. Gr.). Widely distrib. in warm countries.

TOURNEFORTIA, L.

T. HIRSUTISSIMA, L. Sp. 140 (1753); Caruel (1), 622. — CHATHAM ISL.: *Chierchia*, acc. to Caruel. Further distrib. trop. Am. I suspect that Chierchia's specimen will on examination prove to be *T. rufo-sericea*, Hook. f.

T. LAURIFOLIA, Vent. Choix Pl. 2 (1803). *T. syringaefolia*, Vahl, Symb. iii. 23 (1794); Anderss. (1), 206, & (2), 84, not Miq.; ? Caruel (1), 622. — CHATHAM ISL.: *Chierchia*, acc. to Caruel, sterile and doubtful. JAMES ISL.: *Andersson.* Further distrib. Mex., trop. S. Am. A doubtful member of the Galapageian flora.

T. PSILOSTACHYA, HBK. Nov. Gen. & Sp. iii. 78 (1818); Cham. Linnaea, iv. 470; DC. Prodr. ix. 525; Hook. f. (3), 198 (*psilostachys*); Anderss. (1), 208, & (2), 85; Rob. & Greenm. (1), 147. *? T. difformis*, Anderss. (1), 206, & (2), 85. — ALBEMARLE ISL.: Iguana Cove, the commonest shrub, everywhere from the beach to 650 m., *Snodgrass* & *Heller*, nos. 81 (hb. Gr.), 120 (hb. Gr.). CHARLES ISL.: *Lee* (hb. U. S. Nat. Mus.); *Snodgrass* & *Heller*, no. 436 (hb. Gr.). CHATHAM ISL.: *Andersson* (*T. difformis*, Anderss.). HOOD ISL.: *Baur*, no. 227 (hb. Gr.). JAMES ISL.: *Douglas* & *Scouler*, acc. to Hook. f.; common near sandy beach, *Snodgrass* & *Heller*, no. 363 (hb. Gr.). Further distrib. trop. S. Am. This species appears to have recently become abundant in the Galapagos Islands.

T. PUBESCENS, Hook. f. (3), 198; Anderss. (1), 206, & (2), 84; Rob. & Greenm. (1), 147. *T. opaca*, Anderss. (1), 205, & (2), 84; Rob. & Greenm. (1), 147. — ALBEMARLE ISL.: Iguana Cove, abundant near beach, and to 650 m. alt., *Snodgrass* & *Heller*, no. 119 (hb. Gr.); bushes 2 to 4 m. high, *Snodgrass* & *Heller*, nos. 135 (hb. Gr.), 870 (hb. Gr.); mountain east of Tagus Cove, *Snodgrass* & *Heller*, no. 255 (hb. Gr.); southern part, *Baur*, no. 225 (hb. Gr.). CHARLES ISL.: *Lee* (hb. Gr.). CHATHAM ISL.: *Darwin*; southwest end, middle region, *Baur*, no. 206 (hb. Gr.). DUNCAN ISL.: *Baur*, no. 226 (hb. Gr.). INDEFATIGABLE ISL.: in wooded places of the lower region, *Andersson* (hb. Gr.). Endemic. I am quite unable to separate *T. opaca*, Anderss., which appears to be only a glabrate state.

T. RUFO-SERICEA, Hook. f. (3), 197; Anderss. (1), 205, & (2), 84; Rob. & Greenm. (1), 147. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 816 (hb. Gr.). ALBEMARLE ISL.: Iguana Cove, up to 650 m. alt., *Snodgrass* & *Heller*, nos. 54 (hb. Gr.), 853 (hb. Gr.), 868 (hb. Gr.); southern part, *Baur*, no. 207 (hb. Gr.); Tagus Cove, 1300 m. alt., *Snodgrass* & *Heller*, nos. 883 (hb. Gr.), 884 (hb. Gr.). CHARLES ISL.: *Andersson* (hb. Gr.); *Lee* (hb. Gr. & hb. U. S. Nat. Mus.). JAMES ISL.: *Darwin*. Endemic.

T. STRIGOSA, Anderss. (1), 207, & (2), 85, t. 9, f. 3; Rob. & Greenm. (1), 147. — ALBEMARLE ISL.: southern part, *Baur*, no. 224 (hb. Gr.). CHARLES ISL.: wooded places, lower region, *Andersson*. CHATHAM ISL.: *Andersson*. Endemic. This species is reduced to *T. psilostachya* by Hook. f. & Jacks. (Ind. Kew. ii. 1091), but it differs from that species considerably in pubescence and inflorescence, if (as seems probable) Andersson's plant is represented by Baur's no. 224.

VERBENACEAE.

AVICENNIA, L.

A. OFFICINALIS, L. Sp. 110 (1753). *A. tomentosa*, Jacq. Stirp. Am. 178, t. 112 (1763); Hook. f. (3), 195; Anderss. (1), 201, & (2), 82; Rob. & Greenm. (1), 147. — GALAPAGOS IDS.: *Edmonston*; *Habel*. ALBEMARLE ISL.: Elizabeth Bay, about lagoons with *Rhizophora*, acc. to Mr. Heller. CHARLES ISL.: Cormorant Bay, *Baur*, no. 171 (hb. Gr.). CHATHAM ISL.: *Darwin*. INDEFATIGABLE ISL.: forming swamps in lagoons on the north coast, acc. to Mr. Heller. JAMES ISL.: *Andersson*; tree 3 to 6 m. high, along margin of a salt pond, James Bay, *Snodgrass* & *Heller*, no. 368 (hb. Gr.). NARBOROUGH ISL.: fringing lagoons on the east coast, acc. to Mr. Heller. SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 605 (hb. Gr.). Further distrib. general on trop. shores.

CLERODENDRON, L.

C. MOLLE, HBK. Nov. Gen. & Sp. ii. 244 (1817); Hook. f. (3), 195; Anderss. (1), 201, & (2), 82; Caruel (1), 622; Rose (1), 137; Rob. & Greenm. (1), 147. — GALAPAGOS IDS.: *Habel*. ALBEMARLE ISL.: southern portion, *Baur*, no. 168 (hb. Gr.); Iguana Cove, rather common from beach to 300 m. alt., *Snodgrass* & *Heller*, nos. 59 (hb. Gr.), 116 (hb. Gr.), 855 (hb. Gr.). CHARLES ISL.: *Darwin*; *Andersson*; *Lee* (hb. U. S. Nat. Mus.); *A. Agassiz*; *Snodgrass* & *Heller*, no. 443 (hb. Gr.). CHATHAM ISL.: *Andersson*; *Chierchia*; *A. Agassiz* (hb. Gr.); in upper wooded region, *Baur*, no. 170 (hb. Gr.). JAMES ISL.: *Scouler*; *Andersson*; common on lava coast, *Snodgrass* & *Heller*, no. 369 (hb. Gr.). Further distrib. Ecuador.

C. sp. Hook. f. (4), 261, Anderss. (1), 201, & (2), 82.— CHARLES ISL.: *Edmonston*, acc. to Hook. f., l. c.

C. sp. Hook. f., l. c.; Anderss. ll. cc. — CHARLES ISL.: *Edmonston*, acc. to Hook. f., l. c.

DURANTA, L.

D. PLUMIERI, Jacq. Stirp. Am. 186, t. 176, f. 76 (1763). — ALBEMARLE ISL.: mountain east of Tagus Cove, alt. 925 m., *Snodgrass* & *Heller*, nos. 248 (hb. Gr.), 906 (hb. Gr.). Widely distributed in trop. Am. The form on Albemarle has entire leaves and is closely matched by some specimens from Mexico.

LANTANA, L.

L. PEDUNCULARIS, Anderss. (1), 200, & (2), 81; Rob. & Greenm. (1), 147; *L.* sp. Rose (1), 137. Reduced to *L. odorata*, L., by Griseb. Fl. Brit. W. Ind. 496, and with scarcely a doubt the *L. recta* and *L. canescens* of Hook. f. (3), 195, and Anderss. ll. cc. and consequently *L. odorata*, Anderss. ll. cc. — GALAPAGOS IDS.: *Andersson*, no. 215 (hb. Gr.). ABINGDON ISL.: *Snodgrass* & *Heller*, no. 813 (hb. Gr.). ALBEMARLE ISL.: *Darwin* (*L. recta*, Hook. f.); *Andersson;* mountain north of Elizabeth Bay, *Snodgrass* & *Heller*, no. 293 (hb. Gr.); eastern portion, Cowley Bay, *Baur*, no. 174 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no 64 (hb. Gr.); Tagus Cove, from beach to 300 m. alt., not common, *Snodgrass* & *Heller*, no. 201 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 476 (hb. Gr.). BINDLOE ISL.: *Baur*, no. 187 (hb. Gr.); *Snodgrass* & *Heller*, no. 758 (hb. Gr.). CHARLES ISL.: *Darwin ; Andersson ; Lee* (hb. U. S. Nat. Mus.); *A. Agassiz* (hb. U. S. Nat. Mus.); *Baur*, no. 173 (hb. Gr.); *Snodgrass* & *Heller*, no. 445 (hb. Gr.). CHATHAM ISL.: *Andersson ; Snodgrass* & *Heller*, no. 515 (hb. Gr.), doubtful form. DUNCAN ISL.: *Baur ; Snodgrass* & *Heller*, no. 700 (hb. Gr.). GARDNER ISL.: *Snodgrass* & *Heller*, no. 613 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 736 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson ; Snodgrass* & *Heller*, no. 657 (hb. Gr.). JAMES ISL.: *Andersson ;* the most abundant bush, everywhere in lava soil, 1 to 2 m. high, *Snodgrass* & *Heller*, no. 381 (hb. Gr.). JERVIS ISL.: *Baur*, no. 176 (hb. Gr.). NARBOROUGH ISL.: southern part, 600 m. alt., *Snodgrass* & *Heller*, no. 351 (hb. Gr.). TOWER ISL.: *Snodgrass* & *Heller*, no. 806 (hb. Gr.). Endemic? While treating the plants, here mentioned, provisionally as a single endemic species, I suspect that they may be ultimately identified with one of the continental American species (cf. *L. lilacina* and *L. canescens*, HBK.), or segregated into several more or less distinct forms. The indefiniteness which now prevails regarding the S. American species of the genus is such as to render the further classification of the Galapageian forms, for the present, impracticable.

LIPPIA, Houst.

L. CANESCENS, HBK. Nov. Gen. & Sp. ii. 263 (1817). *L. lanceolata*, Rose (1), 137, not Michx. *L. nodiflora*, Cham. Linnaea, vii. 213 (1832); Rob. & Greenm. (1), 147; not Michx. — CHARLES ISL.: *Lee* (hb. U. S. Nat. Mus.); *Baur*. CHATHAM ISL.: *A. Agassiz* (hb. U. S. Nat. Mus.); *Baur*, no. 178 (hb. Gr.), a green form. DUNCAN ISL.: *Snodgrass & Heller*, no. 709 (hb. Gr.). HOOD ISL.: around a mud lake, *Snodgrass & Heller*, no. 755 (hb. Gr.). Further distrib. S. Am.

L. ROSMARINIFOLIA, Anderss. (1), 198, & (2), 80; Rob. & Greenm. (1), 147. — ABINGDON ISL.: *Snodgrass & Heller*, no. 827 (hb. Gr.). ALBEMARLE ISL.: in very dry places of the middle region, *Andersson*; Elizabeth Bay, *Snodgrass & Heller*, no. 280 (hb. Gr.); eastern part, *Baur*, no. 179 (hb. Gr.); Tagus Cove, not common, 150 to 460 m. alt., *Snodgrass & Heller*, nos. 168 (hb. Gr.), 147 (hb. Gr.); mountain east of Tagus Cove, *Snodgrass & Heller*, nos. 233, 251, 888, 890, & 896 (all in hb. Gr.). Nos. 888, 890 & 896 show that the leaves on some branches become strongly toothed. Endemic.

Baur's nos. 181 and 182 (both in hb. Gr.) from eastern Albemarle are probably branches of this species (sterile).

L. SALICIFOLIA, Anderss. (1), 198, & (2), 80. — CHARLES ISL.: in woods on the side of the mountain summit. *Andersson* (hb. Gr.). Endemic.

STACHYTARPHETA, Vahl.

S. DICHOTOMA, Vahl, Enum. i. 207 (1804); Caruel (1), 622. *S. urticifolia*, Sims, Bot. Mag. t. 1848 (1816). *Verbena dichotoma*, Ruiz & Pav. Fl. Per. i. 23, t. 34, fig. b (1798). *Bouchea* sp. Rob. & Greenm. (1), 147. — CHARLES ISL.: *Chierchia*, acc. to Caruel, l. c.; *Lee* (hb. Gr.); *Baur*, without number (hb. Gr.). Further distrib. trop. and subtrop. Am.

VERBENA, L.

V. CAROLINA, L. Syst. ed. 10, 852 (1760); Mill. Dict. ed. 8, no. 7. *V. polystachya*, HBK. Nov. Gen. & Sp. ii. 274 (1817); Hook. f. (3), 195, as var. *V. caroliniana*, Anderss. (1), 199, & (2), 81. — JAMES ISL.: *Darwin*, acc. to Hook. f. Further distrib. Mex., Andean S. Am.

V. GRISEA, Rob. & Greenm. (1), 142, 147, where by typographical error ascribed to Albemarle Island. — DUNCAN ISL.: *Baur*, no. 180 (hb. Gr.). Endemic.

V. LITORALIS, HBK. Nov. Gen. & Sp. ii. 276, t. 137 (1817); Hook. f. (3), 195; Anderss. (1), 200, & (2), 81; Rob. & Greenm. (1), 147. — CHARLES ISL.: *Darwin; Baur*, no. 172 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.). Further distrib. Mex., S. Am.

V. OFFICINALIS, L. Sp. 20 (1753); Hook. f. (1), 194; Anderss. (1), 199, & (2), 81. — JAMES ISL.: *Darwin*. Tropics of both hemispheres.

LABIATAE.

HYPTIS, Jacq.

H. CAPITATA, Jacq. Ic. Pl. Rar. i. t. 114; Hook. f. (4), 261; Anderss. (1), 197, & (2), 80. — CHARLES ISL.: *Edmonston*. Further distrib. trop. S. Am., Mex., W. Ind.

H. SUBVERTICILLATA, Anderss. (1), 197, & (2), 80. — ALBEMARLE ISL.: in very sterile places, middle region, *Andersson* (hb. Gr.). INDEFATIGABLE ISL.: *Baur*. (Of this plant, called *H. spicigera* by Rob. & Greenm. (1), 147, unfortunately no specimen was retained at herb. Gray, and it is now impossible to examine the material. I have little doubt, however, that it was *H. subverticillata*.) JAMES ISL.: James Bay, common on lava rocks, *Snodgrass & Heller*, no. 356 (hb. Gr.). NARBOROUGH ISL.: northern part, *Snodgrass & Heller*, no. 299 (hb. Gr.). Endemic.

SALVIA, L.

S. OCCIDENTALIS, Sw. Prodr. 14 (1788), & Fl. Ind. Occ. i. 43; Hook. f. (3), 200; Anderss. (1), 196, & (2), 79; Rob. & Greenm. (1), 147. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass & Heller*, nos. 61 (hb. Gr.), 96 (hb. Gr.); Tagus Cove, *Snodgrass & Heller*, no. 217 (hb. Gr.). CHARLES ISL.: *Darwin;* in dry places, upper region, *Andersson*, (hb. Gr.); *Baur*, no. 167 (hb. Gr.); *Snodgrass & Heller*, no. 407 (hb. Gr.). CHATHAM ISL.: southwest end, middle region, *Baur*, no. 166 (hb. Gr.). JAMES ISL.: *Darwin ;* common on rocks near beach, *Snodgrass & Heller*, no. 362 (hb. Gr.). Further distrib. Mex., W. Ind., S. Am. This may perhaps have been the sterile "*Lamiacea*" collected by Chierchia and mentioned by Caruel (1), 622.

S. PROSTRATA, Hook. f. (3), 200; Anderss. (1), 197, & (2), 79. — JAMES ISL.: *Darwin.* Endemic.

S. TILIAEFOLIA, Vahl, Symb. iii. 7 (1794); Hook. f. (3), 200; Anderss. (1), 196, & (2), 79. — CHARLES ISL.: *Darwin*, acc. to Hook. f., l. c. Further distrib. Mex., trop. S. Am.

TEUCRIUM, L.

T. INFLATUM, Sw. Prodr. 88 (1788); Hook. f. (3), 201; Anderss. (1), 197, & (2), 79; Rob. & Greenm. (1), 147. — CHARLES ISL.; *Darwin;* grassy places, upper region, *Andersson* (hb. Gr.). CHATHAM ISL.: southwest end, middle region, *Baur*, no. 164 (hb. Gr.). Further distrib. S. Am.

SOLANACEAE.

ACNISTUS, Schott.

A. ELLIPTICUS, Hook. f. in Miers, Lond. Jour. Bot. iv. 343 (1845), & (3), 203; Anderss. (1), 218, & (2), 91 (*Ancistus*). — CHARLES ISL.: *Darwin.* Endemic.

A. **insularis**, nov. sp., frutex; ramis a cortice griseo-brunnea tectis; ramulis pallidioribus in specimine siccato rugoso-striatis apice foliatis; foliis approximatis alternis obovatis integris penninervatis supra atroviridibus parce pubescentibus subtus pallidioribus molliter pubescentibus basi cuneatis apice rotundatis; pilis indumenti crispis; umbellis sessilibus 2–4-floris; pedicellis elongatis filiformibus nutantibus subglabris apice in calycem incrassatis; calyce campanulato tenui subglabro truncato; corolla tubulosa gradatim ampliata pilis crispis pubescenti; limbo 5-fido; dentibus deltoideis subobtusis; antheris in parte faucium superiori subsessilibus; stylo recto filiforme glabro; stigmate capitato. — *A.* sp. Rob. & Greenm. (1), 147. — CHATHAM ISL.: southwest end, middle region, June, 1891, *Baur*, no. 193 (hb. Gr.). Endemic. Leaf-blade 6 to 8 mm. long, half as broad; petiole 1.5 to 2 cm. long; pedicels 2 to 2.5 cm. long; corolla 3 cm. long; flowers more inclined to be nodding or even pendulous than indicated on the plate. This species must in many points resemble *A. ellipticus*, Hook. f. That, however, is described as having leaves attenuate at both ends and glabrous, the calyx-limb 5-crenate, the style somewhat curved and the stigma obscurely bilobed. PLATE 2, FIG. 3.

CAPSICUM, L.

C. ANNUUM, L. Sp. 188 (1753); Anderss. (1), 215, & (2), 90; Rob. & Greenm. (1), 147. — CHARLES ISL.: *Andersson* (hb. Gr.). CHATHAM ISL.: *Baur*. Widely distrib. in trop. reg.

DATURA, L.

D. TATULA, L. Sp. ed. 2, 256 (1762). — CHARLES ISL.: *Snodgrass* & *Heller*, no. 412 (hb. Gr.). Further distrib. U. S., Mex., S. Am.

D. sp. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 124 (hb. Gr.). Differing from the preceding in the having a fine but rather copious pubescence throughout, and fewer weaker bristles on the fruit.

LYCIUM, L.

L. sp. — HOOD ISL.: *Baur*, and probably the same from SEYMOUR ISL.: south, *Snodgrass* & *Heller*, no. 584 (hb. Gr.). Both sterile and indeterminate.

LYCOPERSICUM, Hill.

L. ESCULENTUM, Mill., var. MINOR, Hook. f. (3), 202 (*Lycopersicon*); Anderss. (1), 216, & (2), 90. *L. peruvianum*, Anderss. (1), 216, & (2), 91, at least as to his own plant from Chatham, which entirely lacks the large foliaceous bracts of the S. Am. plant. *L. esculentum*, Rob. & Greenm. (1), 147. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 843 (hb. Gr.). ALBEMARLE ISL.: *Andersson?* southern part, *Baur*, no. 188 (hb. Gr.); Pt. Christopher, *Snodgrass* & *Heller*, no. 928 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 911 (hb. Gr.). CHATHAM ISL.: *Andersson*. HOOD ISL.: *Baur*, no. 189 (hb. Gr.); *Snodgrass* & *Heller*, no. 741 (hb. Gr.). JAMES ISL.; *Darwin*; *Snodgrass* & *Heller*, no. 399 (hb. Gr.). NARBOROUGH ISL.: Mangrove Point, on rocks above beach, *Snodgrass* & *Heller*, no. 305 (hb. Gr.). Further distrib. S. Am., Polynesia.

L. PERUVIANUM, Mill., var. PARVIFLORUM, Hook. f. (3), 202 (*Lycopersicon peruanum*). — CHATHAM ISL.: *Darwin*, acc. to Hook. f. Further distrib. Andean S. Am. I have seen no specimens from the Galapagos Ids. with the foliaceous bracts said to be characteristic of this species. A variety of *L. peruvianum* was collected on the Galapagos Islands by Habel.

L. PIMPINELLIFOLIUM, Mill. Dict. ed. 8, no. 4 (*Lycopersicon*, 1768); Dun. in DC. Prodr. xiii. pt. 1, 23; Hook. f. (3), 202 (*Lycopersicon*);

Anderss. (1), 216 & (2), 91. *Solanum pimpinellifolium*, L. Amoen. Acad. iv. 268 (1759). — GALAPAGOS IDS.: *Goodridge* (hb. Gr.). CHARLES ISL.: *Andersson*. CHATHAM ISL.: *Darwin*; *Andersson*. JAMES ISL.: *Andersson*. Further distrib. Andean.

L. sp. — CHATHAM ISL.: *Snodgrass* & *Heller*, no. 526 (hb. Gr.). Very likely a mere variety of *L. esculentum*, Mill., but although minutely glandular, quite destitute of the spreading-hirsute character shown by the other Galapageian specimens at hand.

NICOTIANA, L.

N. GLUTINOSA, L. Sp. 181 (1753); Hook. f. (3), 202; Anderss. (1), 215, & (2), 89. — CHARLES ISL.: *Edmonston*; *Darwin*; *Andersson*. Further distrib. Andean S. Am.

N. TABACUM, L. Sp. i. 180 (1753); Caruel (1), 622. — CHARLES ISL.: *Chierchia*, acc. to Caruel, l. c. Further distrib. trop. Am. and widely introduced through cult.

N. sp. Hook. f. (4), 261. — CHARLES ISL.: *Edmonston*, acc. to Hook. f., l. c.

PHYSALIS, L.

P. ANGULATA, L. Sp. 183 (1753); Anderss. (1), 215, & (2), 90; Rose (1), 137. — CHARLES ISL.: *Andersson*; *A. Agassiz*, acc. to Rose, l. c. CHATHAM ISL.: *Snodgrass* & *Heller*, no. 522 (hb. Gr.). Further distrib. general in trop. Am.

P. IXOCARPA, Brot. in Hornem. Hort. Hafn. Suppl. 26 (1819); Rydberg, Mem. Torr. Club, iv. 334. *P. aequata*, Jacq. f., acc. to Nees, Linnaea, vi. 470 (1831). *P. pubescens*, Rob. & Greenm. (1), 147, as to pl. Charles. — CHARLES ISL.: Cuevas Bay, *Baur*, no. 186 (hb. Gr.); *Snodgrass* & *Heller*, no. 434 (hb. Gr.). Widely distrib. in Am.

P. PUBESCENS, L. Sp. 183 (1753); Rob. & Greenm. (1), 147, as to pl. Chatham; Rydberg, Mem. Torr. Club, iv. 322. — ALBEMARLE ISL.: Tagus Cove, abundant in shady places near beach, *Snodgrass* & *Heller*, nos. 185 (hb. Gr.), 187 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 768 (hb. Gr.). CHARLES ISL.: *Snodgrass* & *Heller*, no. 433 (hb. Gr.). CHATHAM ISL.: southwest end, upper region, *Baur*, no. 185 (hb. Gr.). HOOD ISL.: *Snodgrass* & *Heller*, no. 740 (hb. Gr.), a robust large-fruited form of doubtful identity. JAMES ISL.: James Bay, in sand near beach, *Snodgrass* & *Heller*, no. 388 (hb. Gr.). NAR-

BOROUGH ISL.: southern part, *Snodgrass* & *Heller*, no. 303 (hb. Gr.). Widely distrib. in Am.

P. sp. Hook. f. (4), 261. — CHARLES ISL.: *Edmonston.*

SOLANUM, L.

S. EDMONSTONEI, Hook. f. (3), 201; Dun. in DC. Prodr. xiii. pt. 1, 45; Anderss. (1), 216, & (2), 90. — CHARLES ISL.: *Edmonston.* Endemic.

S. NIGRUM, L. Sp. 186 (1753); Hook. f. (3), 201; Anderss. (1), 216, & (2), 90; Rob. & Greenm. (1), 147. This is, with little doubt, the *S. Berterii* of Caruel (1), 622, perhaps also of "Hort. Par. 1835" [1829 ?], a nomen nudum, so far as I have learned. — ALBEMARLE ISL.: southern part, *Baur*, no. 192 (hb. Gr.). CHARLES ISL.: *Darwin; Andersson* (hb. Gr.). CHATHAM ISL: *Chierchia; Snodgrass* & *Heller*, no. 521 (hb. Gr.). DUNCAN ISL.: *Baur*, no. 191 (hb. Gr.). JAMES ISL.: *Scouler; Darwin;* James Bay, scattered on sandy beach, *Snodgrass* & *Heller*, no. 387 (hb. Gr.). Further distrib. general. Various forms have been distinguished as to foliage and pubescence.

[S. TUBEROSUM, L., was included by Andersson in his second work (p. 90) but only on the basis of cultivated specimens.]

S. VERBASCIFOLIUM, L. Sp. 184 (1753); Jacq. Hort. Vindob. i. t. 13; Hook. f. (3), 201; Anderss. (1), 215, & (2), 90. — CHARLES ISL.: *Andersson.* JAMES ISL.: *Darwin.* NARBOROUGH ISL.: southern part, rare, 650 m. altitude, *Snodgrass* & *Heller*, nos. 329 (hb. Gr.), 353 (hb. Gr.). Further distrib. wide in trop. reg. Andersson, ll. cc., distinguishes two Galapageian forms on foliar differences.

S. sp. Hook. f. (4), 261. — CHARLES ISL.: *Edmonston*, acc. to Hook. f.

S. sp. Hook. f., l. c. — CHARLES ISL.: *Edmonston*, acc. to Hook. f.

THINOGETON, Benth.

T. HOOKERI, Anderss. (1), 217. — INDEFATIGABLE ISL.: *Andersson*, Endemic. Omitted by Andersson from his second paper.

T. MIERSII, Miers, Ann. & Mag. Nat. Hist. ser. 2, iv. 359 (1849); Anderss. (1), 217, & (2), 91; Dun. in DC. Prodr. xiii. 689. *Dictyocalyx Miersii*, Hook. f. (3), 203; Dun. in DC. Prodr. xiii. 538. *Cacabus Miersii*, Wettst. in Engl. & Prantl, Nat. Pflanzenf. iv. Ab. 3b, 16 (1891). *Solanacea*, Rob. & Greenm. (1), 147. — ALBEMARLE ISL.: *Macrae;* Black Bight, *Snodgrass* & *Heller*, no. 258 (hb. Gr.); Iguana

Cove, *Snodgrass* & *Heller*, no. 42 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 167 (hb. Gr.). BARRINGTON ISL.: *Snodgrass* & *Heller*, no. 475 (hb. Gr.). CHARLES ISL.: *Darwin; Andersson*. CHATHAM ISL.: *Andersson; Snodgrass* & *Heller*, no. 523 (hb. Gr.). CULPEPPER ISL.: *Snodgrass* & *Heller*, no. 2 (hb. Gr.), a tomentose form with more sharply toothed leaves. HOOD ISL.: *Baur*, no. 194 (hb. Gr.). NARBOROUGH ISL.: Mangrove Point, *Snodgrass* & *Heller*, no. 307 (hb. Gr.); northern part, *Snodgrass* & *Heller*, no. 302 (hb. Gr.). Endemic. Mr. Heller remarks that this species as it occurs upon Culpepper Island recalls a cucumber vine and bears large brown berries. This seems to be a well marked genus, not to be united with *Cacabus*.

SCROPHULARIACEAE.

CAPRARIA, L.

C. BIFLORA, L., var. PILOSA, Griseb. Fl. Brit. W. Ind. 427 (1861). *C. biflora*, Anderss. (1), 218, & (2), 91; Rob. & Greenm. (1), 147. — CHARLES ISL.: in very dry places, *Andersson* (hb. Gr.); *Baur*, no. 184 (hb. Gr.). CHATHAM ISL.: *Snodgrass* & *Heller*, no. 509 (hb. Gr.). Further distrib. trop. and subtrop. regions of the New World.

C. PERUVIANA, Benth. in DC. Prodr. x. 430 (1846). — CHARLES ISL.: *Lee* (hb. U. S. Nat. Mus.). Further distrib. coast of Ecuador to the uplands of Peru. Well marked, although by Hook. f. & Jacks. Ind. Kew. referred to *C. biflora*.

SCOPARIA, L.

S. DULCIS, L. Sp. 116 (1753); Hook. f. (3), 200; Anderss. (1), 218, & (2), 91; Caruel (1), 622; Rob. & Greenm. (1), 147. — CHARLES ISL.: *Darwin;* in arid grassy places of the middle and upper region, *Andersson; Baur*, no. 183 (hb. Gr.). CHATHAM ISL.: *Chierchia*, acc. to Caruel, l. c. Further distrib. general in trop. and subtrop. Am.

SCROPHULARIACEA, Hook. f. (3), 200. A dwarf indeterminate plant. — JAMES ISL.: *Darwin*, acc. to Hook. f. (3), 200.

BIGNONIACEAE?

TECOMA, Juss.?

T. sp.? Caruel (1), 622. — CHATHAM ISL.: *Chierchia*, acc. to Caruel, l. c. Sterile and doubtful even as to family.

ACANTHACEAE.

DICLIPTERA, Juss.

D. PERUVIANA, Juss. Ann. Mus. Par. ix. 268 (1806); Nees in DC. Prodr. xi. 478; Hook. f. (3), 195; Anderss. (1), 219, & (2), 92. *Dianthera mucronata*, Ruiz & Pav. Fl. Per. i. 11, t. 16, fig. a (1798). *Justicia peruviana*, Lam. Dict. i. 633 (1783). — JAMES ISL.: *Darwin*, acc. to Hook. f., l. c. Further distrib. Andes of Peru.

JUSTICIA, L.

J. (LEPTOSTACHYA) **galapagana**, Lindau, nov. sp., "herbacea tota glanduloso-pilosa caulibus sexangularibus vel subteretibus, patente pilosis. Folia petiolis 1–2 cm. longis ovata basi rotundata et subito angustata, apice sensim acuminata, acutiuscula, margine integro, cystolithis vix visibilibus. Inflorescentiae pauciflorae, axillares, dichotomae, folia aequantes, apice spiciformes, floribus in axillis bractearum alternantibus. Pedicelli subnulli. Bracteae bracteolaeque subulatae, 0.2 mm. longae, pubescentes pilis glandulosis intermixtis. Calycis laciniae 4, subulatae, 4 mm. longae, 0.75 mm. latae, pubescentia ut in bracteis. Corolla extus dense pubescens tubo 4 mm. longo, basi 2 mm. diametro et apice usque ad 3 mm. ampliato. Labium superum 4 mm. longum, basi 4 mm. latum, dentibus apice minimis, intus rugula[1] instructum, inferum 4 mm. longum, lobis 3 rotundatis, 2 mm. diam. metientibus. Filamenta glabra, 3 mm. longa, antherum loculo supero 1 mm. longo obtuso, infero 1 mm. longo basi in calcar 0.5 mm. longum producto. Pollinis granula pro genere typica, 46–58 μ longa et 27–30 μ diam. Discus adest. Ovarium 1.5 mm. altum, pubescens. Stylus 6 mm. longus, pilosus. Capsula 12 mm. longa, 4 mm. lata, usque ad medium fere in stipitem contracta, pubescens pilis glanduligeris intermixtis, jaculatoribus hamatis, 0.2 mm. longis. Semina 4, brunneo-tomentosa, lentiformis, 0.3 mm. diametro." — ABINGDON ISL.: alt. 520 m., *Snodgrass* & *Heller*, no. 820 (hb. Gr.). "Obs. Proxima *Justiciae Pringlei* Robins., a qua differt pubescentia densiore et magis glutinosa, stylo ovarioque pilosis et maxime seminibus, quae sunt in *J. Pringlei* foveolata et pubescentia, sed in *J. galapagana* laevis et brunneo-tomentosa."

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[1] "Rugula, i. e., ruga longitudinalis a duobus lobis membranae labii superi formata, in qua stylus est inclusus."

TETRAMERIUM, Nees.

T. HISPIDUM, Nees in DC. Prodr. xi. 468 (1847); Gray, Syn. Fl. ii. 330; Hemsl. Biol. Cent.-Am. Bot. ii. 525. Probably the *Tetramerium*, n. sp.? Hook. f. (3), 195; Anderss. (1), 195 & (2), 78. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, nos. 34 (hb. Gr.), 87 (hb. Gr.); mountain north of Elizabeth Bay, *Snodgrass* & *Heller*, no. 288 (hb. Gr.); Tagus Cove, from the coast to 450 m. alt.; *Snodgrass* & *Heller*, no. 213 (hb. Gr.). JAMES ISL.: *Darwin* (identity doubtful). Further distrib. Cent. Am., Mex., southwestern U. S.

PLANTAGINACEAE.

PLANTAGO, L.

P. MAJOR, L. Sp. 112 (1753); Caruel (1), 624. — CHATHAM ISL.: *Chierchia*. Further distrib. cosmop.

P. TOMENTOSA, var. (?) PUMILA, Hook. f. (3), 194 (where by typographical error published as *Plumbago*), & (4), 262; Anderss. (1), 171, & (2), 65. — JAMES ISL.: *Darwin*, acc. to Hook. f., l. c. Identity doubtful.

RUBIACEAE.

BORRERIA, Meyer.

B. BASALIS, Anderss. (1), 191, & (2), 76, t. 8, f. 4. — CHATHAM ISL.: in densely grassy regions, *Andersson*. Endemic.

B. BAURII, Rob. & Greenm. (1), 140, 146. *Spermacoce Baurii*, Rob. & Greenm. (1), 141. — CHATHAM ISL.: lower region, southwest end, *Baur*, no. 144 (hb. Gr.). Endemic.

B. DISPERSA, Hook. f. (3), 217; Anderss. (1), 191, & (2), 76, t. 8, f. 1; Rob. & Greenm. (1), 146. — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 137 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 215 (hb. Gr.). CHARLES ISL.: *Darwin*; *Baur*. CHATHAM ISL.: southwest end, middle region, *Baur*, no. 743 (hb. Gr.). INDEFATIGABLE ISL.: grassy places of the lower region, *Andersson* (hb. Gr.). JAMES ISL.: *Darwin*. Endemic.

B. DIVARICATA, Hook. f. (3), 219; Anderss. (1), 193, & (2), 77; Rob. & Greenm. (1), 146. — CHARLES ISL.: *Darwin*; Cuevas Bay, *Baur*, no. 141 (hb. Gr.). Endemic.

B. ERICAEFOLIA, Hook. f. (3), 218; Anderss. (1), 192, & (2), 77, t. 8, f. 2; Rob. & Greenm. (1), 146. — Abingdon Isl.: *Snodgrass* & *Heller*, no. 831 (hb. Gr.). Albemarle Isl.: Cowley Bay, *Baur*, no. 139 (hb. Gr.); Elizabeth Bay, *Snodgrass* & *Heller*, no. 275 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 38 (hb. Gr.); mountain east of Tagus Cove, alt. 615 m., *Snodgrass* & *Heller*, no. 236 (hb. Gr.); southern portion, *Baur*, no. 137 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 157 (hb. Gr.). Charles Isl.: Cormorant Bay, *Baur*, no. 138 (hb. Gr.). Chatham Isl.: *Darwin; Andersson.* Narborough Isl.: Mangrove Point, common, scattered in crevices of lava from the beach to 615 m. alt., *Snodgrass* & *Heller*, no. 308 (hb. Gr.). Endemic. The forms here included in this species differ from each other somewhat in the size, acuteness and pubescence of the leaves. It is not improbable that some of them may be *B. basalis*, Anderss., *B. parvifolia*, Hook. f., or *B. falcifolia*, Hook. f., nearly related species which, to judge from the material at hand, do not appear likely to prove distinct in nature.

B. FALCIFOLIA, Hook. f. (3), 219; Anderss. (1), 193, & (2), 77. *B. lancifolia*, Anderss. (1), 191, & (2), 76, evidently a typographical error for *falcifolia*. — Albemarle Isl.: *Macrae?* acc. to Hook. f., l. c. Endemic.

B. GALAPAGEIA, Rob. & Greenm. (1), 140, 146. *Spermacoce galapageia*, Rob. & Greenm. (1), 141. — Duncan Isl.: *Baur*, no. 145 (hb. Gr.). Endemic.

B. LINEARIFOLIA, Hook. f. (3), 217; Anderss. (1), 191, & (2), 76. — James Isl.: *Darwin.* Endemic.

B. OVALIS, Anderss. (1), 192, & (2), 76, t. 8, f. 3. Leaves 6 to 8 mm. long, rounded at the apex. — Charles Isl.: in dry and grassy places on the upper part of the island, *Andersson* (hb. Gr.). Endemic.

Forma **abingdonensis.** *B. ovalis*, Rob. & Greenm. (1), 149, as to pl. Abingdon. Foliis ad 1.5 cm. longis plus minusve acutis. — Abingdon Isl.: *Baur*, no. 142 (hb. Gr.). Endemic.

B. PACIFICA, Rob. & Greenm. (1), 140, 146. *Spermacoce pacifica*, Rob. & Greenm. (1), 141. — Indefatigable Isl.: south of Conway Bay, *Baur*, no. 146 (hb. Gr.). Endemic.

B. PARVIFOLIA, Hook. f. (3), 218; Anderss. (1), 193, & (2), 77. — Galapagos Ids.: *Habel.* Albemarle Isl.: *Macrae.* Endemic. A pubescent form of *B. ericaefolia?*

B. PERPUSILLA, Hook. f. (3), 218; Anderss. (1), 192, & (2), 77. — JAMES ISL.: *Darwin*. Endemic.

B. ROTUNDIFOLIA, Anderss. (2), 77. — INDEFATIGABLE ISL.: in grassy places, *Andersson*. Endemic.

B. SUBERECTA, Hook. f. (3), 217; Anderss. (1), 191, & (2), 76. *B. linearifolia*, Rob. & Greenm. (1), 146 (at least in part), not Hook. f. — ALBEMARLE ISL.: *Macrae;* Iguana Cove, *Snodgrass* & *Heller*, no. 129 (hb. Gr.); southern part, *Baur*, nos. 135 (hb. Gr.), 140 (hb. Gr.); Tagus Cove, near beach, *Snodgrass* & *Heller*, no. 204 (hb. Gr.), probably the forma β of Hook. f., l. c. BARRINGTON ISL.: *Baur* (identity doubtful). Endemic.

B. sp. Slender simple stems 5 to 7 cm. high from an apparently annual root: leaves oblong-lanceolate, hispidulous on and near the strongly revolute margin. — CHATHAM ISL.: *Snodgrass* & *Heller*, no. 536 (hb. Gr.). Perhaps a distinct species, but more probably a starved state of one of the above.

CHIOCOCCA, P. Br.

C. RACEMOSA, L. Syst. ed. 10, 917 (1760); Jacq. Stirp. Am. 68; Hook. f. (3), 220; Anderss. (1), 195, & (2), 78. *C. trisperma*, Hook. f. (3), 219; Anderss. (1), 194, & (2), 78, t. 9, f. 2. *C.?* Hook. f. (3), 220. — GALAPAGOS IDS.: *Habel*. ABINGDON ISL.: *Snodgrass* & *Heller*, no. 823 (hb. Gr.). ALBEMARLE ISL.: *Macrae; Andersson;* Elizabeth Bay, *Snodgrass* & *Heller*, no. 267 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, nos. 60 (hb. Gr.), 874 (hb. Gr.); Tagus Cove, 150 to more than 650 m., *Snodgrass* & *Heller*, nos. 144 (hb. Gr.), 172 (hb. Gr.), 899 (hb. Gr.); mountain east of Tagus Cove, alt. 620 m., *Snodgrass* & *Heller*, no. 235 (hb. Gr.), alt. 930 m., *Snodgrass* & *Heller*, no. 234 (hb. Gr.). BINDLOE ISL.: *Snodgrass* & *Heller*, no. 789 (hb. Gr.). CHARLES ISL.: *Andersson* (hb. Gr.). CHATHAM ISL.: *Darwin; Andersson*. JAMES ISL.: *Scouler*. NARBOROUGH ISL.: southern part, 300 to 600 m. alt., scattered but common, 3 to 5 m. in height, *Snodgrass* & *Heller*, no. 350 (hb. Gr.). Widely distrib. in trop. Am. The size of the leaves varies greatly, and changes somewhat with the age of the plant; the fruit is often 2-carpelled and 3-carpelled on the same individual.

DIODIA, L.

D. RADULA, Cham. & Schlecht. Linnaea, iii. 342 (1828); Schumann in Mart. Fl. Bras. vi. pt. 6, 25. *Borreria* (*Spermacoce asperifolia*, Mart.

& Gal. ?), Rob. & Greenm. (1), 146. — CHATHAM ISL.: southwest end, upper region, *Baur*, no. 147 (hb. Gr.). Further distrib. Brazil.

PSYCHOTRIA, L.

P. ANGUSTATA, Anderss. (1), 193, & (2), 78, t. 9, f. 1. Probably *P.* sp. ? Hook. f. (3), 220. — CHARLES ISL.: *Darwin ?;* in most fertile forest regions of the island, *Andersson*. Endemic.

P. RUFIPES, Hook. f. (3), 220; Anderss. (1), 193, & (2), 77; Rob. & Greenm. (1), 146. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 817 (hb. Gr.), ALBEMARLE ISL.: Iguana Cove, below 300 m. alt., *Snodgrass* & *Heller*, nos. 117 (hb. Gr.), 864 (hb. Gr.), 865 (hb. Gr.). CHARLES ISL.: in the most fertile forests, *Andersson; Lee* (hb. U. S. Nat. Mus.). CHATHAM ISL.: southwest end, middle region, *Baur*, no. 148 (hb. Gr.). JAMES ISL.: *Darwin*. Endemic.

RELBUNIUM, Benth. & Hook. f.

R. sp. *Rubia* sp. Hook. f. (3), 216; Anderss. (1), 190, & (2), 75; near *R. Relbun*. — CHARLES ISL.: *Darwin*. Further distrib. Peru, acc. to Hook. f., l. c.

SPERMACOCE, L.

S. TENUIOR, L. Sp. 102 (1753), excl. syn. Dill.; Hook. f. (3), 219; Anderss. (1), 193, & (2), 77; Rob. & Greenm. (1), 146. — CHATHAM ISL.: southwest end, upper region, *Baur*, no. 136 (hb. Gr.). JAMES ISL.: *Darwin*. Widely distrib. in trop. and subtrop. Am.

CUCURBITACEAE.

CITRULLUS, Neck.

C. VULGARIS, Schrad. ex Eckl. & Zeyh. Enum. 279 (1836), & Linnaea, xii. 412; Cogn. in DC. Monogr. iii. 508. *Cucurbita Citrullus*, L. Sp. 1010 (1753). *Cucumis Citrullus*, Ser. in DC. Prodr. iii. 301 (1828); Anderss. (1), 224, & (2), 95. — CHARLES ISL.: *Andersson*, no. 166. Widely distrib. in warm countries.

CUCURBITA, L.

C. PEPO, L. Sp. 1010 (1753); Cogn. in DC. Monogr. iii. 545. *C. Melopepo*, L. l. c.; Anderss. (1), 223, & (2), 94. CHARLES ISL.: cultivated ground, upper region, *Andersson*. Cosmopolitan.

ELATERIUM, Jacq.

E. CORDATUM, Hook. f. (3), 224; Anderss. (1), 224, & (2), 95; Cogn. in DC. Monogr. iii. 859. — JAMES ISL.: *Darwin* (hb. Kew); *Andersson*. Endemic.

MOMORDICA, L.

M. CHARANTIA, L. Sp. 1009 (1753); Sims, Bot. Mag. t. 2455; Cogn. l. c. 436; Rob. & Greenm. (1), 146 (*Charanta*). — ALBEMARLE ISL.: southern part, *Baur*, no. 158 (hb. Gr.). Widely distrib. in trop. reg.

SICYOS, L.

S. VILLOSUS, Hook. f. (3), 223; Anderss. (1), 224, & (2), 95; Cogn. l. c. 874. — CHARLES ISL.: *Darwin*, acc. to Hook. f. Endemic.

CAMPANULACEAE.

LOBELIA, L.

L. XALAPENSIS, HBK. Nov. Gen. et Sp. iii. 315 (1818); Hook. f. (3), 206; Anderss. (1), 190, & (2), 75. — CHARLES ISL. and JAMES ISL.: acc. to Hook. f., probably collected by *Darwin* as suggested by *Andersson*. Further distrib. Mex., W. Ind., S. Am. to Peru.

GOODENIACEAE.

SCAEVOLA, L.

S. LOBELIA, Murr. Syst. ed. 13, 178 (1774). *S. Plumieri*, Vahl, Symb. ii. 36 (1791); Hook. f. (3), 205; Anderss. (1), 190, & (2), 75. *Lobelia Plumieri*, L. Sp. 929 (1753). — GALAPAGOS IDS.: *Edmonston* (hb. Gr.). CHATHAM ISL.: *Darwin*; *Andersson*. Widely distrib. in warm countries.

COMPOSITAE.

ACANTHOSPERMUM, Schrank.

A. LECOCARPOIDES, Rob. & Greenm. (1), 141, 146. — HOOD ISL.: *Baur*, no. 128; *Snodgrass* & *Heller*, no. 744 (hb. Gr.). Endemic.

A. **microcarpum**, nov. sp., annuum erectum ramosum 3 dm. altum; caule terete albide hirsuto; foliis oppositis obovato-rhomboideis crenulatis vel obsolete dentatis 3-nervatis parce hirsutis subtus haud vel paulo pali-

dioribus apice obtusis basi longiuscule angustatis sessilibus 3–4.5 cm. longis 1.5–2 cm. latis; capitulis sessilibus; involucri squamis exterioribus 4–6 oblongis obtusis conspicue ciliatis 4 mm. longis; fructibus triangularibus 6 mm. longis compressis pallidis minute granulatis et spinis brevibus conicis haud vel paulo uncinatis armatis; floribus disci circa 7; corollis pallidis, tubo gracile faucibus subnullis limbo ampliato. — CHARLES ISL.: May, 1899, *Snodgrass* & *Heller* no. 446 (hb. Gr.). Endemic. Nearly related to *A. hispidum*, DC., but differing considerably in the fruit, as will be seen from PLATE 1, FIGURES 3 and 4, where the fruits of both species are contrasted.

AGERATUM, L.

A. LATIFOLIUM, Hemsl. Biol. Cent.-Am. Bot. ii. 82 (1881), not Cav. (*Coelestina latifolia*, Benth. in Oerst. Vidensk. Meddel. 1852, p. 71; Anderss. (1), 175, & (2), 67; Caruel (1), 623. *A. conyzoides*, Hook. f. (3), 207; Anderss. (1), 175, & (2), 67; Rob. & Greenm. (1), 146; not L. — CHARLES ISL.: *Darwin; Andersson; Baur; Snodgrass* & *Heller*, no. 423 (hb. Gr.). CHATHAM ISL.: *Chierchia*, acc. to Caruel, l. c. Further distrib. Costa Rica.

APLOPAPPUS, Cass.

A. LANATUS, Hook. f. (3), 215; Anderss. (1), 177, & (2), 68 (*Haplopappus*). — GALAPAGOS IDS.: *Du Petit-Thouars*. Endemic.

BACCHARIS, L.

B. PILULARIS, DC. Prodr. v. 407 (1836); Hook. f. (4), 261; Anderss. (1), 178, & (2), 69; Gray, Syn. Fl. i. pt. 2, 222. — CHARLES ISL.: *Edmonston*, acc. to Hook. f., l. c. Further distrib. coastal region of California and Oregon. I cannot avoid a suspicion that the Galapageian plant (of which I have seen no specimen) will prove to be not the Californian species but one of several habitally similar plants of trop. S. Am.

B. PINGRAEA, DC., var. ANGUSTISSIMA, DC. Prodr. v. 420 (1836); Rob. & Greenm. (1), 146. — ALBEMARLE ISL.: eastern part, Cowley Bay, *Baur*, no. 124 (hb. Gr.); mountain east of Tagus Cove, alt. 960 m., *Snodgrass* & *Heller*, no. 242 (hb. .Gr). Identification doubtful. Further distrib. coast of Chili.

B. STEETZII, Anderss. (1), 177, & (2), 68. — CHARLES ISL.: in dry places in the middle of the island, *Andersson* (hb. Gr.).

BIDENS, L.

B. CHILENSIS, DC. Prodr. v. 603 (1836). — ALBEMARLE ISL.: Tagus Cove, 1230 m. alt., *Snodgrass & Heller*, no. 887 (hb. Gr.). Further distrib. Chili.

B. PILOSA, L. Sp. 832 (1753). *B. leucantha*, Willd. Sp. iii. 1719 (1804); Anderss. (1), 187, & (2), 74; Caruel (1), 623; Rob. & Greenm. (1), 147. — ALBEMARLE ISL.: *Andersson*. CHARLES ISL.: *Andersson*. CHATHAM ISL.: *Andersson; Chierchia;* southwest end, middle region, *Baur* (hb. Gr.); *Snodgrass & Heller*, no. 534 (hb. Gr.). Widely distrib. in trop. reg.

B. REFRACTA, Brandegee, Zoe, i. 310 (1890). — ALBEMARLE ISL.: Iguana Cove, *Snodgrass & Heller*, no. 92 (hb. Gr.); Tagus Cove, not common inland at 150 to 300 m. alt., *Snodgrass & Heller*, no. 178 (hb. Gr.). CHARLES ISL.: *Snodgrass & Heller*, no 420 (hb. Gr.). HOOD ISL.: *Snodgrass & Heller*, no. 732 (hb. Gr.). JAMES ISL.: James Bay, most abundant herb, growing everywhere in shade of bushes on lava soil, *Snodgrass & Heller*, no. 380 (hb. Gr.). NARBOROUGH ISL.: southern part, abundant at 600 m. alt., *Snodgrass & Heller*, nos. 316 (hb. Gr.), 338 (hb. Gr.). Probably a recent introduction from Mexico or Lower California. Palmer's no. 923 (coll. of 1890) from Manzanillo, Mexico, appears to be identical. PLATE 2, FIG. 4 (drawn from a Galapageian specimen).

BLAINVILLEA, Cass.

B. RHOMBOIDEA, Cass. Dict. xxix. 493 (1823); Anderss. (1), 178, & (2), 69; Rob. & Greenm. (1), 146. *Verbesina dichotoma*, Murr. Comm. Goett. ii. 15, t. 4 (1779). — ALBEMARLE ISL.: *Lee* (hb. U. S. Nat. Mus.); Iguana Cove, *Snodgrass & Heller*, no. 99 (hb. Gr.); Tagus Cove, growing everywhere in tufa soil up to 300 m. alt., *Snodgrass & Heller*, no. 184 (hb. Gr.). BARRINGTON ISL.: *Snodgrass & Heller*, no. 482 (hb. Gr.). CHARLES ISL.: *Baur*, no. 118 (hb. Gr.); *Snodgrass & Heller*, no. 442 (hb. Gr.). CHATHAM ISL.: *Andersson* (hb. Gr.); *Snodgrass & Heller*, no. 535 (hb. Gr.). DUNCAN ISL.: *Baur*, no. 119 (hb. Gr.); *Snodgrass & Heller*, no. 698 (hb. Gr.). HOOD ISL.: *Baur*, no. 117 (hb. Gr.); *Snodgrass & Heller*, no. 720 (hb. Gr.). INDEFATIGABLE ISL.: *Andersson;* northern part, *Snodgrass & Heller*, no. 681 (hb. Gr.). JAMES ISL.: James Bay, common in lava soil, *Snodgrass & Heller*, no. 389 (hb. Gr.). NARBOROUGH ISL.: southern part, tolerably common at 615 m. alt., *Snodgrass & Heller*, no. 314a (hb.

Gr.). Seymour Isl.: north, *Snodgrass* & *Heller*, no. 563 (hb. Gr.); south, *Snodgrass* & *Heller*, no. 600 (hb. Gr.). Widely distrib. weed in trop. reg. Apparently of recent introduction in the Galapagos Ids.

B. tenuicaulis, Benth. & Hook. f. Gen. Pl. ii. 370. *Wedelia tenuicaulis*, Hook. f. (3), 213; Anderss. (1), 179, & (2), 69; and doubtfully acc. to Hook. f. & Jacks. Ind. Kew. ii. 1226, *Wedelia frutescens*, Hook. f. (4), 261; Anderss. (1), 179, & (2), 69; not Jacq. — Albemarle Isl.: *Macrae*. Charles Isl.: *Edmonston* (identity doubtful). Endemic.

Brickellia, Ell.

B. diffusa, Gray, Pl. Wright. i. 86 (1852). — Albemarle Isl.: Iguana Cove, *Snodgrass* & *Heller*, nos. 51 (hb. Gr.), 72 (hb. Gr.), 93 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 214 (hb. Gr.). Further distrib. Mex. to Braz. Apparently of recent introduction on the Galapagos Islands.

Chrysanthellum, Rich.

C. erectum, Anderss. (1), 188, & (2), 74. *C. pusillum*, Rose (1), 137, in part (as to pl. Chatham Isl.). — Chatham Isl.: *A. Agassiz* (hb. U. S. Nat. Mus.); *Snodgrass* & *Heller*, nos. 505 (hb. Gr.), 538 (hb. Gr.). Indefatigable Isl.: in the most grassy places of lower region. *Andersson* (hb. Gr.). Narborough Isl.: southern part, *Snodgrass* & *Heller*, no. 314 (hb. Gr.). Endemic.

C. pusillum, Hook. f. (3), 214; Anderss. (1), 188, & (2), 74, t. 6, f. 2; Rose (1), 137, in part (only as to pl. Charles). — Albemarle Isl.: *Darwin*. Charles Isl.: in dry somewhat grassy places of lower region, *Andersson* (hb. Gr.); *A. Agassiz* (hb. Gr.); *Snodgrass* & *Heller*, no. 413 (hb. Gr.) Chatham Isl.: *Andersson*. Endemic.

Eclipta, L.

E. erecta, L. Mant. ii. 286 (1771); Caruel (1), 623. *E. alba*, Hassk. Pl. Jav. Rar. 528 (1848); Rob. & Greenm. (1), 146. *E. procumbens*, Michx. Fl. Bor. Am. ii. 129 (1803). — Charles Isl.: *Snodgrass* & *Heller*, no. 417 (hb. Gr.). Chatham Isl.: *Chierchia*, acc. to Caruel, l. c.; southwest end, middle region, *Baur*, no. 106 (hb. Gr.); *Snodgrass* & *Heller*, no. 539 (hb. Gr.). Hood Isl.: on margin of a mud lake, *Snodgrass* & *Heller*, no. 721 (hb. Gr.). A widely distributed weed, probably of recent introduction on the Galapagos.

ELVIRA, Cass.

E. **inelegans.** *Desmocephalum inelegans*, Hook. f. (3), 209; Anderss. (1), 178, & (2), 69. — CHARLES ISL.: *Darwin.* Endemic.

E. **repens.** *Micrococcia repens*, Hook. f. (3), 209; Anderss. (1), 179, & (2), 69. — ALBEMARLE ISL.: Tagus Cove, alt. 1230 m., *Snodgrass* & *Heller*, no. 880 (hb. Gr.). JAMES ISL.: *Darwin* (hb. Gr.). Endemic. The plant from Albemarle Island differs from the original material from James Island in not rooting at the lower nodes. Furthermore, the outer bract of the involucre is broadly obovate and subtruncate instead of acuminate as described by Hooker. Mr. W. B. Hemsley has been so kind as to compare the Albemarle plant with some of the original material at Kew and regards them as the same species. A fragment of the Darwin plant, sent many years ago by Mr. Bentham to Dr. Gray, is unfortunately sterile, but agrees closely with the Albemarle plant in all vegetative features except in its evident repent character.

ENCELIA, Adans.

E. HISPIDA, Anderss. (1), 186, & (2), 73. — CHARLES ISL.: *Andersson.* CHATHAM ISL.: in dry grassy places of lower region, *Andersson* (hb. Gr.). Endemic.

ERIGERON, L.

E. LANCIFOLIUS, Hook. f. (3), 208; Anderss. (1), 176, & (2), 67. — ALBEMARLE ISL.: *Darwin;* Elizabeth Bay, *Snodgrass* & *Heller*, no. 278 (hb. Gr.); Point Christopher, *Snodgrass* & *Heller*, no. 930 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 909 (hb. Gr.). NARBOROUGH ISL.: southern part, common, bushes 6 to 9 dm. high, 150 to 600 m. alt., *Snodgrass* & *Heller*, no. 328 (hb. Gr.), leaves acute; and no. 344 (hb. Gr.), leaves obtusish to rounded. (Obviously only foliar forms of the same species.) Endemic.

E. LINIFOLIUS, Willd. Sp. iii. 1955 (1804); Gray, Syn. Fl. i. pt. 2, 221. *E.* sp. Rob. & Greenm. (1), 146. *Conyza ambigua*, DC. Fl. Fr. Suppl. 468 (1815). — CHARLES ISL.: *Baur*, no. 120 (hb. Gr.). Widely distrib. in trop. and subtrop. reg.

E. TENUIFOLIUS, Hook. f. (3), 207; Anderss. (1), 176, & (2), 68 (as to pl. Darwin); Rob. & Greenm. (1), 146. — ABINGDON ISL.: *Snodgrass* & *Heller*, no. 841 (hb. Gr.). ALBEMARLE ISL.: mountain north of Elizabeth Bay, *Snodgrass* & *Heller*, no. 295 (hb. Gr.); mountain east of Tagus Cove, *Snodgrass* & *Heller*, nos. 237 (hb. Gr.), 253 (hb.

Gr.); southern part, *Baur*, no. 121 (hb. Gr.); Tagus Cove, alt. 1250 m., *Snodgrass* & *Heller*, no. 889 (hb. Gr.). CHARLES ISL.: *Darwin.* DUNCAN ISL.: *Baur*, no. 122 (hb. Gr.). JAMES ISL.: *Darwin;* James Bay, scattered along the edge of new lava flow, *Snodgrass* & *Heller*, no. 370 (hb. Gr.). Endemic. Leaves varying in length.

E. sp. *E. tenuifolius*, Steetz (not Hook. f.) in Anderss. (1), 176, & (2), 68; Rob. & Greenm. (1), 146. — CHARLES ISL.: *Andersson* (hb. Gr.); *Baur*, no. 123 (hb. Gr). Endemic. Unfortunately known only from sterile specimens.

EUPATORIUM, L.

E. FILICAULE, Sch. Bip. in Gray, Proc. Am. Acad. xxi. 384 (1886). — ALBEMARLE ISL.: Iguana Cove, *Snodgrass* & *Heller*, no. 29 (hb. Gr.). Further distrib. Mex. to Venezuela. The Galapageian plant has a slightly denser inflorescence and more copious and sordid indumentum than the Mexican, but good floral differences do not appear.

E.? sp. Hook. f. (4), 261. — CHARLES ISL.: *Edmonston*, acc. to Hook. f., l. c. Endemic? Andersson (1), 189, & (2), 74, suggests the possibility that this may be *Flaveria Contrayerba*, Pers.

FLAVERIA, Juss.

F. CONTRAYERBA, Pers. Syn. ii. 489 (1807); DC. Prodr. v. 635; Anderss. (1), 189, & (2), 74. — CHARLES ISL.: *Andersson.* Further distrib. trop. Am.

HEMIZONIA, DC.

H. SQUALIDA, Hook. f. (3), 208; Anderss. (1), 190, & (2), 75. — GALAPAGOS IDS.: *Du Petit-Thouars*, acc. to Hook. f., l. c. Endemic.

JAEGERIA, HBK.

J. GRACILIS, Hook. f. (3), 213; Anderss. (1), 179, & (2), 69. — CHARLES ISL.: *Darwin.* Endemic. From the description of the involucre this can hardly be a *Jaegeria.*

J. PROREPENS, Hook. f. (3), 214; Anderss. (1), 179, & (2), 69; Robinson, Proc. Am. Acad. xxxv. 318. — JAMES ISL.: *Darwin.* Endemic. Neither this nor the preceding has been secured by any other collector.

LECOCARPUS, Decaisne.

L. FOLIOSUS, Decaisne, Bot. Voy. Venus, 20 (1864). *L. pinnatifidus*, Decaisne, l. c. t. 14 of Atlas; Hook. f. (3), 210 (identity of the forms with less cut leaves may be doubted); Anderss. (1), 186, & (2), 73; Rob. & Greenm. (1), 146. — GALAPAGOS IDS.: (presumably CHARLES) *Edmonston* (hb. Gr.); *Habel.* CHARLES ISL.: *Darwin; Du Petit-Thouars;* in dry places, upper region, *Andersson* (hb. Gr.); *Baur.* CHATHAM ISL.: *Darwin,* acc. to Hook. f. Endemic.

LIPOCHAETA, DC.

L. LARICIFOLIA, Gray, Proc. Am. Acad. v. 131 (1862); Rose (1), 137. *Macraea laricifolia,* Hook. f. (3), 210; Anderss. (1), 186, & (2), 72; Caruel (1), 623; Rob. & Greenm. (1), 147. *Trigonopterum Ponteni,* Anderss. (1), 184, & (2), 72, t. 6, f. 1. *Lippia?* Rob. & Greenm. (1), 147. — GALAPAGOS IDS.: *Habel.* ABINGDON ISL.: *Snodgrass & Heller,* no. 830 (hb. Gr.). ALBEMARLE ISL.: *Darwin; Macrae; Andersson;* Cowley Bay, *Baur,* nos. 181 (hb. Gr.), 182 (hb. Gr.); Tagus Cove, *Snodgrass & Heller,* nos. 146 (hb. Gr.), 913 (hb. Gr.); common bush on hills near the cove and to 180 m. alt., also inland to 450 m., *Snodgrass & Heller,* no. 166 (hb. Gr.). CHARLES ISL.: *Darwin;* wooded region, middle of the island, *Andersson* (hb. Gr.); *Lee* (hb. U. S. Nat. Mus.); *A. Agassiz; Baur,* no. 126 (hb. Gr.); *Snodgrass & Heller,* no. 408 (hb. Gr.). CHATHAM ISL.: *Chierchia;* southwest part, *Baur,* no. 127 (hb. Gr.). NARBOROUGH ISL.: southern part, bushes 1.3 to 2 m. high at 320 to 650 m. alt., *Snodgrass & Heller,* no. 335 (hb. Gr.). Endemic. A species of special interest, showing the strongest of several on the whole rather slight traces of affinity between the flora of the Galapagos Islands and that of the Sandwich Islands (on which occur the remaining species of this small genus).

PECTIS, L.

P. Anderssonii. *P. linearis,* Rob. & Greenm. (1), 147, not La Llave. *Lorentia linearis,* Anderss. (1), 174, & (2), 66. — INDEFATIGABLE ISL.: alt. 60 m., *Andersson;* south of Conway Bay, *Baur,* no. 125 (hb. Gr.). Endemic.

P. Hookeri. *Lorentia gracilis,* Hook. f. (3), 206; Anderss. (1), 174, & (2), 66. *Pectis gracilis,* Rob. & Greenm. (1), 147 (excl. pl. Chatham), not Baker. — ALBEMARLE ISL.: *Macrae.* BARRINGTON ISL.: *Baur,* no. 115 (hb. Gr.); *Snodgrass & Heller,* no. 465 (hb. Gr.). CHARLES ISL.: *Baur.* HOOD ISL.: *Snodgrass & Heller,* no. 726 (hb. Gr.). JAMES ISL.: James Bay, abundant on lava rocks near beach, *Snodgrass & Heller,* no. 378 (hb. Gr.). JERVIS ISL.: *Baur,* no. 114 (hb. Gr.). SEYMOUR ISL.: south, *Snodgrass & Heller,* no. 592 (hb. Gr.). Endemic.

P. linifolia, L. Syst. Nat. ed. 10, 1221 (1760); Fernald, Proc. Am. Acad. xxxiii. 85. *P. punctata*, Jacq. Enum. Pl. Carib. 28 (1762), & Stirp. Am. 216, t. 128; Gray, Syn. Fl. i. pt. 2, 362. *Verbesina linifolia*, L. Syst. Nat. ed. 10, 1226 (1760). *Pectidium punctatum*, Less. Linnaea, vi. 707 (1831); DC. Prodr. v. 98; Anderss. (1), 172, & (2), 65. *P. subciliaris*, Anderss. (1), 174, & (2), 66. — Chatham Isl.: *Andersson*. Indefatigable Isl.: *Andersson*. Seymour Isl.: south, *Snodgrass & Heller*, no. 591 (hb. Gr.). Further distrib. Mex., W. Ind., northern S. Am. Several forms are distinguished by Andersson.

P. subsquarrosa, Sch. Bip. in Seem. Bot. Herald, 309 (1852–1857). *Lorentia subsquarrosa*, Hook. f. (3), 206. — Galapagos Ids.: *Habel*. Chatham Isl.: *Darwin*. Endemic.

P. tenuifolia, Sch. Bip. in Seem. Bot. Herald, 309 (1852–1857); Rob. & Greenm. (1), 147. *P. gracilis*, Rob. & Greenm. (1), 147, as to pl. Chatham. *Lorentia tenuifolia*, DC. Prodr. v. 103 (1836); Hook. f. (3), 206; Anderss. (1), 174, & (2), 66. *L. subsquarrosa*, Anderss. (1), 175, & (2), 67, as to pl. Albemarle. — Albemarle Isl.: *Macrae* (type); *Andersson;* Black Bight, *Snodgrass & Heller*, no. 256 (hb. Gr.); eastern portion, Cowley Bay, *Baur*, no. 116½ (hb. Gr.); Elizabeth Bay, *Snodgrass & Heller*, no. 274 (hb. Gr.); southern part, *Baur*, no. 116 (hb. Gr.); Tagus Cove, *Snodgrass & Heller*, no. 148 (hb. Gr.). Charles Isl.: *Andersson* (hb. Gr.), labelled in Andersson's hand but not recorded in his published papers; *Snodgrass & Heller*, no. 437 (hb. Gr.). Chatham Isl.: *Andersson* (hb. Gr.); northern part, *Baur*, no. 112 (hb. Gr.), leaves minutely pubescent; *Snodgrass & Heller*, no. 537 (hb. Gr.). Indefatigable Isl.: northern part, *Snodgrass & Heller*, no. 665 (hb. Gr.), leaves minutely pubescent, involucral bracts acutish. Seymour Isl.: north, *Snodgrass & Heller*, no. 573 (hb. Gr.). Further distrib. shores of Peru? *P. Burchellii*, Bak. in Mart. Fl. Bras. vi. pt. 3, 287 (1884), is habitally similar, but differs in pappus.

Porophyllum, Vaill.

P. ellipticum, Cass. Dict. xliii. 56 (1826); Anderss. (1), 189, & (2), 75; Rose (1), 137; Rob. & Greenm. (1), 147. *P. ruderale*, Cass. l. c. *Cacalia Porophyllum*, L. Sp. 834 (1753); Cav. Ic. iii. 11, t. 222. — Galapagos Ids.: *Habel*. Abingdon Isl.: *Snodgrass & Heller*, no. 848 (hb. Gr.). Albemarle Isl.: Tagus Cove Mountain, *Snodgrass & Heller*, no. 224 (hb. Gr). Charles Isl.: *Andersson; Baur; Snodgrass & Heller*, no. 430 (hb. Gr.). Chatham Isl.: *Andersson* (hb. Gr.);

A. Agassiz (hb. Gr.); southwest end, lower region, *Baur*, no. 133 (hb. Gr.); *Snodgrass* & *Heller*, no. 508 (hb. Gr.). DUNCAN ISL.: *Snodgrass* & *Heller*, no. 688 (hb. Gr.). HOOD ISL.: *Baur*, no. 131 (hb. Gr.); *Snodgrass* & *Heller*, no. 745 (hb. Gr.). INDEFATIGABLE ISL.: *Snodgrass* & *Heller*, no. 654 (hb. Gr.). JAMES ISL.: Orchilla Bay, *Baur*, no. 134 (hb. Gr.); James Bay, abundant on all kinds of soil, *Snodgrass* & *Heller*, no. 358 (hb. Gr.). JERVIS ISL.: *Baur*, no. 133 *bis* (hb. Gr.). Widely distrib. in trop. Am. Now one of the most abundant and generally distributed plants of the islands, but apparently of recent introduction, as it was not noted by Darwin or any of the earlier collectors. Andersson in 1852 found it upon two islands, Baur in 1891 upon five, and Snodgrass & Heller in 1899 upon seven.

SCALESIA, Arn.

S. AFFINIS, Hook. f. (3), 212; Anderss. (1), 182, & (2), 71; Hemsl. in Hook. f. Ic. Pl. xxviii. t. 2718. — CHARLES ISL: *Darwin*. Endemic.

S. ASPERA, Anderss. (1), 180, & (2), 70, t. 7, f. 3. — INDEFATIGABLE ISL.: alt. 65 m., *Andersson* (hb. Gr.). Endemic.

S. ATRACTYLOIDES, Arn. in Lindl. Introd. Nat. Ord. ed. 2, 264, 443 (1836); DC. Prodr. vii. 308; Hook. & Arn. in Hook. Jour. Bot. iii. 312; Hook. f. (3), 210; Anderss. (1), 179, & (2), 69. — GALAPAGOS IDS.: *Cuming*. A plant, from description highly characteristic, never rediscovered on the islands.

S. BAURII, Rob. & Greenm. (1), 141, 146. — DUNCAN ISL.: *Baur*, no. 129 (hb. Gr.). Endemic.

Var. (?) **glabrata**, foliis brevioribus magis ovatis cordatisque supra minute ruguloso-venosis glabratis; lobulis obtusis. — DUNCAN ISL.: *Snodgrass* & *Heller*, no. 706 (hb. Gr.). Very near *S. Snodgrassii* but leaves shorter, more ovate, finely rugulose, less pubescent and nigrescent in drying.

S. DARWINII, Hook. f. (3), 211; Anderss. (1), 179, & (2), 70; Rob. & Greenm. (1), 146; Hemsl. in Hook. f. Ic. Pl. xxviii. t. 2719. — CHARLES ISL.: Cormorant Bay, *Baur*, no. 107 (hb. Gr.). JAMES ISL.: *Darwin*. Endemic.

S. DECURRENS, Anderss. (1), 182, & (2), 71. — ALBEMARLE ISL.: *Baur*. CHARLES ISL.: wooded region of the interior, *Andersson* (hb. Gr.); *Snodgrass* & *Heller*, no. 410 (hb. Gr.). Endemic.

S. divisa, Anderss. (1), 179, & (2), 70, t. 7, f. 1. — Galapagos Ids.: *Habel.* Chatham Isl.: frequent on the lower stony parts, *Andersson* (hb. Gr.). Endemic.

S. gummifera, Hook. f. (3), 212; Anderss. (1), 182, & (2), 71, t. 7, f. 2. Galapagos Ids.: *Habel.* Albemarle Isl.: *Macrae;* in the driest places of the middle region, *Andersson;* Elizabeth Bay, *Snodgrass* & *Heller*, no. 266 (hb. Gr.); Tagus Cove, *Snodgrass* & *Heller*, no. 150 (hb. Gr.). Endemic.

S. **Helleri**, nov. sp., frutex; ramulis molliter pilosis; novellis albopilosissimis, foliis oppositis bi–tripinnatisectis supra viridibus pilis brevissimis conicis scabriusculis subtus paulo pallidioribus glanduloso-puberulis et in nervis hirsutis; pedunculis ad apicem ramuli saepe tribus quam folii multo brevioribus; capitulis eradiatis; involucri hemisphaerici squamis oblongis apice rotundatis obsolete striatis; floribus omnibus ☿ numerosis: corollae puberulae tubo propriori gracile brevi in fauces conspicuas longas rectas ampliato; achenio oblongo valde compresso apice truncato calvo; paleis apice 2–3-dentatis, dentibus laciniato-ciliatis ad apiculum abrupte angustatis. — Barrington Isl.: May, 1899, *Snodgrass* & *Heller*, no. 466 (hb. Gr.). Endemic. Leaves 8 cm. long, 5 cm. broad; petioles 1.5 to 2 cm. long covered with rather soft and widely spreading pilosity: heads 1.8 cm. in diameter. Well marked by its very deeply cut foliage and rounded involucral bracts. Plate 1, figs. 9 and 10.

S. **Hopkinsii**, nov. sp., frutex; ramis teretibus pallide brunneis pilosis in specimine siccato striatulis; ramulis pilosissimis foliosis plus minusve gummiferis; foliis ovatis planis tenuibus molliter pubescentibus subtus pallidioribus subargute pinnati-lobatis, apice acutis basi subintegris acutatis; lobulis dentatis; pedunculo gracile a foliis superato piloso; capitulis eradiatis; involucri hemisphaerici squamis oblongis obovatisve acutis villosis leviter striatis; floribus omnibus ☿; corollae tubo gracili saepe curvato haud vel paulo in fauces ampliato pubescenti; achaenio glabro compresso griseo calvo. — Abingdon Isl.: 520 m. alt., June, 1899, *Snodgrass* & *Heller*, no. 851 (hb. Gr.). Endemic. Leaves 8 cm. long, half as broad. Petioles 2 cm. long. Heads 1.2 cm. in diameter. With its thin flat soft-pubescent leaves this can scarcely be Mr. Hemsley's *S. retroflexa* nor does it agree with any of the other species heretofore characterized. Although near *S. incisa*, Hook. f., it has more ovate leaves and acuter involucral bracts than that species. Plate 3, fig. 1.

S. incisa, Hook. f. (3), 210; Anderss. (1), 179, & (2), 70; Hemsl. in Hook. f. Ic. Pl. xxviii. t. 2716. — Chatham Isl.: *Darwin.* Endemic.

S. **microcephala**, nov. sp., gummifera; ramulis teretibus gracilibus brunneis tenuiter griseo-puberulis, apice foliosis et corymbiferis; foliis ovato-lanceolatis vel lanceolato-oblongis integerrimis attenuato-acutissimis basi rotundato vel ad petioli insertionem subacuminatis dense tomentellis penninervatis subtus pallidioribus 4 to 6 cm. longis 2 cm. latis; petiolis gracilibus exalatis 1.5 cm. longis; capitulis pluribus graciliter pedicellatis corymbos terminales sessiles a foliis superatos formantibus eradiatis 1 cm. longis 7 mm. latis gummiferis; squamis involucri auguste campanulati lanceolato-linearibus attenuatis laxe imbricatis subaequalibus sordide pubescentibus; flosculis ♀ nullis, ☿ 9–12; corollae tubo gracillimo in fauces non ampliato 5 mm. longo minutissime puberulo, limbo 5-dentato recurvato; achenio compresso sparse pubescenti nigrescente lineari-oblongo 4 mm. longo 0.8 mm. lato summo dentes duas nunc breves nunc in aristas parvas productas gerenti; paleis conduplicatis angustis apice argute 3-dentatis puberulis. — ALBEMARLE ISL.: Tagus Cove, 20 January, 1899, *Snodgrass* & *Heller*, no. 910 (hb. Gr.); mountain east of Tagus Cove, alt. 770 m., *Snodgrass* & *Heller*, no. 254 (hb. Gr.). NARBOROUGH ISL.: abundant in southern part at 600 m. alt., forming bushes 2.6 to 4 m. high, *Snodgrass* & *Heller*, no. 343 (hb. Gr.). This last specimen is sterile and somewhat doubtful. It has leaves 7 cm. long and 3 cm. broad. Endemic. PLATE 3, FIGS. 2 and 3.

S. microcephala is well marked among its congeners by its small and more numerous heads, also by the presence of an abortive pappus. These features, however, in the presence of an otherwise close resemblance to the genus *Scalesia* do not appear to form any good ground for generic separation.

S. **narbonensis**, nov. sp., ramis fistulosis foliosis dense breviterque pubescentibus vel subvelutinis teretibus striatis; foliis alternis late ovatis, praeter basis integerrimae acuminati-attenuatae serratis acute acuminatis molliter adpresse pubescentibus, in pagina inferiore pallidioribus supra basem 3-nervatis, nervis lateralibus ramosis; petiolis distinctis semiteretibus exalatis; pedunculis in dichotomis supremis solitariis teretibus gracilibus erectis unibracteatis a foliis superatis: capitulis solitariis involucro campanulato, squamis circa 13 oblongis acutis dense pubescentibus; flosculis ♀ circa 12 squamas involucri paulo excedentibus corollis tubo gracili pubescenti in ligulam breviter oblongam pallidam 5-nervatam dilatatis, achenio pergracili tereto, abortivo: paleis oblongis puberulis valde conduplicatis apice argute 2–3-dentatis: flosculis ♀ numerosis, corollo breviter pubescenti a tubo brevi gracile in fauces teretes longiores 5-nervatis

ampliato: limbo breviter 5-dentato albo-puberulo: achenio puberulo compresso breviter oblongo apice calvo. — NARBOROUGH ISL.: northern part, *Snodgrass* & *Heller*, no. 297 (hb. Gr.); southern part, common at 650 m. alt., *Snodgrass* & *Heller*, no. 341 (hb. Gr.). Endemic. Leaf-blade 6 to 11 cm. long, half as broad, petioles 1 to 1.5 cm. long: heads 2 cm. in diameter; ray-flowers including the achene 12 mm. long. This species approaches *S. affinis*, Hook. f., but differs in its alternate more finely toothed leaves, in the branched lateral leaf-nerves which (if the figure in Hook. Icon. t. 2718 be correct) are simple and marginal in *S. affinis*. *S. affinis* would from the figure seem furthermore to have a longer more hirsute type of pubescence. PLATE 3, FIGS. 4, 5, 6, and 7.

S. OVATA, Anderss. (1), 181, & (2), 70. — CHARLES ISL.: upper wooded region, *Andersson*; *Lee* (hb. U. S. Nat. Mus.). Endemic.

S. PEDUNCULATA, Hook. f. (3), 211; Anderss. (1), 181, & (2), 71; Hemsl. in Hook. f. Ic. Pl. xxviii. t. 2717. — JAMES ISL.: *Darwin*. Endemic.

S. RETROFLEXA, Hemsl. in Hook. f. Ic. Pl. xxviii. t. 2715 (1901). — INDEFATIGABLE ISL.: *Habel*. Endemic.

S. **Snodgrassii**, nov sp. *S. Baurii* et *S. retroflexae* affinis, fruticosa, 6–9 dm. alta; ramis teretibus glabrescentibus foliosis in specimine siccato brunneis tenuiter striatis ramulis parce pilosis et tenuissime glanduloso-puberulis; foliis planiusculis alternis ovato-oblongis penninervatis supra pilis minutis conicis albis obstitis subtus paulo pallidioribus tenuiter pubescentibus pinnatisectis; lobis obtusissimis et obtuse dentatis sinis angustatis; petiolis exalatis puberulis; pedunculis ad apicem ramuli subtribus gracilibus saepius unibracteatis puberulis et parce pilosis; capitulis eradiatis; involucri hemisphaerici squamis obovato-oblongis brevissime acuminatis 3–5-nervatis pubescentibus et breviter ciliatis; floribus omnibus ☿; corollae tubo gracili hispidulo-puberulo saepius arcuato in fauces dilatato; limbo 5-dentato recurvato; achenio compresso griseo glabro apice subtruncato obsolete 2-dentato; paleis striatis conduplicatis argute 2–3-dentatis tenuiter parceque puberulis. — WENMAN ISL.: 8 December, 1898, *Snodgrass* & *Heller*, no. 10 (hb. Gr.). Endemic. This species is probably nearest *S. retroflexa* recently described by Mr. Hemsley from Indefatigable Island. That, however, has the leaves recurved and somewhat crisped, of slightly different dentation and a corolla decidedly ampliate in the throat; also villous petioles. *S. Baurii* differs in its rugulose leaves, *S. Hopkinsii* in its very different indumentum, and *S. incisa* in leaf contour and the bluntly toothed chaff. PLATE 3, FIG. 8.

S. n. sp. ? — ALBEMARLE ISL. : Iguana Cove, below 300 m. alt., *Snodgrass* & *Heller*, no. 856 (hb. Gr.); 300 to 600 m. alt., *Snodgrass* & *Heller*, no. 869 (hb. Gr.); Tagus Cove, 1230 m. alt., *Snodgrass* & *Heller*, no. 875 (hb. Gr.). Differing from all the other species in its large broadly ovate more or less cordate subentire leaves. Unfortunately all the specimens are sterile so that there is a slight doubt as to the genus. Endemic.

SONCHUS, L.

S. OLERACEUS, L. Sp. 794 (1753). — ALBEMARLE ISL. : *Snodgrass* & *Heller*, no. 908 (hb. Gr.); Iguana Cove, *Snodgrass* & *Heller*, no. 50 (hb. Gr.). CHARLES ISL. : *Snodgrass* & *Heller*, no. 404 (hb. Gr.). Cosmop. weed.

SPILANTHES, L.

S. ACMELLA, Murr. Syst. ed. 13, 610 (1774); Hook. f. (4), 261; Anderss. (1), 188, & (2), 74. — CHARLES ISL. : *Edmonston,* acc. to Hook. f., l. c. NARBOROUGH ISL. : southern part, *Snodgrass* & *Heller*, no. 315 (hb. Gr.). Sterile and doubtful. Widely distrib. in trop. reg.

S. DIFFUSA, Hook. f. (3), 214; Anderss. (1), 188, & (2), 74. — CHARLES ISL. : *Darwin.* JAMES ISL.: in a smaller form, *Darwin,* acc. to Hook. f., l. c. Endemic.

TAGETES, L.

T. ERECTA, L. Sp. 887 (1753); Caruel (1), 623. — CHATHAM ISL. : *Chierchia,* acc. to Caruel, l. c. Widely distrib. in warm reg. as a result of cult.

TABLE I. — BOTANICAL COLLECTIONS[1] UPON THE GALAPAGOS ISLANDS.

[The numbers relate exclusively to Pteridophytes and Spermatophytes.]

Collector.	Vessel, etc.	Date.	Islands visited.	Collection determined by.	Remarks.	No. specimens recorded (excl. duplicates).	Plants[2] new to Archipelago.	Plants[2] new to Science.	Specimens examined in preparing this paper.
David Douglas. Dr. John Scouler.	Brig *William & Ann*, Capt. Hanwell (British).	9-12 Jan., 1825.	James.	Sir J. D. Hooker.	Cf. Comp. Bot. Mag. ii. 86-87.	23	23	8	1
James Macrae.	Frigate *Blonde*, Command. Lord George Anson Byron (British).	26 Mar.-2 Apr., 1825.	Albemarle.	Sir J. D. Hooker.	Cf. Trans. Hort. Soc. Lond. v. p. v., & vi. p. iii.	41	34	20	2
Hugh Cuming.	Private yacht.	1829.	Unascertained.	Ferns by Sir W. J. Hooker.	Cf. Journ. Bot. iii. 325-326.	9	4	2	0
Charles Darwin.	H. M. S. *Beagle*, Capt. Fitz Roy (British).	15 Sept.-20 Oct., 1835.	Albemarle. Charles. Chatham. James.	Sir J. D. Hooker; bryophytes by Wilson.	Plants chiefly in herb. Cambridge Univ.	209	154	78	1
Adm. Abel Du Petit-Thouars. Adolphe-Simon Néboux.	Frigate *Vénus* (French).	21 June-15 July, 1838.	Charles.	Joseph Decaisne. Sir J. D. Hooker. Cactaceae by Weber.	Plants in herb. Musée d'Hist. Nat., Paris.	5	3	3	1
Thomas Edmonston. Dr. John Goodridge.	H. M. S. *Herald*, Capt. Beechey (British).	6-16 Jan., 1846.	Charles. Chatham. James.	Sir J. D. Hooker.		41	21	3	13
Capt. James Wood, R.N.	H. M. S. *Pandora* (British).	6-16 Jan., 1846.	Charles. Chatham.	Ferns by Sir W. J. Hooker.	44 ferns received at the Kew Garden in 1854.	27	15	0	0

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[1] In addition to the collections here recorded I find references to two plants collected on the Galapagos Islands by a Mr. King, and there is one specimen in the Gray Herbarium said to have been collected on the Galapagos Islands by a Mr. Brace.

[2] By plants are here meant species, varieties, and named forms of pteridophytes and spermatophytes.

TABLE I. — *Continued.*

Collector.	Vessel, etc.	Date.	Islands visited.	Collection determined by.	Remarks.	No. specimens recorded (excl. duplicates).	Plants new to Archipelago.	Plants new to Science.	Specimens examined in preparing this paper.
Prof. N. J. Andersson.	Frigate *Eugenie*, Rear Adm. C. A. Virgin (Swedish).	11-20 May, 1852.	Albemarle. Charles. Chatham. Indefatigable. James.	Phanerogams, ferns, etc., by Prof. N. J. Andersson. Mosses by J. Ångström.		325	111	50	99
Dr. A. Habel.		22 July, 1868–Jan., 1869.	Abingdon. Bindloe. Hood. Indefatigable.	69 species named at Kew.	28 specimens now in hb. Kew.	69	8	2	0
Dr. Thomas Hill. Dr. Franz Steindachner.	*Hassler*, Capt. Philip C. Johnson (American).	10-19 June, 1872.	Albemarle. Charles. Indefatigable. James. Jervis.	A few plants cited in scattered monographs; the lichens determined by Tuckerman.	Plants of Steindachner in hb. Naturh. Mus. Vienna; lichens and algae of Hill in Crypt. hb. Harvard Univ.	96	2	1	2
Dr. Theodor Wolf.	*Venecia*, Capt. Petersen (Ecuadorian).	Aug., 1875.	Albemarle. Charles. Chatham. Indefatigable.	Dr. T. Wolf.	Dr. Wolf states that all his plants were destroyed during storage in Guayaquil.	7	6	0	0
Lieut. Gaetano Chierchia. Lieut. Cesare Marcacci.	Sloop *Vettor Pisani*, Capt. G. Palumbo (Italian).	21-31 Mar., 1884.	Charles. Chatham.	Phanerogams by Prof. T. Caruel. Algae by A. Piccone.	Lieut. Marcacci appears to have collected only algae.	44	22	2	0

Prof. Leslie A. Lee.	S. S. *Albatross*, Lieut. Comm. Z. L. Tanner, U. S. N. (American).	5–11 Apr., 1888.	Albemarle. Charles. Chatham. Hood. James.	B. L. Robinson.	Plants in U. S. Nat. Museum, a few in herb. Gray.	42	4	0	42
Alexander Agassiz.	S.S. *Albatross*(American).	28 Mar.–4 Apr., 1891.	Charles. Chatham. Duncan. (Also James, on which no plants seem to have been collected.)	Dr. J. N. Rose.	Plants chiefly at U. S. Nat. Museum; some duplicates in herb. Gray.	41	1	0	25
Prof. Georg Baur. C. F. Adams.	Local sloops from Guayaquil.	9 June–6 Sept., 1891.	Abingdon. Albemarle. Barrington. Bindloe. Charles. Chatham. Duncan. Gardner. Hood. Indefatigable. James. Jervis. Tower.	Phanerogams and ferns by B. L. Robinson and J. M. Greenman; lichens by Miss C. E. Cummings.	A nearly complete set in herb. Gray.	385	66	37	346
R. E. Snodgrass. Edmund Heller.	*Julia E. Whalen* (American). Hopkins Exped. from Leland Stanford Jr. Univ.	10 Dec. 1898–June, 1899.	Abingdon. Albemarle. Barrington. Bindloe. Charles. Chatham. Culpepper. Duncan. Gardner. Hood. Indefatigable. James. Narborough. Seymour. Tower. Wenman.	Phanerogams and ferns chiefly by B. L. Robinson; thallophytes and mosses by W. G. Farlow; hepatics by A. W. Evans; Cactaceae by K. Schumann.	A complete set of vascular plants in herb. Gray.	949	78	19	949

TABLE II.

The Distribution of the Pteridophytes and Spermatophytes upon the Galapagos Islands.

[The sign + indicates that a specimen has been examined from the island in question; the sign —, that a record has been found.]

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Acrostichum aureo-nitens . .			+			+	—								+			
aureum	—																	
muscosum													—					
viscosum													—					
Adiantum aethiopicum . . .	—																	
concinnum		+	+										+		+			
Henslovianum						—	+						—					
incisum	—																	
Kaulfussii							—											
parvulum.						—												
patens	—																	
prionophyllum							—											
Aspidium coriaceum . . .													—					
semicordatum	—																	
Asplenium anisophyllum . .													—					
auritum		+				—	+						—					
var. macilentum. .	—																	
cicutarium							+											
formosum		+	+			+	—											
furcatum			+										—					
laetum.							—											
lunulatum						+												
rhizophyllum													—					
rutaceum	—																	
Serra	—																	
serratum							—											
Blechnum occidentale . . .		+	+			—	+						—					
var. caudatum .	—																	
Cheilanthes microphylla . .			+			—	+											
myriophylla			+															
Cystopteris fragilis						—												
Gleichenia dichotoma . . .	—																	
Gymnogramme chaerophylla						—												
leptophylla						+												
tartarea			+		+	+												
Hypolepis repens	—																	
Nephrodium brachyodon . .	—																	
macrophyllum	—																	
molle			+			+							—					
unitum						—												

TABLE II.—*continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Nephrodium villosum							—											
Nephrolepis acuta					+													
pectinata							+						—					
Nothochlaena sulphurea			+										—					
Pellaea geraniaefolia	—																	
Polypodium angustifolium													—					
aureum													—					
crassifolium	—																	
incanum			—			—												
lanceolatum			+			—							—					
lepidopteris	—																	
loriceum	—																	
paleaceum			+			—	+											
pectinatum			+			—	+						—					
percussum	—																	
Phyllitidis													—					
pleiosorum													—					
rude	—																	
squamatum		+	+		+	+	+		+						+			
sp.																		—
Pteris aquilina, var. esculenta			+				+											
incisa	—																	
pedata			+			+	+						—					
propinqua, var Cumingiana	—																	
Taenitis angustifolia	—																	
Azolla caroliniana						—												
Salvinia sp.						—												
Lycopodium clavatum							+											
dichotomum	—																	
sp.						—												
sp.						—												
Potamogeton pectinatus			+															
Ruppia maritima			+															
Najas marina, var. latifolia			+															
Anthephora elegans						+	+					—	+					
Aristida divulsa		+			+		+											
repens													—					
subspicata			+	+	+							+	+			+		
villosa														+				
Bouteloua pilosa		+	+	+			—					+	+	+		+		
Cenchrus distichophyllus			+															
granularis			+			+	+									+		
platyacanthus		+	+	+	+	+	+			+	+	+	+		+			
sp.							+											
Chloris anisopoda						+												
elegans						+										+		
radiata						+												
Chusquea sp.			+															

TABLE II. — *continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Eleusine aegyptica							+				+							
indica						+	+											
Eragrostis bahiensis			+															
ciliaris		+	+		+	+	+				+		+		+		+	
major				+		+	+			+	+					+	+	
pilosa													+					
Eriochloa distachya							+											
Leptochloa albemarlensis			+															
filiformis									+							+		
Lindleyana			+		+		—											
mucronata				+														
virgata						+												
Oplismenus setarius							+											
Panicum colonum						—												
fasciculatum						+						+	+					
fluitans							+				+							
fuscum						—	—											
hirticaulum				+		—	+			+	+	+				+		
var. minus						+	—											
molle							—											
multiculmum			+	+		+			+	+								
sanguinale							+											
serotinum						—												
Paspalum canescens			+			+									+			
conjugatum						—	+											
distichum													+					
longe-pedunculatum			+		+	+												
penicillatum						—												
scrobiculatum							—											
sp.												+						
Pennisetum pauperum			+															
Setaria floriana						—												
setosa			+	+			+		+		+		+		+			
sp.			—															
Sporobolus domingensis		+	+								+							
indicus						+	—											
virginicus			+				+											
Stenotaphrum glabrum			+				+											
Stipa rostrata							+											
Cyperus aristatus			+			—	+						—		+			
brachystachys		+	+			—	+						—				+	
confertus			+		+	+	+		+		+	—	—					
esculentus			+				+											
fugax							+											
galapagensis							—											
grandifolius							—											
laevigatus			+															
ligularis			+															
Mutisii			+			+	—			+					+	+		
rotundus			—															

TABLE II.—*continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Cyperus rubiginosus						—	+		+									
var. cornutus						+	+		+		+					+		
strigosus						—	—											
suranimensis													—					
tristachyus							+											
sp.															+			
sp.																		—
Dichronema leucocephala							+											
Eleocharis fistulosa							—											
mutata			+															
Fimbristylis capillaris					+										+			
diphylla											+							
Hemicarpha subsquarrosa							+											
Kyllinga pumila						+												
Scleria pratensis							+											
Lemna sp.						—												
Tillandsia insularis	—								+									
Commelina nudiflora			+			—			+				—					
Commelinacea ?						—												
Hypoxis decumbens						—	+											
Epidendrum spicatum						+							—					
Peperomia flagelliformis													—					
galapagensis													—					
galioides		+																
petiolata													—					
ramulosa						—												
Snodgrassii			+															
n. sp.			+															
sp.							+											
Fleurya aestuans			+			+	+						—		+			
Parietaria debilis						—							—					
Pilea Baurii						+	+											
muscosa													—					
peploides													—					
Phoradendron florianum						—												
galapageium							—											
Henslovii		+	+			—	+											
uncinatum															+			
Polygonum acuminatum	—																	
galapagense							—											
Atriplex sp.												+				+		
sp.																		+
Alternanthera radicata							+				+							
rigida													+					
subscaposa						—												
Amaranthus caracasanus			+			+	+					—						
celosioides						—	—											
sclerantoides				+		—									+			
forma chathamensis							+											
" hoodensis											+							

TABLE II. — *continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Amaranthus spinosus . . .						–												
squarrulosus			+			+	+		+			+		+		+		
urceolatus												–						
viridis				+			+											
Froelichia juncea			+															
lanigera			+															
nudicaulis						+	–											
scoparia															+			
Iresine Edmondstonei . . .						–												
Pleuropetalum Darwinii . .			+										–					
Telanthera echinocephala . .		+	+	+		+	+		+	+	+	+	+					
filifolia													–					
flavicoma		+				–				+	+							
frutescens							–											
glaucescens						–	+											
Helleri								+										
var. obtusior . . .																		+
nudicaulis			+			+	+		+				+					
rugulosa							+											
Snodgrassii																+		
strictiuscula			+				+								+			
vestita												+						
Batis maritima													+					
Boussingaultia baselloides .						–												
Phytolacca decandra . . .													–					
Boerhaavia erecta			–				+					–						
paniculata													–					
scandens						+	+					–	–					
viscosa			+			+	+			+		+	+					
Cryptocarpus pyriformis . .			+	+	+	+	–				+		+		+			
Pisonia floribunda			+										–					
Nyctaginacea ?																+		
Mollugo flavescens			+			+	+					+						
var. floriana . .						+												
gracillima			+			+							+					
Snodgrassii			+												+			
Sesuvium Edmonstonei . .				+		–				+						+		
Portulacastrum . . .			+													+		
Trianthema Portulacastrum .				+		+	–		+	+	+	–	–			+		
Portulaca oleracea			+			+	+			+	+							
Portulaca sp.																		+
Drymaria cordata							+						–					
Cissampelos Pareira		+	+			+	+						–					
Brassica campestris						+												
Sinapistrum						–												
Raphanus sativus						–												
Senebiera pinnatifida . . .													–					
Sanguisorbea ?						–												
Acacia farnesiana			–															
macracantha						+	–					+	–					

TABLE II.—*continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Acacia tortuosa						—	—						+					
sp.			+															
sp.						—												
sp.													+					
Astragalus Edmonstonei						—												
Caesalpinia Bonducella			+															
pulcherrima						—	—											
Canavalia obtusifolia					+													
Cassia hirsuta						+												
occidentalis			+			+	+											
picta			+				+											
sericea							—					—				+		
Crotalaria glabrescens			+				—											
pumila			+			+	+						+		+	+		
setifera			+															
Dalea parvifolia			+			+	+					+	+					
tenuicaulis			+				—											
Desmanthus depressus						—	+		+	+	+							
Desmodium galapagense													—					
incanum							+											
molle		+	+		+	—				+	+	+		+				
spirale			+		+	+				+	+		+			+		
uncinatum							+											
Erythrina velutina													+					
Galactea Jussiaeana, var. volubilis			+			—	+					+			+	+		
var. glabrescens							+											
n. sp.						—												
Geoffraea superba											+							
Mimosa asperata						—												
Neptunia plena						+	+			+		+		+		+		
Parkinsonia aculeata			+			+	+		+		+					+		
Phaseolus adenanthus											+							
mollis													—	+				
semierectus						+	+											
Piscidia Erythrina							+						—					
Prosopis dulcis		+				+	+		+	+	+	+	+			+		
Rhynchosia minima		+	+	+	+	+	+					+			+			
reticulata							—											
sp.			+															
sp.					+													
Stylosanthes scabra		+	+		+	+						+		+				
Tephrosia cinerea			+	+	+	+	+					+			—	+		
Vigna owahuensis													—					
Oxalis carnosa						—			+									
Cornelli			+	+		+	+		+	+	+	+	+					
corniculata							+											
Linum oligophyllum			+															
Kallstroemia adscendens						—	+		—	+	+							
Tribulus cistoides			—			+					+	+	—			+		

TABLE II. — *continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Tribulus cistoides var. anacanthus			+															
sericeus						+	+											
sp.								+										
Zanthoxylum Pterota . . .		+	+			+	+						—		—			
Castela galapageia							—											
forma albemarlensis			+															
" bindloensis .					+													
" carolensis .						+												
" duncanensis .									+									
" jacobensis .													+					
" jervensis . .														+				
Bursera graveolens		+	+	—		—	—					—	—		+		—	
malacophylla																+		
Polygala Andersonii . . .												+						
galapageia		+	+		+	+	+					+		+				
var. insularis . . .			—			—	+							+				
Acalypha Adamsii							+											
albermarlensis			+															
Baurii							+											
chathamensis							+											
cordifolia						—	—											
diffusa			+				—											
flaccida													—					
parvula			+															
reniformis						—												
sericea		+	—		+													
spicata						+	+		+	+	+			+				
strobilifera			+				+											
velutina						+	—											
var. minor						+												
sp				+														
sp.				+														
sp.				+														
sp.												+						
sp.									+									
Croton Scouleri				+	+	+	+				+		+		+		+	
var. albescens . .			+		+	+							+					
forma microphyllus			+															
var. brevifolius . .			+			—		+	+	+						+		+
" grandifolius .		+				+	+						—				+	
" Macraei . . .			+										+					
Euphorbia amplexicaulis . .					+		—									+	+	+
apiculata							—											
articulata		+	+		+	+	+						+			+		
diffusa			—									+		+				
flabellaris		+		+		—				+		+	+			+	+	
galapageia						+												
nesiotica																+		
nummularia							+											

TABLE II.—*continued.*

	Galapagos Is.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Euphorbia nummularia, var. glabra . . .						+												
pilulifera						+	+						—					
punctulata			—						+		+							
recurva							+											
viminea			+															
forma barringtonensis . . .				+	+													
" carolensis . .						+												
" castellana . .																	+	
Euphorbia viminea, forma chathamensis .							+											
forma jacobensis .													+					
" jervensis .														+				
var. abingdonensis		+																
sp.					+													
sp.							+											
sp.						—												
Hippomane Mancinella . .			+				—											
Manihot utilissima							—											
Phyllanthus carolinensis . .			+			+	+						+		+			
Ricinus communis						—												
Callitriche sp.						—												
Maytenus obovata			+	+		+	+		+		+		+	+	—	+		
Cardiospermum Corindum .			+			—	+					—	+					+
galapageium			+									+						
Dodonaea viscosa			+															
var. spathulata . .			+															
Sapindus Saponaria			+															
Discaria pauciflora			+	+			+		—		+	—	—			+		
Cissus sicyoides			+		—	+									—			
Vitis vinifera						—												
Corchorus pilobulus			+							+								
Triumfetta semitriloba . . .			+															
Abutilon Anderssonianum .		+		+	+	—	+		+	+		+					+	
depauperatum			+	+		—												
Anoda hastata						+												
Bastardia viscosa			+				+											
Gossypium barbadense . . .		+	+	+		+	+		—	+	+		—			+		
Klotzschianum			—		+	—	—					+						
Hibiscus tiliaceus						—												
Malachra capitata													—					
Sida acuta, var. carpinifolia .						—												
angustifolia			+			+	+		+	+		+	—		+			
cordifolia			+															
paniculata			+			+												
rhombifolia						+	+											
spinosa			+															
veronicaefolia, var. humilis			+															

TABLE II. — *continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Waltheria reticulata			+			+							+	—				
forma acamata												+						
" Anderssonii				+			+								+		+	
" intermedia		+			+	+				+					+			
Turnera ulmifolia						+												
Passiflora foetida						—	+											
lineariloba						—				+	+		—		+			
puberula													—					
Carica Papaya						—												
Mentzelia aspera			+			+	+		+	+	+	+					+	
Sclerothrix fasciculata			+										+		+			
Cereus galapagensis						—												
nesioticus			+												+			
sclerocarpus			+		?		?						?		?			
Thouarsii						—												
Opuntia galapageia						—							—	—				
Helleri																		+
myriacantha			—			—						—						
sp.		—		—	—		—	—	—		—				—	—	—	
Cuphea patula							—											
Rhizophora Mangle			—				—		—			—			+			
Psidium galapageium			+										—					
Conocarpus erectus			+				—						+					
Laguncularia racemosa			+												+			
Miconia Robinsoniana							+											
Myriophyllum sp.						—												
Apium laciniatum						—												
leptophyllum			+										—					
Centella asiatica							—											
Hydrocotyle galapagensis							+											
Petroselinum sativum						—												
Plumbago scandens			+			+	+		+			—						
Vallesia cymbaefolia			+				—				+							
pubescens						+	+											
Asclepias angustissima		+	+															
Vincetoxicum sp.						+												
Calystegia Soldanella						—												
Cuscuta acuta					+	—	—								+			
gymnocarpa			+										—					
Evolvulus glaber			+			+	+		+		—	—	—			+		
simplex						+	+					+	+					
Ipomoea biloba			+				—											
Bona-nox			+															
campanulata			+															
galapagensis			+			+	—		+		+		+			+		
Habeliana					+				+	+	+						+	
Kinbergi		+					—					+		+			+	+
linearifolia													—					
Nil						+	+											
pentaphylla		+				—	—		+	+	+	+	—	+		+	+	

TABLE II.—*continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Ipomoea tubiflora													−					
Coldenia Darwini			−		+		+					+	+					
fusca			+	+		−	+				+	+				+		
Cordia Andersoni						+	+											
galapagensis			+						+			+			+			
Hookeriana			+			+							−		+			
leucophlyctis			−										+					
lutea		+	+	+	+	+	+		+	+	+	−	+	+		+	+	
revoluta						−												
var. nigricans			−															
Scouleri							+						−					
n. sp.?						+												
Heliotropium Andersonii												+						
curassavicum					+		+				+	+				+		
indicum						+												
parviflorum		+	+	+		+	+			+	+	−	+		+		+	
Tournefortia hirsutissima							−											
laurifolia							−						−					
psilostachya			+			+	−				+		+					
pubescens			+			+	+		+			+						
rufo-sericea		+	+			+							−					
strigosa			+			−	−											
Avicennia officinalis			−			+	−					−	+		−	+		
Clerodendron molle			+			+	+						+					
sp.						−												
sp.						−												
Duranta Plumieri			+															
Lantana peduncularis		+	+	+	+	+	+		+	+	+	+	+	+	+		+	
Lippia canescens						+	+		+		+							
rosmarinifolia		+	+															
salicifolia						−												
Stachytarpheta dichotoma						+												
Verbena carolina													−					
grisea									+									
litoralis						+	+											
officinalis													−					
Hyptis capitata						−												
subverticillata			+									+	+		+			
Salvia occidentalis			+			+	+						+					
prostrata													−					
tiliaefolia						−												
Teucrium inflatum						+	+											
Acnistus ellipticus						−												
insularis							+											
Capsicum annuum						+	+											
Datura Tatula						+												
sp.			+															
Lycium sp.											+					+		
Lycopersicum esculentum, var. minor		+	+				−				+		+		+			

TABLE II. — *continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Lycopersicum peruvianum, var. parviflorum .							—											
pimpinellifolium . . .						—	—						—					
sp.							+											
Nicotiana glutinosa						—												
Tabacum						—												
sp.						—												
Physalis angulata						—	+											
ixocarpa						+												
pubescens			+		+	+					+		+		+			
sp.						—												
Solanum Edmonstonei . . .						—												
nigrum			+			+	+		+				+					
verbascifolium						—							—		+			
sp.						—												
sp.						—												
Thinogeton Hookeri . . .												—						
Miersii			+	+		—	+	+			+				+			
Capraria biflora						+	+											
Scoparia dulcis						+	—											
Scrophulariacea													—					
Tecoma sp.?							—											
Dicliptera peruviana . . .													—					
Justicia galapagana		+																
Tetramerium hispidum . . .			+										—					
Plantago major							—											
tomentosa, var.? . . .													—					
Borreria basalis							—											
Baurii							+											
dispersa			+			+	+					+	—					
divaricata						+												
ericaefolia		+	+			+	—								+			
falcifolia			—															
galapageia									+									
linearifolia													—					
ovalis						+												
forma abingdonensis		+																
pacifica												+						
parvifolia			—															
perpusilla													—					
rotundifolia												+						
suberecta			+	+														
sp.							+											
Chiococca racemosa		+	+		+	+	—						—		+			
Diodia Radula							+											
Psychotria angustata . . .						—												
rufipes		+	+			+	+						—					
Relbunium sp.						—												
Spermacoce tenuior							+						—					
Citrullus vulgaris						—												

TABLE II. — *continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Cucurbita Pepo						—												
Elaterium cordatum													—					
Momordica Charantia			+															
Sicyos villosus						—												
Lobelia xalapensis						—							—					
Scaevola Lobelia	+						—											
Acanthospermum lecocarpoides											+							
microcarpum						+												
Ageratum latifolium						+	—											
Aplopappus lanatus	—																	
Baccharis pilularis						—												
Pingraea, var. angustissima			+															
Steetzii						+												
Bidens chilensis			+															
pilosa			—			—	+											
refracta			+			+					+		+		+			
Blainvillea rhomboidea			+	+		+	+		+		+	+	+		+	+		
tenuicaulis			—			—												
Brickellia diffusa			+															
Chrysanthellum erectum						+	+					+			+			
pusillum			—			+	—											
Eclipta erecta						+	+				+							
Elvira inelegans						—												
repens			+										+					
Encelia hispida						—	+											
Erigeron lancifolius			+												+			
linifolius						+												
tenuifolius		+	+			—			+				+					
sp.						+												
Eupatorium filicaule			+															
sp.?						—												
Flaveria Contrayerba						—												
Hemizonia squalida	—																	
Jaegeria gracilis						—												
prorepens													—					
Lecocarpus foliosus						+												
Lipochaeta laricifolia		+	+			+	+								+			
Pectis Anderssonii												+						
Hookeri			—	+		—					+		+	+		+		
linifolia							—					—				+		
subsquarrosa							—											
tenuifolia			+			+	+					+				+		
Porophyllum ellipticum		+	+			+	+		+		+	+	+	+				
Scalesia affinis						—												
aspera												+						
atractyloides	—																	
Baurii									+									
var. glabrata									+									

TABLE II. — *continued.*

	Galapagos Ids.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Scalesia Darwinii						+							—					
decurrens			—			+												
divisa							+											
gummifera			+															
Helleri				+														
Hopkinsii		+																
incisa							—											
microcephala			+												+			
narbonensis															+			
ovata						+												
pedunculata													—					
retroflexa												—						
Snodgrassii																		+
n. sp.?			+															
Sonchus oleraceus			+			+												
Spilanthes Acmella						—									+			
diffusa						—							—					
Tagetes erecta							—											

General Features of the Flora.

The habital traits of the vegetation on the Galapagos Islands have been graphically although rather gloomily pictured by Darwin (2), 140, and by Wolf (1), 277. A. Agassiz (1), 57–62, and Baur, who visited the islands at a more favorable season, describe the flora as somewhat more luxuriant. The lower slopes of all the islands, however, are relatively sterile, arid, and rough, much of the surface being covered with irregular blocks of lava. The air, although not excessively hot, is very dry. Trade winds are said to be moderate in force, but almost constant. The perennial vegetation of these lower parts of the islands is predominatingly of a small-leaved xerophytic type, being composed of scattered and often sparse, stemmy shrubs and undershrubs, or wiry herbs and grasses, over which rise the bulky and grotesque trunks of arborescent species of *Cereus* and *Opuntia*. On those islands, which, like Gardner, Hood, Tower, and Bindloe, are entirely low, not attaining an altitude of 300 m., this is the only kind of perennial vegetation, except a few halophytes along the shores. On the higher islands, however, like Albemarle, Charles, Chatham, James, and Indefatigable, the upper parts penetrate higher, moister strata of the air, and support a much more luxuriant vegetation of a mesophytic type. Between these chief types of vegetation there are perceptible but ill-defined belts of an intermediate nature. A few lakes and ponds, some muddy and others briny, support a littoral vegetation, chiefly of *Cyperaceae*, a family represented by seventeen species and varieties of *Cyperus* and one or two each of *Dichronema*, *Eleocharis*, *Fimbristylis*, *Hemicarpha*, *Kyllinga*, and *Scleria*. A cold spring on Charles Island contains an *Azolla* (Andersson), and in a small brook on the same island a *Salvinia*, a *Callitriche*, and a *Lemna* have been found (Wolf); but as both springs and streams are rare and small, such true hydrophytes are few and relatively unimportant in the archipelago. Halophytes are more abundant, and occur not only on the sandy beaches in the brackish marshes along the shore, but to some extent around the saline lakes of the interior portions of the islands. The chief halophytes are *Cissampelos Pareira*, *Tephrosia cinerea*, *Rhizophora Mangle*, *Laguncularia racemosa*, *Sesuvium Portulacastrum*, *S. Edmonstonei*, *Calystegia Soldanella*, *Ipomoea biloba*, *I. Habeliana*, *Batis maritima*, *Avicennia officinalis*, *Verbena litoralis*, *Heliotropium curassavicum*, *Scaevola Lobelia*, *Ruppia maritima*, *Najas marina*, var. *latifolia*, and probably *Thinogeton Miersii*, *T. Hookeri*, and two undetermined species of *Atriplex*.

The lower arid slopes support not only the stemmy, suffrutescent vegetation described above, but also are covered from time to time by an ephemeral growth of desert annuals, which spring up shortly after the rainy season begins, mature rapidly, and quickly dry up completely. Examples of this type of plants are found in *Porophyllum ellipticum*, *Evolvulus simplex*, and several species of *Boerhaavia*, *Kallstroemia*, *Tribulus*, and *Bidens*.

Trees, if we except the arborescent cacti, occur chiefly upon the upper parts of the islands and never attain great size. In many cases, however, the same species which form a tree-like growth in the upper region extend well into the lower or even to the shore as shrubs, stunted by the extreme drought and sterility of their environment. The trees and shrubs are in great part armed with spines or thorns, e. g. *Mimosa*, *Acacia*, *Parkinsonia*, *Discaria*, *Castela*, *Zanthoxylum*, *Cereus*, and *Opuntia*, — genera, all of which, it will be noticed, belong to the choripetalous dicotyledons. On the other hand, species protected by poisonous juices or stinging hairs appear to be few. The climbing plants of the islands are chiefly of the genera *Boussingaultia*, *Cissampelos*, *Galactea*, *Rhynchosia*, *Cardiospermum*, *Ipomoea*, *Elaterium*, *Momordica*, and *Sicyos*.

They are neither so numerous nor conspicuous as in the tropical parts of continental America, and it may be noticed that the climbers of the Galapagos Islands are herbaceous, the true woody liana, so common in tropical jungles, being almost unknown upon these islands.

Epiphytes occur only at the higher altitudes and are neither abundant nor showy. Their ecological class is chiefly represented by one *Tillandsia*, one *Epidendrum*, and several *Peperomiae*. Of phanerogamic parasites there are four species of *Phoradendron* and two of *Cuscuta*.

The phanerogams of the Galapagos Islands have prevailingly small and inconspicuous flowers, although exceptions are not infrequent. It is also worthy of remark that the flowers are in most cases regular and of a rather simple structure. Zygomorphic flowers are not numerous, and even in such genera as *Epidendrum, Salvia*, etc., where zygomorphy is universal, the Galapageian species show this trait only moderately developed. Similarly, flowers with other highly developed mechanisms for securing cross-pollination seem to be very rare in the islands. Fruits with hook apparatus or spines to aid their distribution by mammals are found in *Cenchrus, Tribulus, Acanthospermum, Bidens*, and *Lecocarpus*; but by the decided reduction in the spines of most of the species of *Cenchrus*, and in a variety of *Tribulus cistoides*, as well as in *Acanthospermum microcarpum*, it is easy to remark a tendency toward abortion in this apparatus, — a fact readily explained by the paucity of indigenous mammals. Of course the settlement of the islands has brought introduced mammals in considerable quantity, but it is too recent to have had a perceptible influence in this matter.

AFFINITIES OF THE FLORA.

The flora of the Galapagos Islands is almost wholly American in character. It contains, it is true, a very few plants which are not found in America. Thus a slight relationship to the flora of the Hawaiian Islands might be inferred from the genus *Lipochaeta*, of which one species is Galapageian and the others Hawaiian. There is also a report of a *Vigna* common to the Hawaiian Islands, the Galapagos, and Chili, — a matter which I have been unable to verify. *Ipomoea campanulata* of the East Indies and Malayan Archipelago occurs on the Galapagos Islands, but not to my knowledge upon the continent of America. This is probably a chance introduction. Several species of the Old World, such as *Vitis vinifera*, mentioned by Caruel (1), 623, *Brassica campestris, B. Sinapistrum*, and *Raphanus sativus*, are of course relics of cultivation or introduced weeds. *Eleocharis fistulosa*, ascribed by Caruel

(1), 622, to Chatham Island, is with scarcely a doubt an erroneous determination.

While it is thus clear that the Galapageian flora is only an outlying portion of the American flora with a strong specific differentiation, it is impossible to trace its relationship closely to any one section of the Pacific American vegetation. It can only be said in a general way that nearly all the plants of the archipelago are identical with, or obviously related to, species of the Sierras and Andes or of the Pacific Slope between Lower California on the one hand and northern Chili on the other. The xerophytic elements in the Galapageian flora show a considerable resemblance to the desert flora of southern Peru and the drier parts of the Andes. The mesophytes, on the other hand, correspond most nearly to plants of Ecuador, Colombia, Central America, and southern Mexico.

Those who have written upon the phytogeography of the Galapagos Islands have frequently mentioned the West Indian affinities of the flora, but here I can find no close resemblance or significant relationship. It is to be noticed that Hooker, who first employed the term "West Indian" regarding the flora of the Galapagos Islands, either used it to include, or expressly qualified it by the addition of, the flora of Panama and the adjacent lowlands of the continent, — a qualification which has not always been sufficiently regarded by subsequent authors. But, on the other hand, the discoveries of the last half century have shown a much greater difference between the flora of the Antilles and of the Panama region than was to be inferred when Hooker wrote; so although a definite relationship can be traced between the Galapageian flora and that of the lower slopes of Colombia, it does not follow that there is any marked affinity to the flora of the West Indian Islands. Indeed, of the species common to the Galapagos and the Antilles there are none (if we except a sterile and doubtfully identified specimen of the Cuban *Cenchrus distichophyllus*) which do not also occur upon the adjacent parts of the continent, and nearly all, like the halophytes of the shores, are species of wide tropical distribution.

Hooker (4), 239, 250, drew attention to what appeared to be an interesting double relationship between the plants of the Galapagos Islands and those of other regions, as follows: "Here, as in other countries, the vegetation is formed of two classes of plants, — the one peculiar to the group, the other identical with what are found elsewhere. In this there are even indications of the presence of two nearly equal Floras, — an indigenous and introduced, — and these are of a somewhat different stamp;

for the introduced species are for the most part the plants of the West Indian Islands and of the lower hot parts of the South American coast, whilst the peculiar Flora is chiefly made up of species not allied to the introduced, but to the vegetation which occurs in the Cordillera or the extra-tropical parts of America." But, after repeated efforts, I am unable to verify this double relationship, and must infer that subsequent discoveries, both upon the mainland and upon the islands, have done much to weaken the grounds upon which these conclusions once rested. Many cases could now be cited to show that the endemic plants of the Galapagos Islands, far from forming a marked or peculiar class, are often the nearest allies of species or varieties which are common to the islands and the continent. Both classes include alike the most widely diverse elements, — xerophytic, mesophytic, and halophytic types, annuals and perennials, herbs, shrubs, and trees, climbers and epiphytes, — and both occur in common at all altitudes and in every sort of habitat the islands afford. Accordingly it is not remarkable to find their closest congeners occupying the same diverse habitats upon the mainland from the hot, moist lowlands about Guayaquil and Panama to the cool parts of the Andes, dry regions of Peru or western Mexico, and in a few instances the more fertile uplands of Colombia, Central America, and Mexico. Moreover, the endemic forms show all grades of differentiation from their continental allies; some are well marked specific types, others mere varieties, while still others are scarcely distinguishable forms.

It appears, therefore, that very diverse floral elements have reached the archipelago, probably at different times and from widely different habitats. Presumably all have been subjected on the islands to influences of a kind to bring about change in their nature; and in two-fifths of the plants now known on the islands more or less pronounced evidences of such change can be observed. These plants, which show modification, form, as we have seen, no sharply marked class, but pass over very imperceptibly into nearly related forms which it is impossible to differentiate from plants of the mainland. That some plants have reached the Galapagos from the West Indian Islands during the subsidence of the Isthmus of Panama is by no means impossible, but as we should expect, these plants, if such there were, have established themselves in like manner upon the western coast and slopes of the continent, so that it is now quite impossible to trace any direct floral affinity between the West Indies and the Galapagos which the latter do not exhibit even in a higher degree with the western parts of the mainland.

Mr. Hemsley has already commented upon the wide divergence between the flora of the Galapagos group and that of Cocos Island. The latter is situated about three-sevenths of the way from the coast of Costa Rica to the Galapagos Archipelago. Until very recently the flora of Cocos Island has been scarcely known at all. Largely through the efforts of Professor H. Pittier and Messrs. Snodgrass & Heller, about eighty plants have now been secured on the island, and of these only the following eighteen are common to the flora of the Galapagos group:

Plagiochila Anderssonii
Acrostichum aureum
Asplenium rhizophyllum
Nephrolepis acuta
Polypodium aureum
Polypodium lanceolatum
Polypodium Phyllitidis
Eleusine indica
Panicum sanguinale
Paspalum conjugatum
Paspalum distichum
Caesalpinia Bonducella
Euphorbia pilulifera
Ricinus communis
Hibiscus tileaceus
Ipomoea biloba
Ipomoea Bona-nox.

It will be noticed that with the exception of the hepatic (*Plagiochila Anderssonii*) all of these species are weeds or plants of wide tropical distribution.

COMPOSITION OF THE FLORA.

As with most insular floras, the vegetation of the Galapagos Islands is striking rather by the absence of certain great groups than by the number and diversity of the genera and families represented. For instance, in the pteridophytes there is a total lack of arborescent forms on the one hand and of the filmy ferns (*Trichomanes*, *Hymenophyllum*, etc.) on the other. There are no gymnosperms; and among the monocotyledons there are no palms, aroids, rushes, or *Liliaceae*. Indeed, if we except the grasses and sedges (both well represented), the monocotyledons are shown only by some half-dozen scattered species. Among the dicotyledons the families best represented are the *Amarantaceae*, *Nyctaginaceae*, *Aizoaceae*, *Leguminosae* (about 10 per cent of the phanerogamic vegetation), *Euphorbiaceae* (about 12 per cent), *Malvaceae*, *Cactaceae*, *Convolvulaceae*, *Boraginaceae*, *Verbenaceae*, *Labiatae*, *Solanaceae*, *Rubiaceae*, and *Compositae* (about 13.5 per cent). Several great dicotyledonous families, widely distributed and abundant in the tropics of continental America, such as the *Sapindaceae*, *Myrtaceae*, *Melastomaceae*, *Lythraceae*, and *Onagraceae*, are scarcely or not at all represented in the flora of the archipelago. Altogether there have been 72 families[1] of flowering plants and ferns found on the islands. Of these families 39 include endemic forms, and 33 contain only plants common to other regions. Excluding some indeterminate forms, there are 232 genera of pteridophytes and spermatophytes upon the islands.

Notwithstanding the uncertainty with which some plants are here reckoned as species, and others as varieties or forms, the following figures regarding the plants of the Galapagos Islands will have an interest:

	Species.	Varieties.	Forms.	Indeterminate.	Total.
Ferns	52	2	0	1	55
Fern-allies	2	0	0	2	4
Pteridophytes	54	2	0	3	59
Spermatophytes	445	17	19	50	531
Vascular plants	499	19	19	53	590

Of endemic ferns there are only 3, that is, but 5 per cent. Of endemic spermatophytes there are 202 species, 15 varieties, and 19 forms, — a total of 236, that is, 44.4 per cent of the whole flowering flora. The total number of vascular plants which are endemic is 239, or 40.5 per cent. Of these endemic plants 130, that is, more than half, are confined to a single island.

The ratio of (determined) species to genera is as 2.16 : 1.

The ratio of species, varieties, and forms to genera is as 2.55 : 1.

It is noteworthy that, although there is such a high percentage of peculiar forms, varieties, and species, there is no corresponding peculiarity among the genera of these islands.[2] Of the several genera which have from time to time been characterized as exclusively Galapageian, only two, *Scalesia* and *Lecocarpus*, are now maintained, while all the others have been reduced to genera of continental America, with the single exception of *Macraea*, which falls into a genus of the Hawaiian Islands. Even *Scalesia* is not a strong genus, as it is not easy to show very sharp generic distinctions between it and some allied *Helianthoideae* in Mexico and Central America.

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[1] This does not include the *Rosaceae* and *Bignoniaceae*, which rest, so far as their Galapageian occurrence is concerned, upon single and doubtful determinations.

[2] The statement of Darwin (2), 165, regarding the genera of *Compositae*, is, as pointed out by Mr. Hemsley, quite erroneous, and must have rested upon some misapprehension of data furnished him.

FAMILIES CONTAINING VASCULAR PLANTS PECULIAR TO THE ISLANDS.

Family.	Endemic species.	Endemic varieties.	Endemic forms.	Total.
Filices	3	0	0	3
Gramineae	13	1	0	14
Cyperaceae	4	1	0	5
Bromeliaceae	1	0	0	1
Orchidaceae	1	0	0	1
Piperaceae	5	0	0	5
Urticaceae	1	0	0	1
Loranthaceae	4	0	0	4
Polygonaceae	1	0	0	1
Amarantaceae	29	1	2	32
Nyctaginaceae	1	0	0	1
Aizoaceae	4	1	0	5
Leguminosae	6	0	0	6
Oxalidaceae	1	0	0	1
Zygophyllaceae	2	1	0	3
Simarubaceae	1	0	6	7
Burseraceae	1	0	0	1
Polygalaceae	2	1	0	3
Euphorbiaceae	25	7	7	39
Celastraceae	1	0	0	1
Sapindaceae	1	0	0	1
Malvaceae	3	0	0	3
Sterculiaceae	1	0	3	4
Passifloraceae	2	0	0	2
Cactaceae	7	0	0	7
Myrtaceae	1	0	0	1
Melastomaceae	1	0	0	1
Umbelliferae	1	0	0	1
Apocynaceae	1	0	0	1
Asclepiadaceae	1	0	0	1
Convolvulaceae	8	0	0	8
Boraginaceae	12	1	0	13
Verbenaceae	4	0	0	4
Labiatae	2	0	0	2
Solanaceae	5	0	0	5
Acanthaceae	1	0	0	1
Rubiaceae	16	0	1	17
Cucurbitaceae	2	0	0	2
Compositae	39	1	0	40

ABINGDON ISLAND.

Abingdon is, with the exception of the rather remote islets of Wenman and Culpepper, the most northern of the archipelago. It is about 14 km. long, and attains a height of 600 m. It was visited by Dr. Baur for a few hours, September 8th, 1891, and by Messrs. Snodgrass and Heller in June, 1899. Fifty flowering plants and ferns are known to occur on the island, and of these four are peculiar to it, namely: *Euphorbia viminea*, var. *abingdonensis*, *Borreria ovalis*, forma *abingdonensis*, *Justicia galapagana* (with close Mexican congener), and *Scalesia Hopkinsii.* The peculiar element is thus 8 per cent of the flora. *Peperomia galioides* of Mexico and tropical South America occurs upon Abingdon, but upon no other of the Galapagos Islands. The remaining plants are common to other islands of the group, and represent in all 22 families, of which the *Filices*, *Gramineae*, *Rubiaceae*, *Euphorbiaceae*, and *Compositae* have the greatest number of species. Although Abingdon lies, as we have seen, to the northward of the main archipelago and on the side toward Wenman and Culpepper, it has, so far as we yet know, only one plant in common with the former and none with the latter. A little over half the flora of Abingdon is exclusively Galapageian, and the common element is greatest with Charles, Albemarle, and Chatham Islands.

ALBEMARLE ISLAND.

Albemarle is the largest island of the archipelago, and extends through about one and a quarter degrees of latitude. It is L-shaped and crossed by the equator near its northern extremity. There are many craters upon it, some of them having been active within historic times. The five largest range from 770 to 1570 m. in height. The island seems to have been explored chiefly if not exclusively along its western shore, the greater part of the plants collected upon it having been secured about Iguana Cove, Point Christopher, Elizabeth Bay, Tagus Cove, Banks Cove, and Black Bight. The island was first visited for botanical purposes by Macrae (whose name is also written McRae), a Scotch gardener, sent by the London Horticultural Society, on the voyage of the "Blonde," when in 1825 that vessel, under the command of the seventh Lord Byron, conveyed back from England the king and queen of the Sandwich Islands. Macrae remained eight days upon the island, and collected there 41 different kinds of plants. Albemarle has since been visited for botanical purposes by Darwin, Andersson, the Hassler Expedition, Wolf, Lee, Baur, Snodgrass and Heller. The broad southern portion of the island is relatively well watered and possesses a rich and copious vegetation, while the northern parts are described by Darwin as miserably sterile — an account to a great extent confirmed even by those who have visited the island at a more favorable season. In all 205 flowering plants and ferns have been found on Albemarle, and of these, 17 are to our present knowledge confined to this island. Thus the peculiar element (about 8 per cent) is less than that of any of the other large islands. Among the noteworthy plants of Albemarle are a well marked and apparently abundant *Scalesia* (*S. gummifera*) confined to the island, and the problematic *Pleuropetalum Darwinii*, which elsewhere occurs only upon James Island, although close congeners are found in Ecuador and Costa Rica. The different plant families occurring on Albemarle are represented in about the proportion in which they occur in the whole archipelago. Of the species of Albemarle nearly half are common to Charles and Chatham, and about one-third to James, while scarcely more than one-fifth have been found on Indefatigable, although it attains about the same height and lies directly between Albemarle and Chatham.

BARRINGTON ISLAND.

Barrington is a small island about 8 km. long, lying between Indefatigable and Chatham. It rises only 277 m. above sea-level, and possesses only a xerophytic type of vegetation, with no ferns, no sedges, no *Piperaceae*, and only two *Leguminosae.* The island has been visited by Dr. Baur and by Messrs. Snodgrass and Heller, but only 39 plants have been secured there. Of these but one *Scalesia* is confined to the island. The thorny *Discaria*, the *Scalesia*, just mentioned, *Croton Scouleri*, *Cordia lutea*, *C. Hookeriana*, and *Maytenus obovata*, together with a large and abundant *Opuntia*, are its shrubby and arborescent species, the other plants being herbs of a decidedly xerophytic type, grasses and *Euphorbiaceae* predominating as to number. Of its 40 species, 26 occur upon Charles and Chatham Islands, while but 18 have been found on the nearer Indefatigable. *Leptochloa mucronata* of South America, collected on Barrington, is not known to occur on any of the other islands of the archipelago.

BINDLOE ISLAND.

Bindloe lies with Abingdon and Tower to the northward of the other chief islands. It is roughly elliptical and about 15 km. long, rising to a height of only 250 m. It appears to have been visited for botanical purposes only by Dr. Baur, 4–5 September, 1891, and Messrs. Snodgrass and Heller, 29 June, 1899. Only 42 different plants have been collected or recorded there, and only 1, a mere leaf form of *Castela galapageia*, is peculiar to the island, although the continental *Canavallia obtusifolia*, found upon Bindloe, has as yet been collected upon none of the other islands. Messrs. Snodgrass and Heller report a gigantic *Cereus* as rare on Bindloe; only one individual was seen. The *Opuntia* of Bindloe is low, only 3 to 9 dm. in height. More than half the plants of Bindloe occur upon Charles, Chatham, and Albemarle respectively, while the proportion found on Abingdon and Tower is considerably less, and but a single plant of Bindloe has been found on Culpepper and Wenman.

BRATTLE ISLAND.

Brattle is a small islet close to the southeastern shore of Albemarle. It forms, according to Dr. Baur, the remains of a single volcano, the southeast part of which has been nearly destroyed. The island has never been botanically explored. Dr. Baur made repeated efforts to land upon it, but was baffled by the precipitous shores. From his account (2), 236, we learn that the island is green as though covered with vegetation, that many ravines run down from the upper part, and that it is the breeding place for numerous sea-birds. Messrs. Snodgrass and Heller sailed within a few hundred yards of Brattle and report it a low, steep, and sterile rim of a tufa crater, the only vegetation being a scattering growth of *Croton* bushes.

Grossman Islands are similar small rocky islets, on which no plants have been observed.

CHARLES OR FLORIANA ISLAND.

Charles is one of the five larger and higher islands, and with the possible exception of Chatham has been the most fully explored botanically. It has yielded the largest number of plants, namely, 267. Of these 33 are peculiar to it, and 105 to the archipelago. Charles was at one time inhabited by a penal colony from Ecuador, and in its flora shows a greater number of obviously introduced plants than are found on any of the other islands except Chatham. Of the 267 plants found on Charles 126 occur also on Chatham and 100 on Albemarle. According to Baur (2), 239, the appearance of Charles is quite different from that of Chatham, the hills being more rounded. He also states that there are no large forest trees on Charles. The desolate coast of Charles at Black Beach is figured by Agassiz (1), t. 19, 20, and the copious vegetation on the way to the hacienda, t. 21.

CHATHAM ISLAND.

Chatham being the most easterly of the islands, is of course the nearest to the mainland. It is relatively large and fertile, and the only one of the group which is now inhabited. Portions of it are covered by forests of large trees, and in other parts are high arable plains, well shown by Agassiz (1), t. 17. When Baur visited the island in 1891, he found two hundred and ten acres under cultivation. The plantations are owned by Mr. Cobos, to whose courtesy and hospitality the visiting naturalists have been repeatedly indebted. Chatham has been relatively well explored, 231 plants having been found upon it. Of these, 24 are peculiar to it, and 82 exclusively Galapageian. Notwithstanding the habital differences spoken of by Baur (2), 230, the flora of Chatham possesses the largest common element with that of Charles. A giant cactus (*Cereus sclerocarpus?*) with red egg-shaped fruit is mentioned by Baur. This is doubtless the one which appears in Agassiz's Plate 16.

CULPEPPER ISLAND.

Culpepper is a small barren islet, the most northern member of the archipelago. It is remote from all the other islands except the similar islet, Wenman. Culpepper rises precipitously from the sea, and attains an altitude of only 169 m. It has been visited for botanical purposes only by Messrs. Snodgrass and Heller, on December 10th, 1898. They report that the main part of the island is inaccessible. It is a gently rounded plateau falling off abruptly on all sides. Observed from below, it was seen to be covered with a fairly dense growth of vegetation, including *Croton* bushes and *Opuntia* thickets. Only 4 plants were secured, and these were gathered on a short strip of talus at the base of the cliffs, on the leeward side of the island. They were *Telanthera Helleri* (the most abundant species seen), a sterile and indeterminate *Tribulus*, *Croton Scouleri*, var. *brevifolius*, and *Thinogeton Miersii*. With the probable exception of the *Tribulus*, all these are confined to the Galapagos Islands, and the *Telanthera* is a marked new species, which in its typical form has been found only upon Culpepper, although a form of the same plant was also collected on Wenman.

DUNCAN ISLAND.

Duncan is a small low circular island, between Indefatigable and Albemarle. Plants have been collected upon it by Mr. Alexander Agassiz early in April, 1891; Dr. Baur, 2 August, 1891; and Messrs. Snodgrass and Heller, 5 May, 1899. Altogether 51 different species have been secured upon the island; of these, 5 are peculiar to it, namely: *Castela galapageia*, forma *duncanensis*, *Verbena grisea*, *Borreria galapageia*, *Scalesia Baurii*, and its var. *glabrata*. About half the flora of Duncan is exclusively Galapageian. The *Opuntia* on this island is said to be scattered and of large size, growing upon the rim of the crater. *Rhizophora Mangle* occurs in a small swamp upon the west coast. It is another of the unaccountable anomalies in the florulae of these islands that the common element between Duncan and Charles or Chatham is greater than between Duncan and the nearer islands of Albemarle, Indefatigable, and James.

GARDNER ISLAND.

Gardner is a very small steep-shored island about 1 km. in length, and only 8 km. east of Charles Island. It attains an altitude of 240 m. It was visited by Dr. Baur in July, 1891, and by Messrs. Snodgrass and

Heller in May, 1899. Thirty-three plants have been found on it, including 4 grasses, 3 *Euphorbiaceae*, and 3 *Malvaceae*, but no ferns, no *Rubiaceae*, and no *Compositae*. Gardner is the only one of the Galapagos Islands which on botanical exploration has yielded no peculiar plant. Baur (2), 233, speaks of the flora of Gardner as being the same as that of Hood, and doubtless there is great habital similarity, but of the 33 plants known to grow on Gardner, only 22 have been found on Hood, while no less than 31 have been observed on Charles.

HOOD ISLAND.

Hood is a desert island of elliptical form, lying to the southeast of the other islands. It is nearest Chatham, Barrington, Gardner, and Charles. According to Dr. Baur, it is a low tableland with few peaks, the highest only 200 m. in altitude, and thus still wholly within the lower dry strata of the atmosphere. There are no trees of size on Hood, and the ground is strewn with large angular masses of rock. Dr. Baur states that cacti are here less abundant than on the other islands, and that the *Opuntia* is short and thick-stemmed. Hood Island has also been visited for botanical purposes by Dr. Habel, Professor Lee, and by Messrs. Snodgrass and Heller. Altogether 55 different plants are known to occur on the island; of these only two are peculiar to it, namely: *Amaranthus sclerantoides*, forma *hoodensis*, a mere leaf form, and *Acanthospermum lecocarpoides*, a well marked species. Of the plants of Hood, 20, or about 36 per cent, are peculiar Galapageian forms. There are no ferns, 8 grasses, 7 *Leguminosae* (including the Brazilian *Geoffraea superba*, not collected elsewhere in the Galapagos), 5 *Convolvulaceae*, 5 *Boraginaceae*, and 6 *Compositae*. There are no *Rubiaceae* and only 3 *Euphorbiaceae*. About three-fourths of the plants of Hood are also found on the lower arid slopes of Charles and Chatham Islands, while only about one-third of them have been observed on Indefatigable.

INDEFATIGABLE ISLAND.

Indefatigable is one of the larger islands, and is centrally situated in the archipelago. It is broadly elliptical, some 32 km. in diameter, and formed of a single, rather symmetrical volcanic mountain, rising to a considerable height. In the basin of the cone is a large area of well-watered land, which, according to Captain Tanner, possesses considerable natural resources. Indefatigable was first explored botanically by Andersson, who could, however, spend only a few hours upon it. It has been subsequently visited by Dr. Habel, the Hassler Expedition, Dr. Wolf, Dr. Baur, 11 to 13 July, 1891, and Messrs. Snodgrass and Heller. Altogether 76 flowering plants have been found on the island. Of these, 8 are *Compositae*, 7 *Boraginaceae*, 7 grasses, and 5 *Amarantaceae*. Nine species and one form are peculiar to the island. Among these are two species of *Scalesia*, and two of *Borreria*. The vegetation so far as yet shown is of a more xerophytic cast than that of the other large islands, although it is to be suspected that this is due to imperfect exploration. Perhaps the most striking feature is the entire absence of ferns. The genus *Croton*, also, although one of the commonest and most widely distributed in the Galapagos, has not been collected or reported upon Indefatigable. Of the 76 plants known to occur on this island, 51 have been found on Chatham, 48 on Charles, and 46 on Albemarle.

James Island.

James is also one of the larger, higher, and more central islands. It has been visited by most of the expeditions which have explored the archipelago, and 153 plants have been collected upon it. Of these 19 are peculiar to the island, and 56 to the archipelago. The most noteworthy feature is the relative abundance of ferns, which here amount to 13 per cent of the whole vascular vegetation, — a striking contrast to their entire absence on the adjacent Indefatigable. The nearest affinities of the flora of James are with Charles, Albemarle, and Chatham.

Jervis Island.

Jervis Island is less than 3 km. long and of low altitude. It is but 9.6 km. south of James, and together with Duncan lies in a small portion of the ocean to a considerable extent inclosed by James, Albemarle, and Indefatigable islands. It was visited by the "Hassler," but so far as I can learn, only Dr. Baur has collected plants upon this island. He secured 22 different kinds, of which 2, *Euphorbia viminea*, forma *jervensis*, and *Castela galapageia*, forma *jervensis*, are foliar forms of species which in slightly different forms are rather widely distributed on the islands. It is a curious fact that of the 22 plants observed on this island only 9 have been found on the adjacent James Island, although 12 have been collected upon Chatham, and no less than 15 on Charles, both much more distant. Fourteen, or about two-thirds, of the plants of Jervis are confined to the Galapagos Archipelago.

NARBOROUGH ISLAND.

In proportion to its size and interest Narborough has received the least botanical attention of any of the islands. This is to be regretted, as it is not only the most remote from the mainland, but is much shielded by the neighboring long and lunate island Albemarle from any direct drift from the other islands or from the continental coast. Although it is to be expected that Narborough would on this account possess a decidedly peculiar flora, the single collection made there by Messrs. Snodgrass and Heller and including 59 different plants, contains only 4 species confined to the island and only 26 which are confined to the archipelago. Thus the percentage of peculiar and strictly Galapageian plants is less rather than more than on the other large islands. This lack of peculiarity may well be due to recent volcanic activity on Narborough, since this would tend to impoverish the flora except as it was replaced by recent immigration from the other islands.

THE SEYMOUR ISLANDS.

North and South Seymour are two islets lying just north of Indefatigable, from which they are separated by channels only 1 or 2 km. broad. They are of relatively low altitude. South Seymour is somewhat larger than North Seymour and lies directly between it and Indefatigable. The Seymour Islands have been visited for botanical purposes only by Messrs. Snodgrass and Heller, who collected upon the north island 18 plants and on the south 34. As upon the adjacent Indefatigable no ferns have been found on the Seymour Islands. On South Seymour there are 2 endemic species not known to occur upon the other islands, namely, *Bursera malacophylla* and *Euphorbia nesiotica,* while North Seymour has an as yet indeterminate plant (*Nyctaginacea?*) which may well prove peculiar to it. It is noteworthy that less than half the plants of the Seymour Islands have as yet been found upon Indefatigable, near as it is; indeed the common element is considerably greater with the much more distant islands of Charles, Chatham, and Albemarle. Halophytes form a noteworthy part of the vegetation of the Seymour Islands.

TOWER ISLAND.

Tower is a small triangular island some 3 km. in breadth. It lies at the northeast of the main archipelago and is nearest Bindloe, which is about 50 km. away. Tower rises only 65 m. above sea level. It has been visited by Dr. Baur, 2 September, 1891, and by Messrs. Snodgrass and Heller, June, 1899, who have together secured only 19 different kinds of plants upon it. Of these, 5 are *Euphorbiaceae,* and 3, *Convolvulaceae.* There are no ferns, and what is more remarkable, no *Amarantaceae, Leguminosae, Rubiaceae,* or *Compositae,* all of which are families exceptionally well represented upon most of the other islands. The only plant peculiar to the island is *Euphorbia viminea,* forma *castellana,* merely a well marked leaf-form of this polymorphous species. It is worthy of note, however, that in the size, shape, and thickness of its leaves, this form on Tower represents the opposite extreme from the variety of the same species found on the adjacent island of Abingdon. Eleven of the 19 plants observed on Tower are confined to the Galapagos Islands.

WENMAN ISLAND.

Wenman is a rocky islet, which, like Culpepper (43 km. distant), lies considerably to the northward of the other islands. For botanical purposes it has been visited only by Messrs. Snodgrass and Heller, who remained there from the 12th to the 18th of December, 1898. Mr. Heller's field notes, so far as they relate to plants, are as follows: "The main part of the island is largely inaccessible, but, like Culpepper, covered with a fair growth of vegetation. Through a glass the larger plants appeared to be *Croton, Opuntia,* and on the north side a cluster of leafless trees, each about a foot in diameter and twenty to thirty feet high, with a smooth brownish bark. In an inaccessible cave a *Polypodium* was seen. Our collecting was confined chiefly to a low detached islet north of the main island. Its vegetation consisted of a low *Opuntia* [*Helleri*], which grew in thick beds near the edges of the cliffs. Many of these were covered with greenish-yellow flowers and others with dry and prickly fruit. The central part of the islet was covered by a heavy growth of *Croton* [*Scouleri,* var. *brevifolius*] bushes. These were largely leafless, although a few were in flower. In habit they resembled saplings, with straight trunks ten feet high and an inch or two thick. An *Ipomoea* [*Kinbergi*] was common, twining among the *Croton* bushes. This had large white flowers. A *Telanthera* [*Helleri,* var. *obtusior*] was also common about the cliffs and near the *Opuntia* thickets. Dried stalks of a *Cyperus* were noted. On a talus slope of the main island the following species were collected: *Scalesia* [*Snodgrassii*], bushes two or three feet high, with a thick head of leaves and flowers; *Cardiospermum* [*Corindum*] a few vines, also an *Atriplex* [undetermined] fairly common." Altogether eleven species were collected on Wenman and the detached islet. Of these three are new and peculiar, three Galapageian occurring also upon other members of the group, two common to the mainland, and three not fully determined.

TABLE III. — SUMMARY OF FLORULAE.

(Relating exclusively to the Pteridophytes and Spermatophytes.)

Island.	Species, varieties, and forms on island.	Peculiar to island.	Per cent peculiar to island.	Peaculir to archipelago.	Per cent peculiar to archipelago.	Common to other regions.	Per cent common to other regions.
Abingdon . .	50	4	8	26	52	24	48
Albemarle . .	205	17	8	84	41	120	59
Barrington . .	40	1	2.5	19	48	21	52
Bindloe	42	1	2.4	19	45	23	55
Charles	267	33	12	105	39	162	61
Chatham . . .	231	24	10	82	42	149	58
Culpepper . .	5	1	20	3	60	2	40
Duncan	51	5	10	25	49	26	51
Gardner . . .	33	0	0	15	45	19	55
Hood	59	2	3.4	20	34	35	66
Indefatigable.	76	10	13	35	47	41	53
James	153	19	12	56	37	97	63
Jervis	22	2	9	16	73	6	27
Narborough .	59	4	7	26	44	33	56
Seymour . . .	47	2	4	16	34	29	66
Tower	19	1	5	11	58	9	42
Wenman . . .	11	2	18	5	45	6	55

TABLE IV. — PTERIDOPHYTES AND SPERMATOPHYTES COMMON TO THE DIFFERENT ISLANDS.

Island.	Abingdon.	Albemarle.	Barrington.	Bindloe.	Charles.	Chatham.	Culpepper.	Duncan.	Gardner.	Hood.	Indefatigable.	James.	Jervis.	Narborough.	Seymour.	Tower.	Wenman.
Abingdon . .	50	34	11	15	36	31	0	10	13	14	18	26	9	14	7	11	1
Albemarle . .	. .	205	23	24	100	93	2	29	17	33	46	74	13	42	25	7	2
Barrington . .	. .	. .	40	11	26	26	2	13	14	20	18	19	5	15	16	10	0
Bindloe. . . .	. .	. .	. .	42	25	22	1	7	8	13	15	14	5	15	8	9	1
Charles	. .	. .	. .	. .	267	126	2	35	31	40	48	78	19	38	30	13	2
Chatham . . .	. .	. .	. .	. .	. .	231	2	34	22	42	51	64	12	33	31	15	4
Culpepper . .	. .	. .	. .	. .	. .	. .	5	2	1	2	0	0	0	2	2	1	1
Duncan. . . .	. .	. .	. .	. .	. .	. .	. .	51	17	26	20	21	8	9	16	7	1
Gardner . . .	. .	. .	. .	. .	. .	. .	. .	. .	33	22	17	15	6	7	13	9	1
Hood	. .	. .	. .	. .	. .	. .	. .	. .	. .	59	20	29	8	15	20	10	0
Indefatigable	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	76	33	12	14	23	9	2
James.	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	153	10	27	19	10	1
Jervis	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	22	1	7	3	1
Narborough .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	59	8	7	0
Seymour . . .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	47	6	2
Tower	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	19	2
Wenman . . .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	11

BOTANICAL EVIDENCE REGARDING THE ORIGIN OF THE GALAPAGOS ISLANDS.

Two views have been advanced regarding the origin of the Galapagos Islands. According to the first they are pelagic islands, built up from the sea-floor by volcanic action, while according to the second view they are continental islands, tops of mountains, formerly a part of the mainland, now separated from it by subsidence. The first view, namely, that the Galapagos are islands of elevation, was held until about 1890 with scarcely any question. It is strongly supported by the following facts: (1) All parts of the islands now visible are obviously of volcanic origin. (2) The islands are separated from the mainland by a very considerable depth of ocean (more than 1500 fathoms). (3) The western coast of South America shows no signs of subsidence, but rather of marked elevation in recent geologic time.

In accordance with this theory of emergence, the flora of the Galapagos Islands is assumed to have been brought to them by the ordinary agents of plant-distribution, namely, the wind, oceanic currents, and migratory birds. Moreover, it is not difficult to see the probable efficiency of these means of seed-transportation in the present case. The islands lie in the course of pretty constant trade winds, doubtless capable of bearing spores and small seeds to a great distance, as winds have been known to carry fine particles of sand and pumice for hundreds of miles. The great ocean current which sweeps along the west coast of Mexico, and the Humboldt Current, which runs northward along the coast of Chili and Peru, both turn westward just in the equatorial belt where the islands lie. That the current between Central America and the Galapagos has considerable seed-carrying power, seems more than probable from the interesting observations of Mr. Alexander Agassiz (1), 59, 69, who while dredging that part of the Pacific found the bottom "strewn thickly with vegetable matter, which came up in great masses in almost every haul of the trawl." He states also that: "The velocity of the currents in the Panama district is very great, sometimes as much as seventy-five miles a day, so that seeds, fruits, masses of vegetation harboring small reptiles, or even large ones, as well as other terrestrial animals, need not be afloat long before they might safely be landed on the shores of the Galapagos." It may here be noted that Hooker (4), 256, has shown that a large proportion of the plants of the Galapagos Islands are provided with special means of seed-dispersal, — a fact of interest in this connection.

But, however probable the pelagic origin of the islands seems from the data above presented, Dr. Baur has pointed out what has appeared to be a fatal defect in this theory. For, although it may be quite possible to explain the presence of plants and some animals upon islands of emergence, it is much more difficult if not impossible to explain upon the same theory the extraordinary biological relations between these islands themselves. An examination of the flora shows that many plants on the different islands are nearly related to each other, without being exactly the same. For instance, each of several islands has a peculiar species of *Scalesia*, a genus confined to the archipelago. There are also on these islands several different forms of *Euphorbia viminea*, several very nearly related species of *Acalypha*, many habitally uniform species of *Borreria*, and of many other genera. In many instances these forms are confined to a single island, and in the majority of cases each form is more closely related to those of the other islands than to any continental ally. The same is true of the fauna, for the animals of particular groups, while exhibiting near affinity, show slight differences, which can in many cases be correlated with their occurrence upon different islands. Thus the flora and fauna of the islands are, to use Dr. Baur's expression, harmonic.

From this peculiar distribution, which seemed to him wholly inexplicable upon the emergence theory, Dr. Baur advanced the view that the Galapagos were islands of subsidence, once attached to the continent by some isthmian connection extending presumably to the Central American coast. In an enthusiastic presentation of this theory, Dr. Baur certainly advanced in support of it some as yet unanswered biological arguments. Thus, it explains perfectly the harmonic relations of the living forms upon the different islands; for if the islands were once united and then by subsidence separated, the remnants of their common flora and fauna, persisting upon the different islands, would have diverged not only from the continental types, but from each other. From the depth of ocean between the archipelago and the mainland, it would naturally be inferred that the islands were cut off from the continent before they were divided from each other. This, on the supposition of a gradual variation, would account not only for the divergence, for instance, of the genus *Scalesia* from its Mexican allies, but for the minor differences which are found between the different species of *Scalesia* on the several islands, where they occur; while, as we have seen, this harmonic distribution of very similar yet slightly differing forms on the different islands has appeared unaccountable, on the theory that these are islands of emergence casually seeded. For, to quote from a discussion of the forms of *Euphorbia viminea*:[1] "The question at once presents itself, if this archipelago is composed of islands of elevation, built up from the sea-floor independently by volcanic action, how has such a distribution been effected. If the vegetation has been derived from the mainland by the chance transportation of seeds, it is quite impossible to believe that each island has received a slightly different form of the same species, and we are forced to the much more natural assumption that racial and varietal divergence has come about after the introduction of the species upon the islands. Now, continuing the supposition that these are islands of elevation, the seeds of *Euphorbia viminea* must have reached them in one of two ways: either each of the nine islands, where we know the species now to occur, must have received its seed directly from the mainland, or, what is much more natural, seed must have reached one or more of the islands and from these spread to the rest. That the same species should have reached all these islands presupposes a considerable facility of transportation. But as soon as this is granted it is impossible to understand the highly individual development of the forms upon the different islands. For relative or complete isolation seems necessary to account for the racially divergent floras of the islands; and especially for the occurrence of only one form upon each island. It would thus appear necessary, in accounting for the present distribution, to assume that at one time in the remote past, the islands were either united, or at least that the channels which separate them were less formidable barriers to seed-transportation than at present, so that a general distribution of species could have been effected; and that subsequently, as the islands separated, or as the channels through some change of currents, or other cause, became less easily passed, an era of *much* greater isolation of the floras of the different islands came about. The divergence of character of the vegetation would then begin at once, and the otherwise unaccountable existence of a single and peculiar form upon each island would be readily intelligible. While not prepared to make any positive assertion regarding the probable origin of the islands, the authors fail to see in the hitherto generally accepted theory of elevation any satisfactory explanation for the harmonic yet divergent floras of the different members of the group."

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[1] Rob. & Greenm. (1), 136.

Finally, for the subsidence theory, it must be admitted that the direct geological arguments for the elevation of the islands are not so forcible as they at first appear. Thus, as Baur has pointed out, the fact that all parts of the islands now visible are volcanic proves little; for if the Andes were sunk until only equivalent land areas remained, they too would appear wholly volcanic; and as to the recent elevation of the South American coast, that, as I am informed by Professor W. M. Davis, is no conclusive proof that areas five hundred miles to the seaward have suffered like elevation or, indeed, that they have not been simultaneously subjected to a sort of counter-balancing subsidence.

Such, in brief, have been the arguments advanced on both sides regarding the origin of the Galapagos Islands. During a re-examination of the whole vascular flora of the islands, I have sought further light upon this question, and now find the peculiar distribution of the plants less difficult to account for on the emergence theory than it seemed when the Baur plants were studied some years ago.

Let us consider theoretically what would be likely to happen to a plant casually introduced in a pelagic archipelago, where no plant of its particular affinity had previously found its way. Let us suppose, for instance, that the ancestral form of *Euphorbia viminea* reached Chatham Island from the continent, and that conditions of seed-transportation were such that subsequent seedings from the mainland would not be likely to happen oftener on the average than once in thirty, fifty, or perhaps one hundred years, — no unreasonable assumption. Now, it is known from observation that a plant introduced into a new region can overrun considerable territory and increase to thousands of individuals, even in a shorter space of time. It is further likely that a plant established upon such an island would be at once exposed to modifying influences and tend toward the formation of a new race particularly suited to its altered environment. Whether this were effected by direct influence or by natural selection is not significant in this case. In the interval between the first and second seeding the change would probably be very slight and taxonomically imperceptible, but that some modification would have taken place seems likely. It is clear that the second and subsequent seedings of the same island by the same species from the continent would tend by the infusion of pure stock to reclaim the incipient insular variation to the typical continental form of the species. But to see how great or rather how slight the influence of these later seedings would be, it is only necessary to consider the numerical relation of both forms. The descendants of the first immigrant might well have increased to many thousands of (slightly altered) individuals before the second seed arrived from the mainland. Thus the reclaiming influence of this second immigrant would not be as one to one but as one against thousands, that is to say, virtually nil. Of course, it might be thought that as the first seed increased in a few years to thousands of individuals, the second might do so likewise, so that their influence would after all be not very unequal. This, however, could scarcely happen for two reasons. In the first place the original seed would have found in the island (before uninhabited by any near relative) a fresh terrain, giving it an opportunity to multiply rapidly. The second immigrant, however, would find its proper habitat on the island no longer free for settlement, but largely if not entirely occupied by countless individuals of a very nearly related stock. It could scarcely fail to cross repeatedly with the insular form and quickly merge into it, the more so because the descendants of the second seed would at once be subjected to the same modifying influences that had affected those of the first.

It is thus clear that the insular form, having once started its divergence from the continental type, would be likely to differentiate itself more and more and not be much affected by the occasional arrival of scattered and isolated seeds from the continent. Furthermore, if seeds of the Chatham form were by any chance carried to other islands of the archipelago, further specialized races would be almost certain to arise bearing much the same relation to that of Chatham that it bore to the continental form, and quite as little affected by subsequent seedings. Thus unchecked, the races would have every opportunity to develop into more and more highly differentiated forms, varieties, and ultimately into well-marked species characteristic of particular islands.

This may all seem purely hypothetical, but it will be seen that the conclusions rest upon only two very natural premises, namely, that seed-transference between the mainland and the islands or between the islands themselves, does not occur in the case of particular plants oftener, on the average, than once in several years, and in the second place that plants have multiplied on the islands as rapidly as they have frequently been observed to multiply elsewhere. When these two not unreasonable postulates are admitted, it is clearly no harder to account for the existence of a "harmonic" flora on islands of emergence than of subsidence. Indeed, it is what is to be expected in an archipelago of either kind where seed-transference is relatively rare. That this condition obtains in the case of the Galapagos is clearly shown by the existing diversity in the floras of the different islands, — a condition which could not continue if seed-transference were very common between the islands. The fact that it is not more frequent is perhaps sufficiently explained by the arid and sterile shores, which would certainly offer to most seeds washed thither by oceanic currents an exceedingly poor chance of surviving.

The great existing difference between the Galapageian flora as a whole and that of tropical America is doubtless due not only to the differentiation of the insular forms, but also and perhaps in a considerable degree to changes which have been simultaneously going on in the continental flora itself. Thus the ancestral forms of many Galapageian plants, for instance of the above mentioned *Euphorbia viminea* (which might reasonably be sought in the deserts of Peru), have probably failed to persist at all upon the mainland. The fact that the florulae of the several islands are on the whole much more like each other than any one of them is like any part of the continental flora is perhaps sufficiently explained by the relative nearness of the islands to each other, but it may well be due in part to the circumstance that these insular florulae have in common been protected from much of the change which has, through a sharper competition in the larger flora of the mainland, been forced upon the vegetation of the adjacent parts of the continent. In this matter again insular floras would fare alike, whether the islands they inhabited were the result of emergence or subsidence.

During the first years, or probably centuries, in the history of islands of emergence their floras, made up as they must be of heterogeneous elements which chance has brought to their shores, would be very unlike the far more homogeneous floras persisting on islands of subsidence; but after islands of each kind had attained considerable age, — that is an age sufficient to have given rise to a flora as specialized as that which now exists on the Galapagos, it is not likely that their floras would show any marked distinction, for, as we have seen, the development would not be unlike in the two cases. Although this negative result is in a way disappointing, it is certainly much better to admit a nullity of botanical evidence in regard to this interesting question than to attribute to the "harmonic" flora of these islands an historic meaning which it may not possess.

There is one point, however, from which botanical evidence can be derived which has a certain bearing upon this matter, namely, the relation which the proximity of the different islands bears to likeness in their florulae. The islands are so different in altitude, climate, and consequent fertility, that diversity in their vegetation is by no means surprising; but the difference certainly reaches a higher degree than we should expect. Thus, the common element between any two of these islands rarely exceeds 75 per cent and is often less than 50 per cent, sinking in some cases to nothing. Of course it is not unlikely that these differences may in some instances be more apparent than real owing to imperfect exploration, and it is probable that further collecting will show at least a small common element between each two of the islands. However, the differences in the recorded floras of the larger repeatedly visited islands cannot be due to our ignorance. Indeed, each new exploration brings quite as much material to demonstrate their diversity as their likeness. The most noteworthy feature of these differences is not, however, their extent, but rather the fact that for the most part they stand in no relation to the distance of the islands from each other or to the depth of the intervening channels. Thus the florulae of Albemarle and Chatham at opposite sides of the archipelago are more alike than either is to that of the intervening Indefatigable; Jervis lying near James has a greater common floral element with the more distant Charles; the florulae of the Seymour Islands have a greater number of plants in common with Charles, Chatham, and Albemarle than with Indefatigable, of which the Seymour Islands are merely a detached spur. Although a high percentage of ferns has been recorded on James, not a single representative of this group has thus far been found on the adjacent Indefatigable. The common floral element between Duncan and the relatively remote Chatham is greater than with any of the three large islands, James, Indefatigable, and Albemarle, which to a considerable extent surround it. In fact, the only cases in which it appears that proximity between two islands has brought about any marked similarity on their floras are on the one hand Narborough and Albemarle, and on the other Gardner and Charles, and even in the former of these pairs, the likeness is by no means close, for not over 71 per cent of the plants of Narborough have been observed on Albemarle.

These anomalies in the different florulae must find their explanation in peculiarities of climate and soil, together with an element of chance — arising partly from imperfect exploration, and partly from the accidents of seed-dispersal. Although they are not fully explained by the theory that these are islands of emergence casually seeded, they are much less in accord with the Baur theory of subsidence; for, were the florulae remnants of a common flora persisting upon islands separated by gradual subsidence, it is evident that those islands would possess the most floral similarity which were nearest together and divided by the shallowest channels, since these would have been separated from each other more recently than the remoter islands, which are cut off by a greater depth of ocean. As we find no such relation prevailing in the Galapagos Islands, but have observed just the reverse, namely, that the more distant islands, separated by relatively deep channels, often show greater floral similarity than the nearer ones, it is necessary to conclude that the botanical evidence, so far as it has yet been made out, is opposed rather than favorable to the subsidence theory.

ADDENDA.

Plants collected on the Galapagos Islands by Dr. Franz Steindachner of the Hassler Expedition.

After the greater part of the present paper was in type, I received through the kindness of Dr. A. Zahlbruchner and Dr. A. Heimerl of

Vienna, a manuscript list of the specimens of plants collected on the Galapagos Islands in 1872 by Dr. Steindachner and now deposited in the Naturhistorisches Hofmuseum in Vienna. These plants were gathered rather as souvenirs of a peculiar flora than as scientific specimens, and many are fragmentary. The list at hand includes the pteridophytes (nos. 21 and 22) and spermatophytes (nos. 23 to 116). The first twenty numbers seem to have been applied to thallophytes and bryophytes, but these are not listed. The only plants mentioned which are not included in the above flora are *Tribulus terrestris*, L., *Crotalaria incana*, L., and *Rhynchosia Senna*, Gill. I have little doubt that the first of these is what Andersson has called *T. sericeus*, a plant which is too near to *T. terrestris* for very satisfactory distinction. Dr. Zahlbruchner writes that the labels furnish no data as to the particular islands upon which the plants were collected. The only new species among Dr. Steindachner's plants was *Tillandsia insularis*, Mez.

Plants collected by Messrs. Snodgrass and Heller on Cocos Islana of the Pacific.[1]

Leptogium azureum (Ach.) Mont.[2]
Macrolejeunea subsimplex (Mont.) Schiffn.[3]
Plagiochila Anderssonii, Angstr.[3]
Platylejeunea pogonoptera (Spruce) Evans (Lejeunea [Platy-Lej.] pogonoptera, Spruce).[3]
Pilotrichum asperifolium, Mitt.[2]
Rhizogonium spiniforme (L.) Br.[2]
Acrostichum caudatum, Hook. (no. 959).
Adiantum intermedium, Sw. (no. 967).
Alsophila armata, Presl (no. 964).
Asplenium rhizophyllum, Kunze (no. 954).
Dicksonia cicutaria, Sw. (no. 952).
Gymnogramme calomelanos, Kaulf. (no. 970).
Nephrolepis acuta, Presl (no. 953).
Polypodium aureum, L. (no. 969).

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[1] With the collaboration of Professor H. Pittier, the author is preparing a catalogue to bring together the now scattered references to the flora of Cocos Island, and unite with them the data derived from recent and very interesting collections. In this subsequent paper the species here listed will be treated in greater detail.

[2] Determined by W. G. Farlow.

[3] Determined by A. W. Evans.

Polypodium chnoodes, Spreng. (no. 948).
" lanceolatum, L. (no. 958).
" Phyllitidis, L. (no. 965).
Trichomanes Prieurii, Kunze (no. 955).
" pyxidiferum, L. (no. 956).
" radicans, Sw. (no. 973).
Selaginella stenophylla, A. Br. (no. 972).
Lycopodium mollicomum, Mart. (no. 971).

Chloris **paniculata**, Scribner, nov. sp., "perennans dense caespitosa 3–5 dm. alta foliis longis angustis, inflorescentia dense paniculata 8–10 cm. longa; vaginis carinatis compressis laxis; laminis foliorum linearibus 3–4 mm. latis longe attenuatis serrulatis praeter marginis scabrae glaber, foliis inferioribus numerosis in specimine inspecta culmo aequantibus vel etiam superantibus; paniculae simplicis ramis numerosis congestis adscendentibus circa 2 cm. longis superioribus brevioribus, axe principali pubescenti rachide secundario valde scabro spiculas unilaterales valde imbricatas biseriales gerente; spiculis 2.5–3 mm. longis, glumis sterilibus lanceolatis acutis translucentibus uninervatis carina scabra, prima circa 1.5 mm. secunda circa 2.5 mm. longa, gluma fertili oblongo-lanceolata truncata 2–2.3 mm. longa in margine carinaque scabra, nervo medio in setam scabridam 1–1.5 mm. longam producto; callo dense barbato, capillis longissimis circa 1 mm. aequantibus: flosculo sterili 1 mm. longo vel breviori setam suae longitudinis gerente." — NUEZ ISL. (an islet near Cocos Isl.): abundant in bunches 3–5 dm. in diameter, covering considerable areas on cliffs near the coast, *Snodgrass & Heller*, no. 968 (hb. Gr., hb. U. S. Dept. Agric.). Endemic. "This grass is somewhat anomalous, having the infloresence of *Leptochloa*, suggesting that of *L. scabra*, while the spikelets are those of *Chloris*, to which it has seemed best to refer it."

Paspalum conjugatum, Berg. (no. 943).
" platycaule, Poir. (no. 942).
Calyptrocarya Palmetto, Nees (no. 947).
Cyperus sphacelatus, Rottb. (no. 946).
Hypolytrum sp. (no. 945).

Kyllinga **nudiceps**, C. B. Clarke, nov. sp., "culmis caespitosis, filiformibus; foliis nullis; spica 1, parva, globosa, pallida; bracteis spica brevioribus —

Rhizoma visum vix 5 mm. longum. Culmi 2–3 dm. longi. Vaginae pallide bruneae, glandulis rubris laete inspersae, apice in altero latere productae, non virides. Spica 5 mm. in diam., multispiculosa; bractea ima interdum 4 mm. longa, apice lineari viridi, saepius inconspicua. Spicula 1-flora (omnino ut *K. odoratae*, Vahl), supra 2 glumas imas vacuas caduca. Nux obovoidea, lateraliter compressa, brunea; styli basis non dilatata, rami 2 lineares." — COCOS ISL.: *Snodgrass & Heller*, no. 944 (hb. Gr.), type; also on shady cliffs, Wafer Bay, *Pittier*, no. 16272 (hb. Gr.)

Catopsis aloides, Baker (no. 962).
Peperomia nigro-punctata, Miq. (no. 957).
Hibiscus tiliaceus, L. (no. 961).
Conostegia lasiopoda, Benth. (no. 963).
Ossea macrophylla, Cogn. (no. 951).
Jussieua linifolia, Vahl (no. 950).
Ardisia cuspidata, Benth. (no. 949).
Rolandra argentea, Rottb. (no. 960).
Wedelia paludosa, DC. (no. 966).

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(Roman numbers indicate pages where the respective genera and families receive their principal treatment; italic numbers show the pages upon which the names are merely mentioned or occur as synonyms.)

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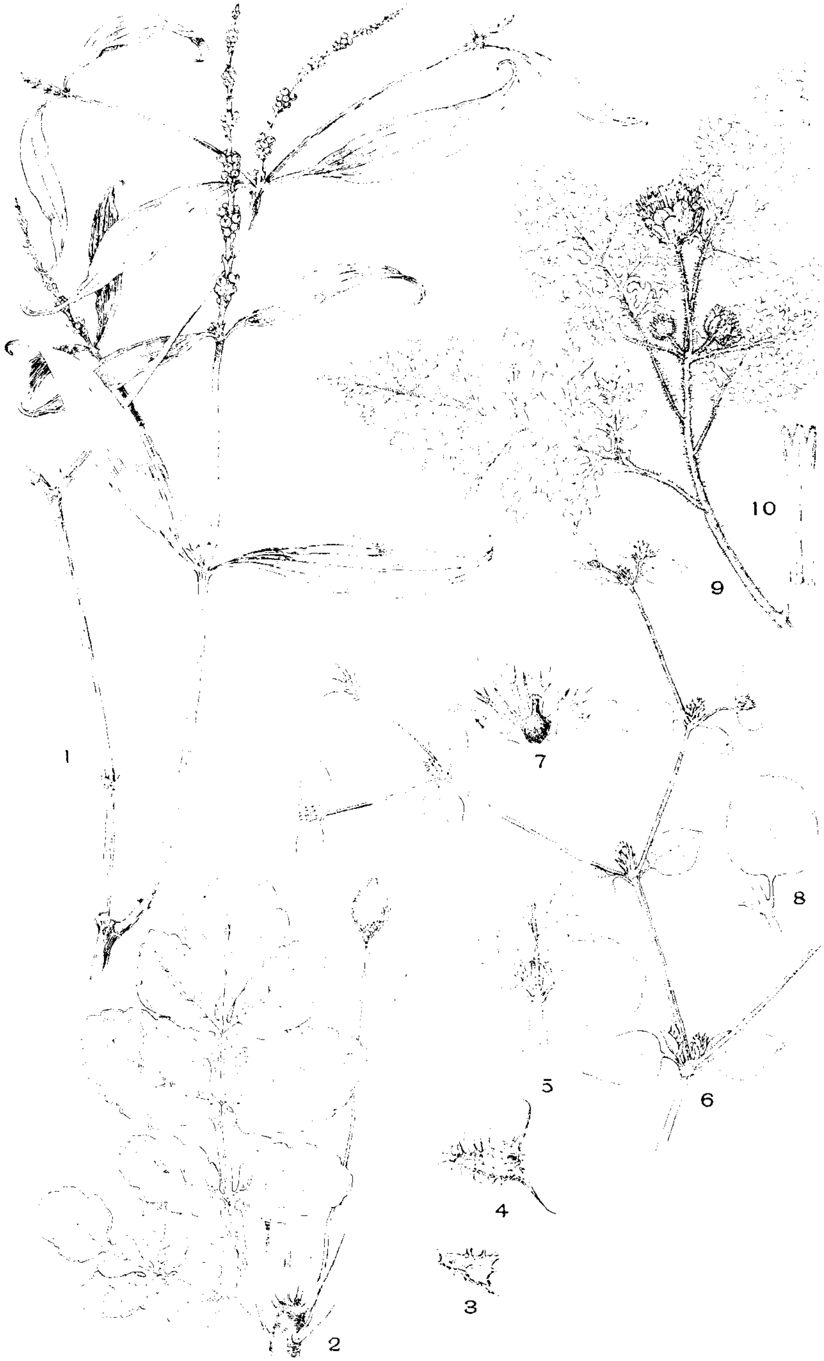
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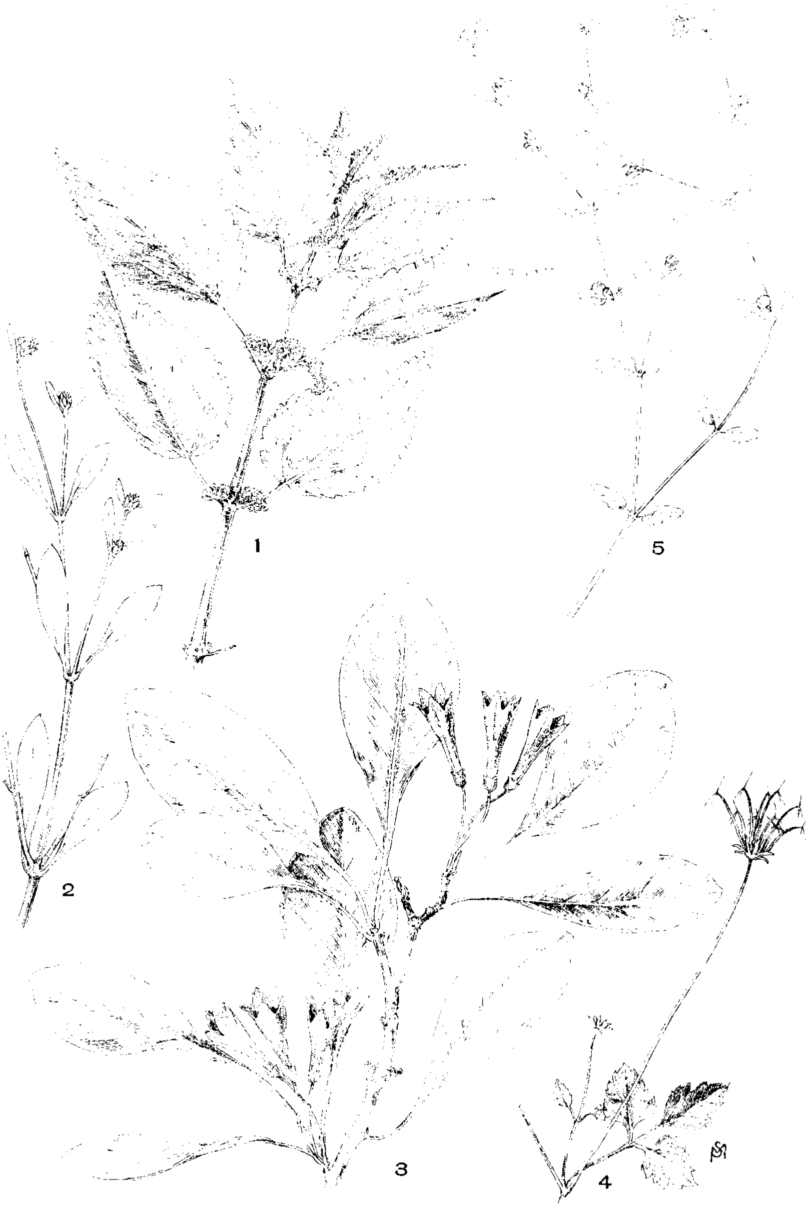

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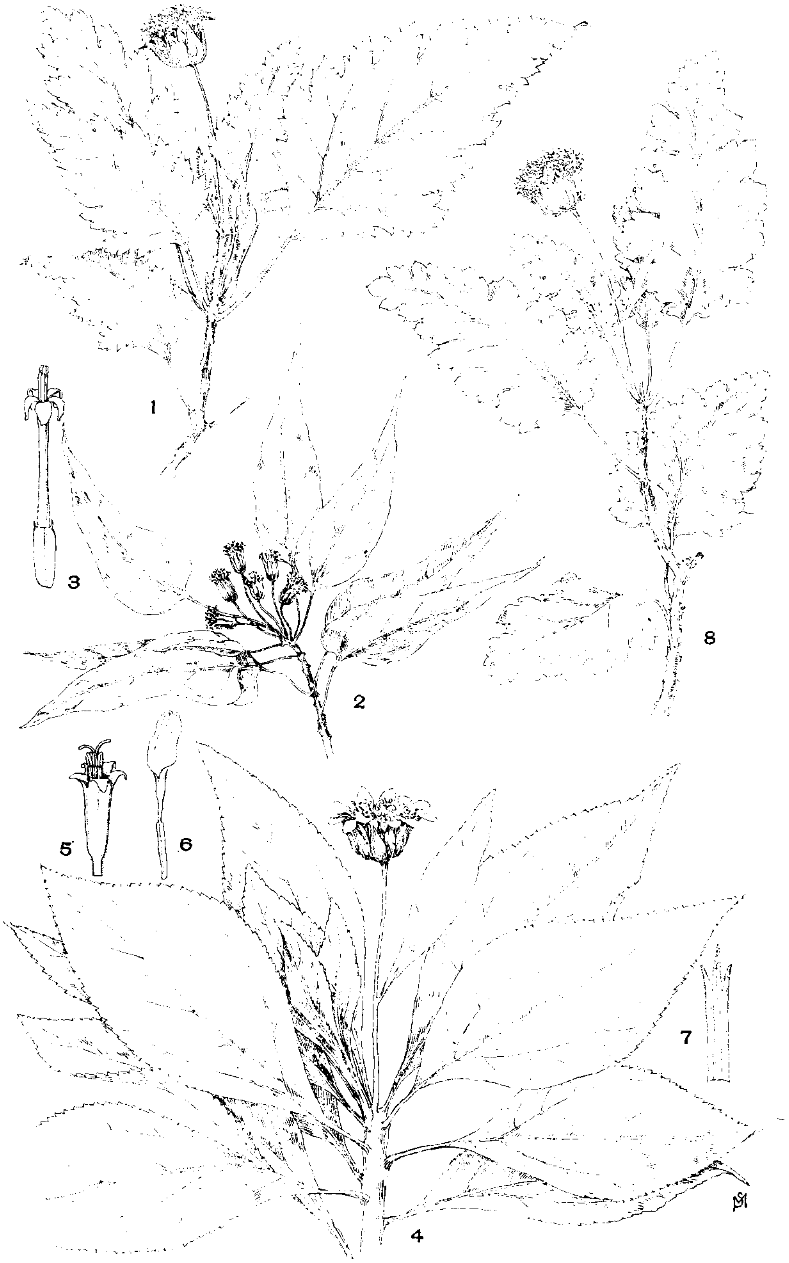
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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

CONCERNING GAS-ANALYSIS BY MEASUREMENT IN CONSTANT VOLUME UNDER CHANGING PRESSURE.

By Theodore William Richards.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

CONCERNING GAS-ANALYSIS BY MEASUREMENT IN CONSTANT VOLUME UNDER CHANGING PRESSURE.

BY THEODORE WILLIAM RICHARDS.

Received June 2, 1902.

IN Hempel's admirable book upon gas-analysis is described a method of exact analysis which has not received the general attention that it deserves. Instead of measuring the changing volume under constant pressure, he suggested measuring changing pressure in constant volume.*

Possibly one reason for the neglect under which this excellent method has suffered is the demand made by it for the use of a large quantity of mercury. One object of this paper is therefore to point out that the same principle may be applied conveniently in a simpler manner.

In Hempel's apparatus the gas to be measured is confined in a bulb immersed in a mercury-trough, and the absorption is effected in an Ettling-Doyère gas-pipette. It is more convenient to measure the gas in a bulb provided with a capillary outlet at the top, and to conduct the absorption in an ordinary Hempel pipette of small size, or in one of the simpler forms of pipette recently used in this laboratory.† This manipulation of course introduces the necessity of using rubber connections, which Hempel desired to eliminate; but if they are properly connected ‡ and securely wired, their introduction is not usually a serious defect.

If not more than 25 per cent of the gas is to be absorbed in any one process, a water column of 2.5 meters, easily obtainable in the laboratory, is enough to furnish the necessary change of pressure; hence water may be used as the liquid in the measuring vessel instead of mercury, provided of course that none of the components in the gas to be analyzed is soluble in water. As will be shown, the use of water greatly simplifies the measurement of pressure, for an appreciable error in the height of the column causes no appreciable effect on the result. Hence no cathetometer is needed, and the simplest apparatus suffices.

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* Hempel's Gas-analysis, translated by Dennis, p. 76 (1902), p. 44 (1892) (Macmillan).

† Richards, These Proceedings, **37**, 273 (1901). Zeitschr. anorg. Chem. **29**, 359 (1902).

‡ Richards, *ibid*, pages 275 and 362, respectively.

Although a marked tube (such as an inverted burette) may be used for the attainment of constant volume, the measuring apparatus consists preferably of a bulb of about 50 c.c. capacity with a single mark upon its stem.* This bulb should have a capillary tube above with an internal diameter not far from 1 mm.,† while below it is provided with a straight tube about 20 cm. long and 7 mm. in diameter, with a sharp line etched just below the bulb. To this tube is attached a levelling bulb by means of a rubber tube about 150 cm. long.‡ The levelling bulb is conveniently hung from a bent stick, which may be clamped to the iron ring-stand, either above or below the table-top. Of course the bulb should be enclosed in a water-jacket, conveniently made from an inverted broken bottle, in order to insure constancy of temperature. The arrangement is represented in the Figure.

In case the full absorption of 25 per cent is expected, it is convenient, although not necessary, to begin with an excess of pressure. So much gas is put into the apparatus for analysis that a pressure of about a meter of water (measured by means of a wooden meter-rule) is necessary to compress the gas exactly to the mark on the stem. About 10 per cent (5 c.c.) more gas than the amount needed to fill the space under atmospheric pressure may be introduced for this purpose. The height of this water-column is reduced to the mercury standard at 0° by multiplying it by about $\frac{1}{13.62}$; § and the reduced value is added to the barometric

---

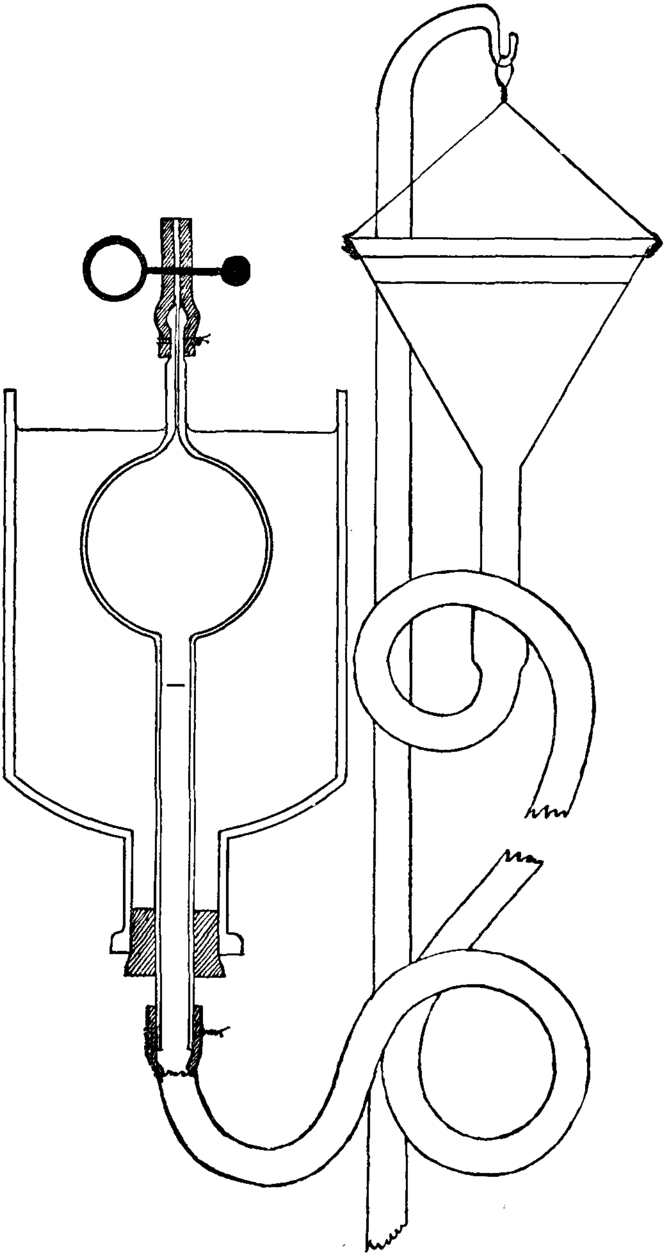
* Mr. W. N. Stull has suggested the use of a tube with several connected bulbs with as many marks in order to attain a wider range with less change of pressure. This apparatus will soon be tested in this Laboratory.

† The rubber tube attached to the capillary should be very thick-walled and have an exceedingly fine bore. It should be wired to the glass in the manner shown in the diagram.

‡ In order to prevent the admission of small air-bubbles into the measuring bulb from the long rubber tube, a trap may be formed by bending the lower glass tube through 100°, or else by making a loop in the rubber tube in the fashion shown in the diagram. This precaution is very rarely necessary, however; it is much better to be certain in the first place that no such air-bubbles are present.

§ The divisors corresponding to various temperatures of water column are as follows: —

10° Divisor = 13.60
15° " = 13.61
20° Divisor = 13.62
25° " = 13.63



height. After the absorption of one of the components a lower pressure is needed to effect the same adjustment. The loss of pressure gives the means of computing the percentage composition of the gas. Of course the tension of aqueous vapor should be subtracted from the *total* readings since the gas is moist; this important correction seems to have been overlooked by Professors Hempel and Dennis in the second English edition, although correctly stated in the first.* The tension of aqueous vapor does not affect the *change* of pressure unless there is a change of temperature.

The following figures, of two trial analyses of air kindly made by Mr. Edward Mallinckrodt, Jr., serve to illustrate the method. The minus sign before the second reading signifies that the counterpoise bulb was below the other, almost as far as the level of the floor.

	I.	II.
Difference of water level at first	+100 0 cm.	+100.0 cm.
Difference of water level after absorption of oxygen	−128.0 cm.	−126.5 cm.
Mercury equivalent to first difference	+ 73.3 mm.	+ 73.3 mm.
Mercury equivalent to second difference	− 93.8 mm.	− 92.7 mm.
Total loss of pressure	167.1 mm.	166 0 mm.
Barometer (corrected to 0°)	752.2	752.2
Initial pressure of moist gas = 752.2 + 73.3 mm. =	825.5	825 5
Tension of aqueous vapor ($t^\circ = 27.6$)	27.4	27.4
Initial pressure of dry gas = ...	798.1	798.1
Percentage of oxygen = $\frac{\text{loss of pressure}}{\text{initial dry pressure}}$ = . .	20.94	20.80

At another time, Mr. W. N. Stull kindly made another test of the apparatus, finding the following percentages of oxygen in the air of the laboratory: —

$$\frac{100 \times 170.2 \text{ mm.}}{818.0 \text{ mm.}} = 20.80\,; \qquad \frac{100 \times 173.5 \text{ mm.}}{831.8 \text{ mm.}} = 20{\cdot}86\,;$$

---

* See page 77 in edition of 1902, and page 69 in edition of 1892 (Macmillan).

$$\frac{100 \times 168.9 \text{ mm.}}{809.7 \text{ mm.}} = 20.86.$$

The mean of all these analyses indicates 20.85 per cent by volume of oxygen in the air of the laboratory, a value which is probably very near the truth.*

It is interesting to note that the difference between the results 20.94 per cent and 20.80 per cent is caused by a difference of reading in the water column of 15 mm. Hence it is clear that a meter-stick, or a rod provided with labels whose distance apart may be measured at leisure, is quite sufficiently accurate as a means of measurement for ordinary purposes.

Attention should be called to several possible errors which must be guarded against in this process.

(1) Either no air should be permitted to remain permanently in the tube connecting the pipette and the measuring bulb, or else the volume of the residual bubble should be suitably corrected, according to circumstances.

(2) None of the absorbing liquid should be run into the measuring bulb; or if by accident some of the absorbing liquid finds its way into this bulb, it should be washed away with pure water, in order that it may not affect the tension of the aqueous vapor.

(3) Care should be taken to allow time for the equalization of the temperature change caused by the compression and expansion of the gas as well as for the running down of adhering water.

(4) The temperature must be kept constant, within 0.05° centigrade, a condition which is easily fulfilled by stirring warmer or cooler water into the open receptacle. If an accurate thermometer is not at hand, a sensitive air thermometer may be improvised for this purpose, — for *change* of temperature must especially be guarded against. The ingenious device suggested by Professor Hempel for this correction serves well.†

Of course all these precautions apply equally to the usual measurement under constant pressure, but they are not always heeded.

Attention may be called also to the fact that neither the constant-volume nor the constant-pressure method necessarily affords the true measure of the volume of the absorbed gas. This would only be true if all gases were perfect gases; as a matter of fact, no accurate measurements of the actual slight contractions or expansions which take place on mixing gases seem to have been made. These are, however, so small, that for ordinary purposes they may be neglected.

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* The method was tried also by a large class of beginners in gas-analysis with successful results.

† Hempel (translated by Dennis) (1902), page 84.

It may be a matter of interest to call attention one by one to the percentage effect of the various probable sources of error in the determination of oxygen in air, supposing in each case that the particular error under consideration is the only one present. The analysis is supposed to be conducted in a room having the atmospheric conditions of 20° temperature and 760 mm. pressure, with the apparatus described above. The error is recorded in percentage of the total volume taken.

(1)	Omission of the correction for aqueous vapor	0.4	per cent.
(2)	Error of 1 mm. in reading water column . .	0.01	"
(3)	Constant error of 1° in thermometer in water-jacket	0.02	"
(4)	Change of 1° in temperature of water-jacket	0.5	"
(5)	Constant error of 1 mm. in reading barometer column	0.02	"
(6)	Change of 1 mm. in atmospheric pressure . .	0.13	"
(7)	Admission of absorbing liquid into measuring bulb, according to amount { from . . . to perhaps .	0 1	" "
(8)	Maximum error from adhering water, perhaps	0.5	"
(9)	Possible error from adiabatic contraction, perhaps	1	"
(10)	The retention of a bubble of gas 1 cm. long in the 1 mm. capillary	0.02	"

That all the significant errors may be sufficiently avoided by reasonably careful work is clear from the actual results; but the enumeration of their widely varying effects may be useful in showing the novice where to employ his precautions. Most of these errors apply equally to any kind of gas-analytical work, but many of them are persistently disregarded in common practice.

If the mixture to be analyzed contains gases easily soluble in water, mercury must of course be employed in the measuring bulb, and a good cathetometer should be used for measuring the changes of pressure. A poor cathetometer is often worse than none.

SUMMARY.

In this paper is described a method of gas-analysis which demands only the simplest apparatus, and yet is capable of yielding results accurate enough for many ordinary purposes. The limitations of this apparatus are compared with those of other gas-analytical methods. In particular the importance of applying the correction for aqueous vapor in all methods which depend upon the measurement of changing pressure in constant volume is emphasized.

It is worthy of remark that this simple method forms a highly instructive exercise in gas-analysis for a class of students in this subject.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF CASE SCHOOL OF APPLIED SCIENCE. — XLIV.

A METHOD FOR DETERMINING THE INDEX OF REFRACTION OF SOLID HYDROCARBONS WITH THE PULFRICH REFRACTOMETER.

INDEX OF REFRACTION OF THE SOLID HYDROCARBONS IN PETROLEUM.

By Charles F. Mabery and Lee Shepherd.

Aid in the Work described in this Paper was given by the Academy from the C. M. Warren Fund for Chemical Research.

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INDEX OF REFRACTION OF THE SOLID HYDROCARBONS IN PETROLEUM.*

By Charles F. Mabery and Lee Shepherd.

Accepted June 6, 1902.

One of the most important physical constants of liquids is the index of refraction from which the molecular refraction may be calculated. Of the two forms of apparatus in use for these determinations, the Abbé and Pulfrich refractometers, the latter is in all respects the most convenient. But neither refractometer as constructed permits of the determination of the index of solid bodies, which requires a rise in temperature sufficient to liquify the solid. So far as we are aware no method has been proposed for determining the index of solids. The only method suggested for ascertaining the index of liquids at increased temperatures is that of Brühl (Ber. d. deutsch. Chem. Gesellsch. 24, 286, 1891), which depends on raising the temperature to the desired point by means of a copper bar of peculiar shape, extending to a sufficient distance from the prism to be heated by a lamp. Sufficient heat is applied to the bar to reach the desired temperature over the prism by conduction. A stirer is also attached to insure equal heating of the liquid.

But this method is not satisfactory, especially for solids, since they require temperatures for melting that are difficult to maintain constant by external application without breaking the joint between the prism and the glass cup which holds the liquid. It occurred to us that a more convenient source of heat for melting solids might be supplied by means of an electric current passing through a wire of suitable size to afford the necessary resistance. After numerous experiments that need not be described in detail it was found that the heat from a coil of wire surrounding the glass cup containing the substance, even with a sheet of asbestos interposed, caused such unequal expansion of cup and prism that the lute cracked, causing leaks at the joint. On trying to maintain the desired temperature by a coil of wire placed within the glass cup it was found difficult to regulate the temperature at just the point desired. Finally it was found that a coil of wire surrounding the glass cup could be relied on to raise the temperature nearly to the point desired, and by means of an inner coil the temperature just required for melting could be obtained. German silver wire was found to be more suitable for the coils on account of its rigidity, and the size selected was No. 25 English standard gauge. The arrangement of outside and inside coils, prism, glass cup, and attachment for stirring the melted solid, are shown in the figure.

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* The solid hydrocarbons used in the preparation of this paper were prepared in the work carried on with aid granted by the Academy from the C. M. Warren Fund for Chemical Research.

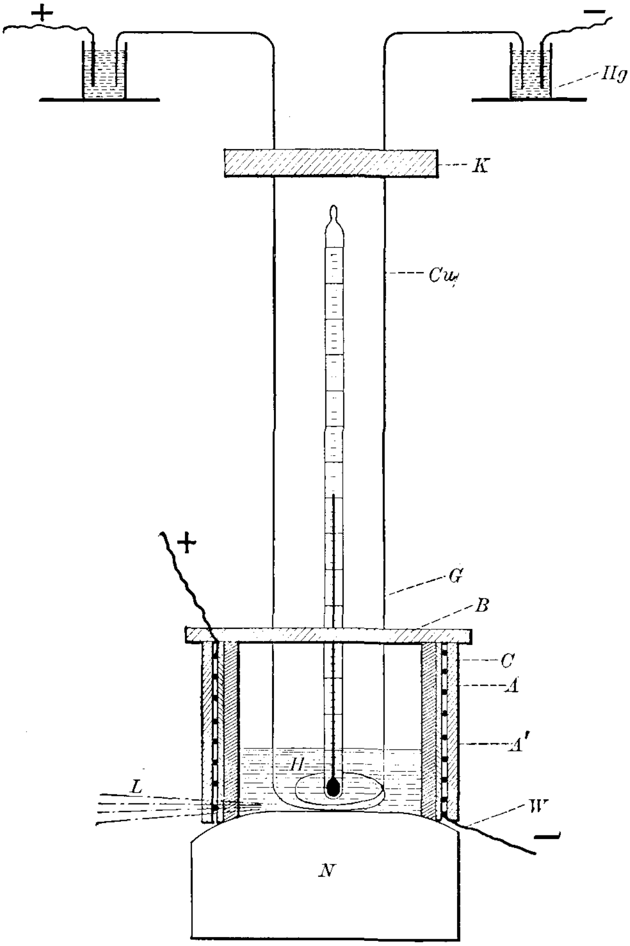
The outer coil of wire W is drawn over a thin sheet of asbestos A which surrounds the glass cup and another thicker sheet is placed outside to prevent radiation. German silver wire of the same size is used for the inner coil and connected above the glass cup to longer copper conductors, which are bent twice so as to dip into mercury cups. This coil is kept in place by means of a cork support K, and the asbestos cover B, which also serves to retain the heat. The inner coil also serves as a stirrer by moving it up and down, which is possible on account of the deep mercury cups. In this manner the solid after melting is very evenly heated. The solid is placed in the glass cup on the face of the prism N, melted by the heated wire, and the temperature kept constant by the inner coil as indicated by the thermometer.

The rays of light L enter the prism through a slit in the asbestos, 0.5 cm. square. A current of 3–5 amperes was sufficient to maintain the temperatures desired in these determinations. With this combination of internal and external heating a temperature of 100° may readily be kept constant without fear of breaking the joint between the prism and glass cup.

The accuracy of this method of heating was determined with water with the following results: —

	20°	30°	40°	50°	60°	70°
Angle,	67° 38′	67° 51′	68° 10′	68° 10′	68° 48′	70° 9′
Index of refraction,	1.3313	1.3304	1.3291	1.3276	1.3241	1.3208
Molecular refraction,	3.692	3.692	3.692	3.692	3.691	3.692

Theoretical molecular refraction of water, 3.692.



This method was applied to the determination of the index of refraction of hydrocarbons separated from Pennsylvania petroleum direct as described by one of us (Mabery) in a former paper,* and also to the same hydrocarbons separated from commercial paraffine. The bodies from crude petroleum had been distilled many times under a tension of 50 mm. and separated from the liquid hydrocarbons by cooling to a low temperature, pressing out the liquid hydrocarbon, and crystallization of the solids from ether and alcohol.

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* These Proceedings, XXXII. 365 (1902).

Attempts were first made to ascertain the index directly by melting the solid on the face of the prism; but on account of lack of uniformity in melting, a thin film of the solid persisted on the face of the prism and obstructed the passage of light. The method of mixture was, therefore, resorted to with perfect success. Readings of angles were taken at temperatures above the melting points of the solids and the density of the solids was taken at the same temperatures. The solvent selected was a petroleum distillate with high boiling point, 202°–203°, 50 mm. that had been fractioned many times. It was essentially pure $C_{18}H_{38}$; on account of its high boiling point there was no danger of loss by vaporization during the observations. The index and specific gravity of the solvent were determined at 60°, 70°, and 80°. The specific gravity was in all cases referred to water at 4°.

The following values were given by the solvent oil: —

	Specific Gravity.	Angle of Refraction.	Index of Refraction.
60°	0.7785	49° 6′	1.4341
70°	0.7722	49° 35′	1.4311
80°	0.7612	50° 16′	1.4277

The results of the observations were calculated by the formula for mixtures: —

$$\frac{n-1}{d}\,100 = \frac{n_1-1}{d_1} + \frac{n_2-1}{d_2}\,(100-p)$$

n = index of mixture
n_1 = " " solvent
n_2 = " " solid
$$\left.\begin{array}{l} d = \text{density of mixture} \\ d_1 = \text{" " solvent} \\ d_2 = \text{" " solid} \end{array}\right\} \text{all taken at } t^\circ$$
p = per cent of solvent
(100 — p) = " " " solid

The mixtures were made of such proportions that each contained approximately 20 % of the solid, and two grams was necessary for each determination, one half the full capacity of the cup, which was necessary to maintain a uniform temperature. The index and specific gravity of each solid were taken at two temperatures, 60° and 70°, or 70° and 80°.

The following results were obtained with the hydrocarbons from Pennsylvania petroleum: —

Distillate, 50mm. 260°–262.° $C_{23}H_{48}$.
Per cent of solvent, 77.86.
“ “ “ solid, 22.14.

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle for mixture.	n.
60°	0.7780	0.7769	49° 9′ 3″	1.4337
70°	0.7711	0.7709	49° 43′	1.4304

Calculated for solid:

	n_2.	Molecular Refraction.	Theoretical.
60°	1.4432	110.61	
70°	1.4260	107.68	107.68

Distillate, 50mm. 272°–274.° $C_{24}H_{50}$.
Per cent of solvent, 82.53.
“ “ “ solid, 17.47.

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle for mixture.	n.
60°	0.7785	0.7771	49° 5′ 3″	1.4341
70°	0.7734	0.7719	49° 40′	1.4307

Calculated for solid:

	n_2.	Molecular Refraction.	Theoretical.
60°	1.4432	113.08	
70°	1.4251	112.00	112.57

Distillate, 50mm. 282°–284°. $C_{25}H_{52}$.
Per cent of solvent, 84.03.
“ “ “ solid, 15.97.

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle for mixture.	n.
70°	0.7724	0.7765	49° 50′	1.4297
80°	0.7618	0.7632	50° 23′ 8″	1.4263

Calculated for solid:

	n_2.	Molecular Refraction.	Theoretical.
70°	1.4241	115.	
80°	1.4212	117.	117.2

Distillate, 50mm. 292°–294°. $C_{26}H_{54}$.
Per cent of solvent, 83.13.
“ “ “ solid, 16.87.

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle for mixture.	n.
70°	0.7726	0.7780	49° 38′	1.4309
80°	0.7618	0.7685	50° 13′ 7″	1.4373

Calculated for solid :

	n_2.	Molecular Refraction.	Theoretical.
70°	1.4320	122.	
80°	1.4305	123.2	121.8

Distillate, 50mm. 300°–301°. $C_{27}H_{56}$.
Per cent of solvent, 77.64
" " " solid, 20.36

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle for mixture.	n.
70°	0.7719	0.7757	49° 55′	1.4292
80°	0.7623	0.7655	50° 29′ 4″	1.4257

Calculated for solid :

	n_2.	Molecular Refraction.	Theoretical.
70°	1.4206	124.2	
80°	1.4194	125.45	126.4

Distillate, 50mm. 312°–314°. $C_{28}H_{58}$.
Per cent of solvent, 86.79.
" " " solid, 13.21.

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle for mixture.	n.
70°	0.7738	0.7770	49° 40′ 6″	1.4297
80°	0.7618	0.7669	50° 14′ 6″	1.4272

Calculated for solid :

	n_2.	Molecular Refraction.	Theoretical.
70°	1.4184	126.6	
80°	1.4170	128.8	131

Having at hand a series of solid hydrocarbons separated from commercial paraffine, that correspond in composition to the solid hydrocarbons from crude Pennsylvania petroleum, determinations of the indices of refraction of these bodies were made for the purpose of comparison. The formulas of the paraffine hydrocarbons are given as they have been determined, although the results have not been published.

Distillate, 50mm. 260°–262°. $C_{23}H_{48}$.
Per cent of solvent, 79.84.
" " " solid, 20.16.

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle for mixture.	n.
60°	0.7780	0.7706	49° 16′ 5″	1.4330
70°	0.7668	0.7641	49° 44′	1.4303

Calculated for solid:

	n_2.	Molecular Refraction.	Theoretical.
60°	1.4256	106.7	
70°	1.4374	111.	108.

Distillate, 50mm. 282°–286°. $C_{25}H_{52}$.
Per cent of solvent, 78.35.
" " " solid, 21.65.

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle for mixture.	n.
60°	0.7784	0.7779	49° 10′	1.4337
70°	0.7721	0.7707	49° 46′	1.4301

Calculated for solid:

	n_2.	Molecular Refraction.	Theoretical.
60°	1.4206	117.6	
70°	1.4194	117.	117.2

Distillate, 50mm. 300°–302°. $C_{27}H_{56}$.
Per cent of solvent, 82.81.
" " " solid, 17.19.

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle for mixture.	n.
70°	0.7727	0.7770	49° 50′	1.4297
80°	0.7625	0.7669	50° 13′	1.4272

Calculated for solid:

	n_2.	Molecular Refraction.	Theoretical.
70°	1.4287	123.4	
80°	1.4276	126.1	126.4

Distillate, 50mm. 312°–314° $C_{28}H_{58}$.
Per cent of solvent, 79.75.
" " " solid, 20.25.

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle of mixture.	n.
70°	0.7737	0.7806	49° 42′	1.4305
80°	0.7630	0.7699	50° 15′	1.4277

Calculated for solid:

	n_2.	Molecular Refraction.	Theoretical.
70°	1.4285	129.3	
80°	1.4226	129.9	131

Commercial Paraffine gave the following indices:
Per cent of solvent, 78.97.
" " " solid, 21.03.

	Sp. gr. of mixture.	Sp. gr. of solid.	Angle for mixture.	n.
60°	0.7784	0.7788	49° 6′ 3″	1.4340
70°	0.7727	0.7732	49° 35′ 6″	1.4311

The results described above show a close agreement in the hydrocarbons separated directly from petroleum, and the corresponding bodies from paraffine, as well as the application of this method for obtaining the molecular refraction of solid bodies.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

THE SIGNIFICANCE OF CHANGING ATOMIC VOLUME.

III. — *THE RELATION OF CHANGING HEAT CAPACITY TO CHANGE OF FREE ENERGY, HEAT OF REACTION, CHANGE OF VOLUME, AND CHEMICAL AFFINITY.*

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Received June 16, 1902.

I. Systems Involving a Minimum of Concentration Effect.

Julius Robert Mayer showed, sixty years ago, that when a gas is compressed, the work of compression reappears almost exactly as heat. One of the circumstances which permitted the discovery of this relationship is the fact that the heat capacity of a gas at constant volume is approximately independent of the volume. If the heat capacity of the gas were diminished by the compression while the other influences remained unchanged, it is clear that some of the heat energy already in the system would be displaced by the compression, and hence that the apparent evolution of heat would be made up of two added quantities, one due to the work put into the system from the outside, and the other due to the lessened heat capacity of the system. Thus the total evolution of heat would be greater than the heat corresponding to the work which had been done upon the system, unless the diminution of heat capacity involved the storing of energy in potential change. In order to develop the reasoning, step by step, this latter possibility will be waived for the present, and taken up again after the facts have been studied.

If $\mathfrak{A}$ is used to represent the unknown total heat energy necessary to raise the original system from the absolute zero to the reaction temperatures, $\mathfrak{A}'$ that necessary to raise the final system through the same interval, W the outside work done upon the system, and U the heat actually evolved, we may represent the possible relationship as follows: —

$$U = W + \mathfrak{A} - \mathfrak{A}'$$

Unfortunately $\mathfrak{A}$ and $\mathfrak{A}'$ are not definitely known, hence the exact solution of the equation is not possible at present. Nevertheless, a qualitative study of the situation in the light of the facts leads to interesting results, as will be shown. In the case of a perfect gas $\mathfrak{U} = \mathfrak{W}'$, hence the above equation reduces to the form $U = W$, the fact pointed out by Mayer. $\mathfrak{U}$ and $\mathfrak{W}'$ are of course functions of the respective heat capacities.

The analogy which has been shown to exist between the work of chemical compression and the work of gaseous compression * seems to afford a means of extending this logic to the liquid and the solid condition, and thus to afford a clue to the vexed question as to the relationship of free energy and total energy.

It is well known that the heat capacity of a solid or liquid system usually changes slightly during the course of a reaction, and it has been shown in numerous isolated instances that compression usually tends to diminish the heat capacity of solids and liquids. † The diminishing specific heat with increasing specific gravity of the allotropic forms of sulphur, phosphorus, silicon, and carbon may be mentioned as other examples of the same general principle.

We should expect, then, if no other complications are involved, to find an excess of heat evolved, over and above the chemical work concerned in the reaction, in all those cases where the heat capacity diminishes during the reaction, and vice versa. But what measures have we of the chemical work involved?

By many the "free energy," or the energy available for *outside* work, is taken to represent the chemical energy. ‡ For the sake of argument, let us assume temporarily that this is the case, always bearing in mind the fact that it is an assumption, and being ready to abandon the position when it has been proved untenable.

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* After the preceding paper was printed, my attention was kindly called to the fact that in 1881 Müller-Erzbach pointed out the parallelism of contraction and heat of reaction in some cases (Ber. d. d. ch. Ges., **14**, 217, 2043; Wied. Ann., **13**, 522.) More recently, Hagemann has called attention to the same relation (a brief paper published privately by Friedländer of Berlin in 1900). These facts were wholly unknown to me at the time of writing. The reason why the discovery has not had more effect upon chemistry is undoubtedly because these investigators did not call attention to the effect of the different compressibility of different substances, and drew no corresponding conclusions from the observations. Unless compressibility is considered, the exceptions to the rule are too frequent to permit satisfactory generalization. For examples of the express or implied denial of the significance of this relation, see Ostwald, Grundriss der allgem. Chem., 187 (1899), also Békétoff, Chem. Centralblatt 1894, II. 229. In neither case was compressibility considered.

† Richards, These Proceedings, **37**, 399 (1902); Regnault, Ann. Chim., **73**, 15 (1840); Thomsen, Thermo-chem. Untersuch., **1**, 52 (1882); Kahlbaum, Zeitschr. anorg. Chem., **29**, 177 (1902).

‡ Ostwald, The Chemometer, Zeitschr. phys. Chem., **15**, 399 (1894).

The immediate problem under investigation then resolves itself into the following question: Has the change of heat capacity any definite causal relationship to the relative magnitudes of the free-energy change and the heat of the reaction?

Throughout this paper the inductive method rather than the deductive one is to be used in all the reasoning. Hence the first step is the comparison of actual data concerning a number of carefully studied reactions.

Unfortunately the change in free energy can be computed from actual data only in the case of easily reversible changes; and these form but a small minority of the cases of chemical reaction. The reversible galvanic cell was shown by Helmholtz * to belong to this class, and may therefore be used as an example.

Among the many galvanic cells which have been measured, not all are immediately available for the present purpose. In the first place, only a few heat capacities of solutions have been accurately determined; and in the next place, in some of the cells perceptible electromotive energy is to be ascribed to the unequal balancing of osmotic pressures, which has nothing to do with the affinities involved. We must then select cells containing two liquids in which the extent of ionization and the migration velocities are similar, the anion identical, and the electromotive force and heat of reaction well known. The series of reversible cells which seems to fulfil these conditions most satisfactorily is that composed of pairs of the sulphates of copper, nickel, iron, zinc, and magnesium. The heat capacities of these solutions have been accurately determined by Marignac † and Thomsen ‡ and the electrical potentials have been determined by Wright and Thompson. §

In order to illustrate more clearly the table containing the results obtained from these cells, a single case may be given here in detail. The common Daniell cell is chosen for this purpose, using solutions of the strength $MSO_4 \cdot 200H_2O$ because the heat capacities of such solutions

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* Helmholtz, Sitz. ber. Berlin Acad., 2 February and 7 July (1882), Ges. Abh. Bd. II.

† Marignac, Ann. Chim. (5), **8**, 410 (1876).

‡ Thomsen, Pogg. Ann., **142**, 337 (1871).

§ Wright and Thompson, Phil. Mag. (5), **17**, 288 (1884), **19**, 1 (1885). This work is revised in connection with others by Wilsmore, Zeitschr. phys. Chem., **35**, 291 (1900).

have in every case been carefully determined. The chemical reaction taking place in the cell may be represented by the following equation: —

$$Zn + CuSO_4 \,.\, 200H_2O = ZnSO_4 \,.\, 200H_2O + Cu.$$

The change in the heat capacity of this system may be computed easily by subtracting the total heat capacity of the factors from that of the products. The "absolute" standard of heat capacity, the *mayer*[*] (or the heat capacity which is raised 1° C by 1 joule of energy), is peculiarly convenient for calculations of this kind; hence it is used below. The following table contains the data in question: —

Factors.	mayers.	Products.	mayers.
$CuSO_4$ $200H_2O$		$ZnSO_4$ $200H_2O$	
$(0.9516) \times (3762.7) \times (4.181) =$	14,970	$(0.9523) \times (3764.5) \times (4.181) =$	14,989
Zn =	26	Cu =	24
Total Factors	14,996	Total products	15,013
		Difference	17

Thus the heat capacity of the system is increased by 0.11 per cent during the reaction.

Since this is the case, we should expect to find that some of the heat energy actually evolved in the reaction would be needed to "fill" the extra capacity, and hence would not become manifest as rise of temperature. As a matter of fact, the observed evolution of heat energy in the reaction (210 kilojoules) is less than the electromotive energy ($1.10 \times 96{,}580 \times 2 = 213$ kilojoules) by about 3 kilojoules. Hence the data are consistent with the theory as to sign; and since both figures are small in proportion to the totals, the order of magnitude might also be consistent.

A single case does of course suffice to prove a point of such importance, hence ten galvanic cells, comprising every combination of the five metals before mentioned, are given in the following table.

Wright and Thompson used amalgamated zinc, but Richards and Lewis[†] have shown that amalgamation makes no essential difference in the potential of this metal, because of a singular compensation of effects. The table explains itself; the fourth and eighth columns contain the significant results to be compared, namely the change in heat capacity and the so-called "bound energy."

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[*] Richards, These Proceedings, **36**, 327 (1900).

[†] Richards and Lewis, Zeitsch. phys. Chem., **28**, 1 (1899).

Factors in Reaction.	Heat Capacities of MSO_4+200H_2O+M'.		Loss of Heat Capacity during Reaction.	Electromotive Force.	Electromotive Energy.	Observed Heat of Chemical Reaction.	"Bound Energy," or Excess of Heat evolved over Electromotive Energy.
	Factors.	Products.					
	mayers.	mayers.	mayers.	volts.	kiloj'les.	kiloj'les.	kilojoules.
$Ni+CuSO_4$	14,997	14,966	+ 31	0.57	111	131	+ 20
$Fe+CuSO_4$	14,996	14,955	+ 41	0.69	134	156	+ 22
$Zn+CuSO_4$	14,996	15,013	− 17	1.10	213	210	− 3
$Mg+CuSO_4$	14,995	14,889	+106	1.81	350	530	+180
$Fe+NiSO_4$	14,967	14,957	+ 10	0.12	23	24	+ 1
$Zn+NiSO_4$	14,966	15,016	− 50	0.53	103	81	− 22
$Mg+NiSO_4$	14,966	14,891	+ 75	1.24	240	400	+160
$Zn+FeSO_4$	14,955	15,015	− 60	0.41	79	57	− 22
$Mg+FeSO_4$	14,955	14,890	+ 65	1.12	217	375	+158
$Mg+ZnSO_4$	15,014	14,890	+124	0.71	138	319	+181

The data in this table are reduced as nearly as possible to the same standards. The heat capacities and electromotive forces correspond to solutions containing two hundred molecules of water to every molecule of salt; the electromotive force scarcely changes with dilution. It is not always easy to ascertain from Thomsen's statements the dilution corresponding to the heat of reaction, but in every case the solutions were at least as dilute as this, being sometimes of twice the dilution. Here also such further dilution can cause but a negligible effect on the result. The atomic weights used in Thomsen's work were unusually crude, even for that day, and all his results have been recalculated to correspond to more accurate values. For example, the values given by him for magnesium correspond to the value 24. instead of 24.36; and accordingly since magnesium itself was weighed, all his data for this element must be raised by one and one-half per cent.* The details of these and other similar calculations may be omitted, since their outcome does not seriously affect the conclusions attained.

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* In passing it may be pointed out that a complete recalculation of *all* heats of reaction would be demanded if the standard of atomic weights were changed to $O = 15.879$.

It will be observed that in every case the loss of heat capacity has the same sign as the excess of heat evolved; and in general, where one is large the other is also large. This relationship is illustrated by the accompanying diagram. In comparing differences between such large numbers one could hardly expect exact parallelism; but the two phenomena run so closely together that one is forced to infer the existence of a fundamental connection between them.

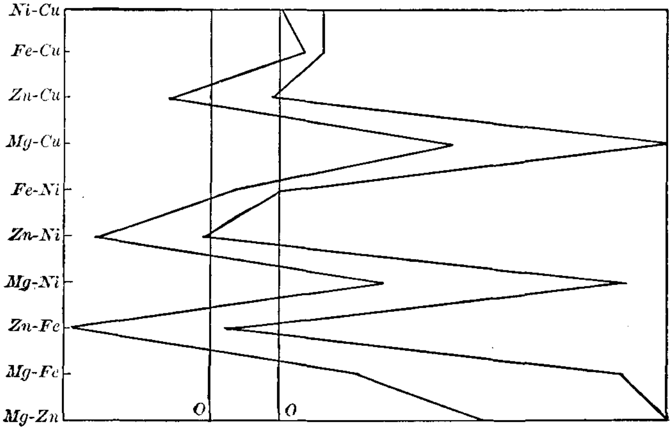

Diagram Illustrating Parallelism of Change of Heat Capacity and "Bound Energy."

Note. The left-hand zigzag line depicts change of heat capacity in the ten cells given in the previous table, and the right-hand zigzag line depicts the "bound energy" or the difference between the heat of reaction and the available free energy of the reaction. In order to avoid confusion, these curves are referred to different ordinate axes, indicated by the two vertical zero lines.

A few more similar cells with other anions might have been added to this list from the data of Wright and Thompson, Marignac, and Julius Thomsen. The following exhibit relationships similar to those already given in detail: zinc-copper nitrate; zinc-lead nitrate; zinc-magnesium nitrate; lead-copper nitrate;* lead-magnesium nitrate; copper-magnesium nitrate; zinc-magnesium chloride. Unfortunately the great bulk of the remaining data cannot be utilized at present because of a lack in each case of one or two of the necessary figures.

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* The heat value for this cell, calculated from the results of Jahn (Wied. Ann., **17**, 593 [1882]), do not at all agree with the results of Thomsen. For the Ag-Cu-nitrate cell Jahn gives 10.6 Cal. per gram equivalent, while Thomsen gives 17.8 Cal.; and for the Ag-Pb-nitrate cell Jahn gives 21.5 Cal., while Thomsen gives 25.6 Cal. The value for the Pb-Cu-nitrate cell would then be 10.9 Cal. according to Jahn, and 7.8 according to Thomsen. I selected the latter value.

More complicated cells, in which also the osmotic tendencies are nearly balanced, show the same tendency. For example, in the cell Hg, HgCl, KCl, KNO_3, KBr, HgBr, Hg, which has been studied by Bugarszky,* there is a loss of heat capacity of about 26 mayers per gram equivalent during the reaction, while the observed heat energy exceeds the electromotive energy by 4.3 kilojoules. When the iodides are substituted for the bromides, the loss of heat capacity is 13 mayers, and the electromotive energy is less than the heat energy by 8.2 kilojoules. On the other hand, when mercurous oxide and potassic hydroxide are combined with the calomel electrode, a cooling reaction yields an electromotive energy algebraically 23.5 kilojoules greater than the heat energy, while the heat capacity gains 44 mayers. These figures are of the same order of magnitude as before, and in the expected direction. They are not included in the table because in them the osmotic energy may amount to an appreciable quantity, which cannot be wholly determined until the solubilities of the nearly insoluble salts are known.

Among all the cells which have been studied, including nearly if not quite all of those for which even moderately accurate data exist, only the cells containing aluminum as one of the metals disagree with the generalization. The solutions of salts of aluminum have usually a very large heat capacity, but yet the heat evolved is greater than the electromotive energy. Great discrepancies exist in the determination of this last quantity, however; for Wright and Thompson give as the potential of the cadmium-aluminum chloride cell only 0.05 volt †, while Neumann's ‡ values indicate 0.87 volt. It is not impossible that even the latter may be too low; for aluminum may be like chromium in its anomalous electrochemical behavior. § In the light of the discrepancy it seems permissible to reject the cells containing aluminum until more certain knowledge is obtained. From the accepted data two important connected inferences may be drawn with considerable security. In the first place, it appears that *unchanging heat capacity is an essential condition in determining the equality of the free electrical and the total heat energy changes, in reactions from*

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* Bugarszky, Zeitschr. anorg. Chem., **14**, 145 (1897).

† Wright and Thompson, Phil. Mag. [4], **19**, 117 (1885).

‡ Neumann, Zeitschr. phys. Chem., **14**, 225 (1894).

§ The use of aluminum as a current rectifier is well known (Graetz, Wied. Ann., **62**, 189; Pollack, Zeitschr. phys. Chem., **24**, 546.). The same question is concerned in this phenomenon.

which osmotic work is eliminated by the balancing of nearly equal pressures. In the next place, as has been stated, the *sign and magnitude of the difference between the free and total energy changes is dependent upon the sign and magnitude of the change of the heat capacity of the system.*

These inferences are of very great interest, for according to the well-known equation of Helmholtz, $n \epsilon_0 \pi - U = n \epsilon\, T \frac{\partial \pi}{\partial T}$ *, we know that the temperature-coefficient of the free energy change is a simple function of the difference between the free and the total energy change. Moreover, the effect of the change of heat capacity of the system on the total energy change was pointed out long ago by Kirchhoff in the well-known equation $-\Delta K = \frac{\Delta U}{\Delta T}$, where Δ represents a finite change.

Hence we may draw the further inference: The *change of the available or free energy of a reaction with the temperature must have some fundamental connection with the change of the total energy with the temperature.*

This fundamental connection becomes manifest on comparing the actual values of $\frac{\partial U}{\partial T}$, with those of $\frac{\partial A}{\partial T}$, the former quantity being given according to the equation of Kirchhoff by the change in the heat capacities, and the latter being calculated from the difference between the total and free energy changes by the equation of Helmholtz.

The following table contains the comparison: —

Factors of Reaction.	$-\Delta K = \frac{\partial U}{\partial T}$.	$\frac{A-U}{T=291^\circ} = \frac{\partial A}{\partial T}$.
	mayers.	mayers.
1. $Ni+CuSO_4$	+ 31	− 69
2. $Fe+CuSO_4$	+ 41	− 75
3. $Zn+CuSO_4$	− 17	+ 10
4. $Mg-CuSO_4$	+106	−620
5. $Fe+NiSO_4$	+ 10	− 3
6. $Zn+NiSO_4$	− 50	+ 75
7. $Mg+NiSO_4$	+ 75	−550
8. $Zn+FeSO_4$	− 60	+ 76
9. $Mg+FeSO_4$	+ 65	−540
10. $Mg+FeSO_4$	+124	−623

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* In this equation n represents the number of equivalents, ϵ_0 represents 96,580 coulombs, π the potential, U the total heat of action, and T the absolute temperature.

The last inference drawn above is supported by this table. Clearly $\frac{\partial U}{\partial T}$ is related to $\frac{\partial A}{\partial T}$. The two always bear opposite signs, and in general one is large when the other is large. The ratio of these two coefficients varies from 1 : 0.13 to 1 : 1.70, neglecting the fifth cell, where the values are too small to be significant. The average is 1 : 0.53 — that is to say, the free energy of a given reaction changes on the average about twice as fast as the total energy with change of temperature, but in the opposite direction. When the total energy increases, the free energy diminishes. Expressed algebraically, $\frac{\partial A}{\partial T} = - M \frac{\partial U}{\partial T}$, in which the value of M averages about 2.

The relation of these two series of data is pointed out yet more clearly by means of the accompanying diagram, in which several of the cells are represented graphically. In this diagram energy is plotted vertically, and temperature in the direction of the axis of abscissae. The lines marked U give the actual quantities of heat evolved in the chemical reactions, the change with the temperature being calculated from Kirchhoff's equation, while the lines marked A give the actual magnitude of the electromotive energy per gram molecule. (See next page.)

On extrapolating the several lines, it is evident that each pair tends to converge at a point not far from absolute zero. There is no reason for surprise at this fact; indeed, such a result is a necessary consequence of the equation of Helmholtz, in which $A - U = 0$ when $T^\circ = 0^\circ$. The interest centres about the fact that the U always increases when A decreases with the temperature, and vice versa.

It will be noticed that in order to converge at the absolute zero these lines must be not exactly straight, but slightly curved, — at least the lines for the free energy must be. This is only natural; for if U steadily increases with the temperature when A is diminishing, $U - A$ will grow at a rate which is greater than that due to the change in A alone. Hence $\frac{U - A}{T} = - \frac{\partial A}{\partial T}$, also will increase as the temperature rises. $\frac{\partial A}{\partial T}$ could be constant, or $A : T$ a linear relationship only in the case of the constancy of U. As a matter of fact Bugarzsky's results enable one to conclude that the temperature coefficient is on the average about 3 per cent higher at 30° than it is at 10°. * Of course more accurate measure-

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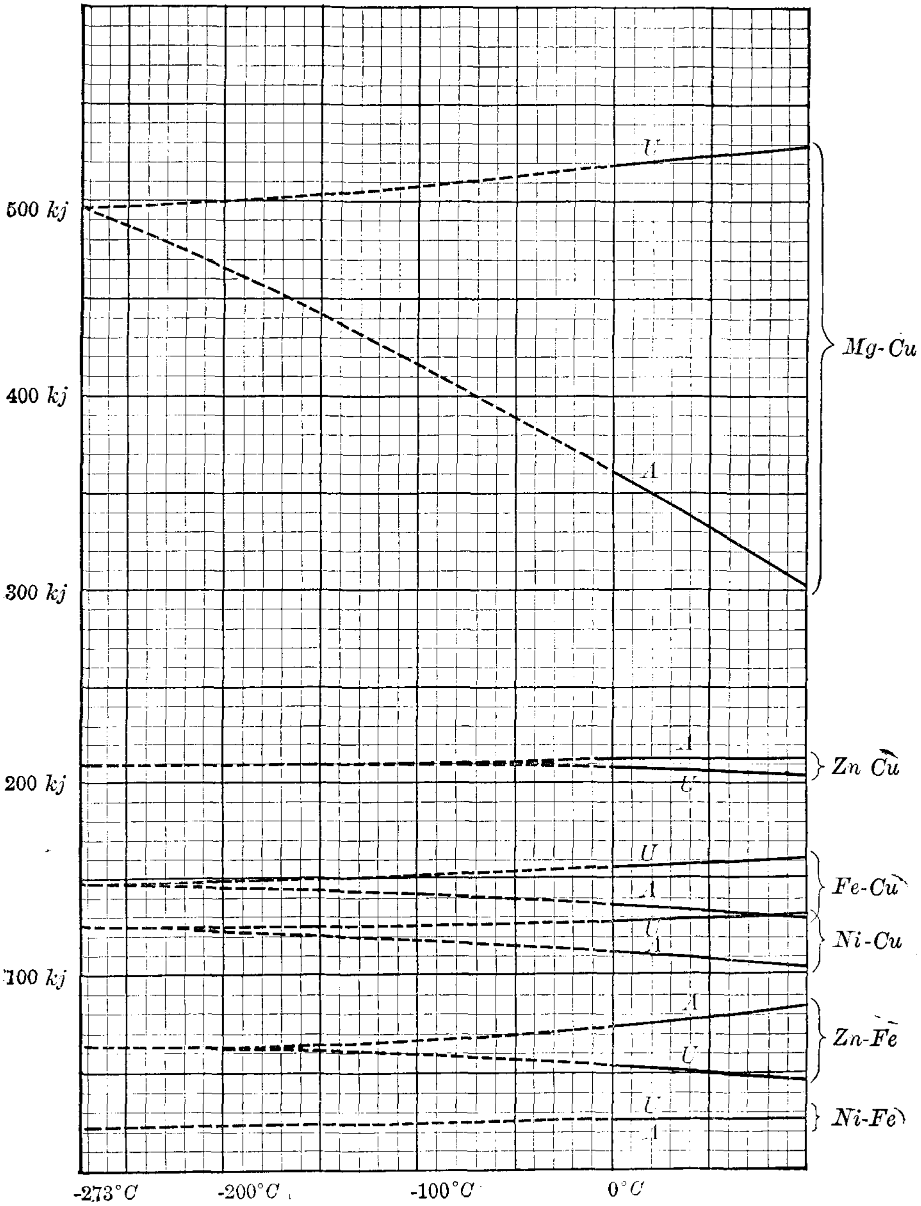
* This calculation is based upon all except one of the cells for which Bugarzsky gives data. The mercuric sulphide cell is rejected because of known irregularity. Zeitschr. anorg. Chem., **14**, 157 (1897).



U = heat of reaction. A = electrical energy.
Ordinates represent energy, abcissae temperature (Centigrade).

ments are necessary to attain a precise knowledge of this relationship of the temperature coefficients; and moreover it probably differs with different cells. The foregoing calculations are merely for the purpose of orientation.

Qualitatively all the results correspond with the theory at first proposed. In every case where the heat capacity diminishes, the heat evolved is greater than the free-energy change. But in some cases the difference is very great; and the suspicion arises that perhaps the free-energy change is at least as much smaller than the sum of the attracting energies as the heat of reaction is greater than this sum. Is there any method of determining quantitatively the heat which has been displaced by the change of heat capacity in order to test this question? If the free-energy change could be proved to be equal to the total-energy change after correction for change of heat capacity, it would be fairly safe to assume that this two-fold result really represented the attracting energies. If, on the other hand, the heat evolved is still the larger, even after correction for the change of heat capacity, there would be valid reason to suspect that the free-energy change is less than the sum of the affinities.

Thus the question is a very fundamental one; but unfortunately the exact computation of how much of the heat energy which has been needed to raise the system from the absolute zero is still present as heat vibration, and how much has been expended in separating the atoms, and thus stored as potential energy, is impossible.* Nevertheless, the heat still present as vibrational activity obviously cannot exceed the total heat which has been put into the system, and this total may be calculated with considerable exactness when the specific heat of ice at very low temperatures is known with accuracy.

For the present, as usual, an approximate calculation must suffice. Making allowance for every circumstance, it is hardly conceivable that more than 2500 kilojoules of heat energy have thus been introduced into the system $Mg + CuSO_4 + 200H_2O$, in order to raise it from absolute zero to 20°C. But in this system as given in the table on p. 297 the heat capacity changed 106 mayers during its subsequent reaction, or 0.71 per cent of the whole. This change, entirely apart from any heat which might be evolved from the affinities concerned, might cause an isothermal displacement of 0.71 per cent of the heat energy present,† or at most $0.0071 \times 2500 = 18$ kilojoules. The actual evolution of heat in excess of the free energy is, however, 170 kilojoules, a figure far greater than the calculated value. Other cells yield the same result; in no case in which osmotic phenomena cause no disturbance is the greatest possible amount of heat displaced enough to correspond to the deficiency of free energy. Two alternatives are now open, — either the free energy does not fully represent the "attracting energy," or else the heat evolved is too large to correspond to the "attracting energy," even after correction for the change of heat capacity.

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* Boltzmann has calculated that about half of the energy is used for each purpose, but the result is doubtful. Sitz. ber. Wien. Akad., **63**, II. 1871.

† This assumption is uncertain, but lack of data forbids greater accuracy (p. 293). The result given is probably a *maximum*, which is all that is desired.

Before going further it is well to define precisely one of the terms employed above. The phrase "attracting energy" is used to represent the sum of the work done by all those tendencies which exert a positive attraction. These tendencies may be three in number, — purely chemical attraction or "*chemism*" (such attraction as binds chlorine to itself in chlorine gas), *gravitation* (which seems to be important chiefly in the heavy metals), and *electrical attraction;* but if any other attractive tendencies exist, they too are included.

In the preliminary study of the phenomena of changing atomic volume, use was made of the possible analogy between an atom and an elastic sphere of gas, in which every portion of the interior gas was supposed to exert an attraction upon every portion of another similar sphere. Further use of this analogy makes it possible to explain the relation between free and total energy in a plausible manner. The illustration must, however, be somewhat modified as it becomes more detailed and concrete; for while the temperature of a sphere of gas is supposed to be traceable to the motion of molecules within it, we must imagine the temperature of an atom to be due to the elastic bodily oscillation of the greater part of its substance.

Two such elastic balls colliding in space would compress one another. Unless some adhesive tendency caused them to cling together, they would immediately fly apart again with their original velocities, tending to absorb as much heat in their consequent expansion as they gave out during compression. Except for the change of direction, their condition would be the same as if they had never met, and no violation of either the first or the second law of thermodynamics would be involved.

If, on the other hand, some attracting or adhesive tendency caused them to cling together, permanently and irreversibly, they would continue in internal oscillation, the additional energy * of which would be a direct measure of the energy of compression. But this additional oscillation would signify an evolution of heat.*

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* That is to say, the energy over and above their original energy of progression before they affected one another.

Suppose now that instead of colliding irreversibly, the two imaginary balls were arrested by an outside mechanism, being allowed to attain gently the same united equilibrium, thus utilizing the energy which would otherwise have become manifest in oscillations as outside work. It is conceivable that in this way all oscillation might be prevented, in which case no heat would have been evolved, and the equivalent of the energy which would otherwise have caused the oscillations will have been used for some outside purpose by the mechanism. The Daniell cell, or better, the Ni-Fe cell, may serve as the actual type of such a conception.

It does not necessarily follow, however, that the mechanism would be capable of using all the attracting energy, or would be able to perform an amount of outside work which would be equivalent to the heat. Indeed, the distortion of both atoms caused by their mutual compression might well be expected to cause an expenditure of energy within the atom; hence less would be available for outside work. Such distortion would naturally tend to restrict the heat vibration, and hence diminish the heat capacity. Hence one might have predicted that when there is a loss of heat capacity in the system, the free energy could not equal the attracting energy, and that some of the attracting energy must inevitably appear as heat, not available for outside work. In symbolic language, according to the first law of energy, $\mathfrak{C} = A' + \mathfrak{P}$, where $\mathfrak{C}$ represents chemical attracting energy, A' free energy, and $\mathfrak{P}$ energy needed to effect a diminution of heat capacity.

On the other hand, the vibrational activity of heat energy may be supposed to be continually striving to overcome the restrictions which circumstances impose upon it. Hence when the opportunity of an enlarged heat capacity is offered by the progress of a chemical change, this heat energy may be able to perform outside work in addition to that capable of being performed by the chemical affinity alone. The result would be a cooling reaction and an excess of free energy. This behavior would not be a contradiction of the second law of thermodynamics, because the law states only that heat cannot *of itself* do work at constant temperature.

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* A perfectly symmetrical elastic ball in collision with another symmetrical elastic ball might be supposed to cause this other to rebound reversibly even if attraction existed between them. As an illustration of this one may conceive of a perfectly elastic ball bouncing on a perfectly elastic surface under the influence of gravity, — its oscillation would continue forever. If, however, one or both of the atoms under consideration is irregular in shape, it is clear that the shock might be so split up into cross-vibrations as to scatter the energy which would otherwise have been used to cause the rebound. Thus heat would be evolved, and the union would be permanent. It is possible that this need of asymmetry in the atom may be the reason why simple pure substances will not combine, but need the catalytic effect of a third substance such as water.

Thus the hypothesis of compressible atoms not only is consistent with the ordinary applications of the two laws of energy, but also affords a conceivable picture of the cause of the newly discovered relation of $\frac{\partial U}{\partial T}$ to $\frac{d A}{d T}$.

On the basis of the present data it is unprofitable to attempt the calculation of the mathematical relation of $\frac{\partial U}{\partial T}$ to $\frac{\partial A}{\partial T}$, or to probe further into the mechanism of the change. When more exact data have been obtained, it may well be possible to arrive at more definite conclusions. If the contraction of volume on combination could only be interpreted in the light of accurate determination of the compressibilities involved, it seems reasonable that this contraction might be a more exact measure of the affinity than either the free or the total energy change.

The foregoing facts and logic seem to lead to the conclusion that the change of free energy of a process does not really represent the attractive energy at work in the process, unless the heat capacity of the system remains unchanged during the reaction. If the heat capacity diminishes during the reaction, the free energy is less than the affinity, and vice versa.

This conclusion is contrary to the common belief. If warranted, it shows that the free energy change is a no more satisfactory guide to the affinity than is the heat evolved in the reaction, even when no concentration effect is present. The free energy seems to represent rather the remainder left after a resisting energy has been subtracted from the attracting energy. Only when the heat capacity does not change during the reaction may we suppose that the attracting energy, the change of free energy, and the heat of reaction are equal. A consequence of this equality is that the attracting energy, like the free and total energy changes, cannot change with the temperature when the heat capacity is invariable.

It is of course true that the change of free energy is the total resultant effect which determines whether or not the reaction will take place. To put the question in another way, the present reasoning seems to afford some basis for separating the change of free energy into two components, one representing the sum of all the attracting energies, and the other a resisting tendency which is connected with the restriction of the heat capacity. It seems to me that further light upon the question of atomic energy is to be had only by means of some such analysis of those composite effects which thermodynamics is content to leave superposed.

The results of this section may be summed up once more in the following sentences: —

When the heat capacity of a system does not change during a reaction, and concentration influences are balanced, the free-energy and total-energy changes of the reaction are equal and unchangeable with the temperature, and each may be supposed to represent the total "attracting energy," — a term which covers gravitational and electrical attraction as well as purely chemical attraction.

When, on the other hand, in such a system the heat capacity of the system changes, it seems reasonable to suppose that the "attracting energy" lies between the free-energy and the total-energy change, one being too small and the other too large.

II. Systems involving Appreciable Concentration Effect.

Allusion has been made more than once to the osmotic or gas-pressure work which results from differences of concentration. Even in the most favorable cases given above, this modifying influence must have had a slight although negligible effect.

The present chapter will show how slight this effect may be supposed to have been, as well as indicate the manner of its action in other cases. It is well known, according to the work of Helmholtz, Nernst, and others, that in dilute solutions the potential due to difference of concentration is approximately represented by the equation $\pi = \frac{RT}{n\epsilon_0} \ln \frac{c_0}{c}$* if the reaction takes place in a constant volume and migration velocities are eliminated. When the solutions are so dilute that there is no heat of reaction, this equation represents quite closely the actual potential.

The effect of this osmotic work on the electrical potential of cells where heat-producing affinity is also at work is, according to the preceding analysis, best seen in those cells which exhibit no change of heat capacity under their operation. Such cells are those which involve simply the dilution of amalgams, which thus assume great theoretical importance. Meyer [*] was the first to measure carefully cells of this type; but more accurate and significant measurements have been made since by Richards and Lewis [†] and by Cady [‡] under the direction of Bancroft.

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* c_0 = initial concentration, c = final concentration.

The results of Cady are especially easy to interpret, because he worked with sodium amalgam, which has a large heat of dilution, even with fairly dilute solutions. He found that the electromotive energy of the cell Na 20.2 Hg — Na 86.7 Hg was equal to the sum of two quantities, the osmotic work and the heat of dilution. Expressed mathematically, $n \pi \epsilon_0 = RT \ln \frac{c_0}{c} + U$, where $\pi \epsilon_0$ is electrical energy, c_0 the concentration of the mercury in the stronger amalgam, c that in the weaker, and U the heat of dilution (135 cal. per gram atom). He showed that the electromotive force was independent of the nature of the solvent between the amalgams and the concentration of the dissolved electrolyte, and concerned the amalgams alone. U was shown to be constant between 4° and 22°.

This result is of the first importance, and taken in connection with the preceding considerations concerning the heat capacity, it seems to afford a new insight into the mechanism of electromotive energy. This energy must be looked upon as the sum of at least two quantities; in the first place of the free energy of the chemical reaction, which happens to be equal to the total energy in Cady's case because the heat capacities are unchanging, and in the second place of the osmotic energies at work. Only when the heat capacity is constant is the change of free energy of the chemical reaction equal to the total energy; hence in general Cady's result,

$$n \pi \epsilon_0 = RT \ln c_0/c + U,$$

is applicable only to such cases. When the heat capacity diminishes during the reaction the second member of the expression will be greater than the first, and vice versa. This possibility was not considered by Cady.

The outcome of all this inductive logic is essentially identical with the result attained deductively by Lewis. He found that from the first and second laws of thermodynamics and the gas law, with the help of a few assumptions, the following equation may be derived: —

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[*] Meyer, Zeitschr. phys. Chem., **7**, 477 (1891).

[†] Richards and Lewis, These Proceedings, **34**, 87 (1898); Zeitschr. phys. Chem., 28, 1 (1899).

[‡] Cady, J., Phys. Chem., **2**, 551 (1898).

$$A = R\,T \ln \frac{c_0^{n_0} \ldots}{c^{n} \ldots} + U - T \int_{T_0}^{T} \frac{K_0 - K}{T}\, dT + H\,T.^*$$

In this equation there is only one unfamiliar symbol H, which signifies a sum of several integration constants. Since the last two terms are hard to determine practically in many cases, we may combine them and write

$$A = R\,T \ln \frac{c_0^{n_0} \ldots}{c^{n} \ldots} + U - f(K,\, T).$$

In words, the available energy of a reaction is equal to the sum of the concentration effect and the heat evolved, minus a quantity which is partially dependent upon change of heat capacity.

This is qualitatively the outcome of the reasoning based purely upon experiment. In one respect the experimental conclusion is more detailed than the theoretical, in that the former indicates the complete elimination of the last term where the heat capacity of the system does not change during the reaction, while the theoretical equation still contains an unknown constant of integration in this case.

A clearer insight into the matter is possibly to be obtained by writing this equation in the following form, in which $\mathfrak{C}$ represents the true although as yet unknown value of the attracting affinity, and $\mathfrak{P}$ is the work needed to change the heat capacity.

$$A = R\,T \ln \frac{c_0^{n_0} \ldots}{c^{n} \ldots} + \mathfrak{C} - \mathfrak{P}.$$

As Lewis has already pointed out from the deductive side, the equation of Nernst for the single potential difference of a reversible electrode should obviously be remodelled on the basis of these conclusions. The Nernst solution-pressure P must include not only the concentration effect, but also the effect of the free energy due to affinity of the solid electrode for the solution.†

The latter is really by far the most important part of the reaction. For the present purpose it is best not to include this effect in the vague and comprehensive P, but to separate the latter into its two important parts, thus

$$n\,\pi\,\epsilon = R\,T \ln \frac{P}{p} = R\,T \ln \frac{p'\,z}{p} = R\,T \ln \frac{p'}{p} + R\,T \ln z.$$

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* G. N. Lewis, These Proceedings, **35**, 1 (1899), Z. phys. Chem., **32**, 364.

† Lewis introduces also a term to cover the nature of the solvent in the cell. While this may be necessary, it seems probable that the value of A includes the effect of changing solvent.

In this equation p' indicates the true concentration effect or "thermal pressure"* of the pure metal, and z represents an imaginable pressure-effect of the affinity (called the "physicochemical potential" in a recent paper on driving energy †). The expression $R\,T \ln z$ indicates of course the outside work $\mathfrak{C} - \mathfrak{P}$ capable of being done by the affinity, and may be indicated by A' for the sake of brevity, according to the equation on p. 305. Then

$$n\,\pi\,\epsilon_0 = R\,T \ln \frac{p'}{p} + A'.$$

When the heat capacity is unchanging $\mathfrak{P} = 0$ and $A' = \mathfrak{C} = U$ and the equation reduces to the result found by Cady.

With the help of this equation it is possible to understand many relationships which would otherwise be inexplicable.

In solutions of equal ion-concentration, $R\,T \ln \frac{p'}{p}$ would be nearly the same for all metals. It is true that we are as yet uncertain concerning the true value to give p', for the so-called "mass-law" often does not apply to concentrated material; but a large change in p' would not cause a serious change in its logarithm. Considering the five metals already treated, if we assume the pressure to be inversely proportional to the volume, and suppose that a normal solution of the sulphate of each metal is about one quarter ionized (four liters containing about a gram-ion), the value of $R\,T \ln \frac{p'}{p}$ will range from $R\,T \ln \frac{4000}{6.7}$ in the case of nickel to $R\,T \ln \frac{4000}{13.3}$ in the case of magnesium. At 20° these values become $2.43 \ln 600 = 15.6$ kilojoules and $2.43 \ln 300 = 13.8$ kilojoules. Thus the range between the extremes is only 2 kilojoules, an amount not greater than a possible error in the heat of reaction in some cases. On the average, the concentration energy (or the pressure energy) of a metallic electrode in a normal solution of its sulphate is probably about 15 kilojoules per gram atom.

The only satisfactory actual data known to me concerning this concentration effect are those of Richards and Lewis ‡ published in the paper already quoted. In this work the electromotive force due to the concentration effect between zinc and a one per cent solution of zinc in mercury was found to be 0.0585, at 20° C; and of cadmium in the same way 0.0615. On diluting the amalgams to the concentration ratio of 400 : 1 to bring them into comparison with the calculation given above about 0.025 would be added to the electromotive forces, corresponding to a total of about 0.085 volt for this dilution. The electromotive energy is then $2 \times 96{,}580 \times 0.085 = 16.4$ kilojoules, or an amount quite close to the above values (14 to 15 kilojoules) calculated theoretically. More extended data of this kind would obviously be highly instructive; and it is our intention to study such cells experimentally in this laboratory in the near future.

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* Lewis, These Proceedings, **36**, 145 (1900); Z. phys. Chem., **35**, 343.

† Richards, These Proceedings, **35**, 477 (1900). In this paper the relation between c and p is dwelt upon. It is by no means certain that anything is gained either in the present equation or in that of Nernst by assuming this chemical energy as a simple function of the temperature.

‡ Richards and Lewis, These Proceedings, **34**, 87 (1898); Z. phys. Chem. **28**, 1.

Obviously then the electromotive energy of most reversible electrodes arise chiefly from the energy of the affinity, only about fifteen kilojoules per gram atom being probably due to the difference in concentration between the pure metal and ion. Since, moreover, this concentration effect cannot be very different with different metals, no great error will result if the appropriate multiple of this value is subtracted from each, in order to find the free energy due to the affinity proper. Evidently this procedure would leave the results on page 297 unchanged, since the same value would be subtracted from each electrode.

In the light of these facts, we should expect to find an approximate parallelism between electromotive force of a single metal in one of its salt solutions and the heat of ionization. Moreover, since the affinity of the metal for the water seems to be the cause of ionization, the heat of formation of the hydroxide and the contraction occurring during its formation should each exhibit a parallelism with each of these quantities. Of course considerable irregularities due as has been shown to changing heat capacity and varying compressibility would be expected, and all that can be done with the present data is to reduce the figures to comparable magnitude by adding or subtracting a constant from each series. This has been done in the following table. It is assumed that the heats of hydration of cupric, argentic, and mercuric oxides are zero.

It is clear that in general the expected parallelism actually exists. The deviations are not frequent enough to obliterate the meaning of the facts. This is especially so, since many of the deviations can be explained. The great compressibility of potassium and sodium and their comparatively expanded condition in the elementary state account for the relatively larger contraction to be observed in their cases. On the other

	Electromotive Force.	Electromotive Energy.	Heat of Ionization. (Ostwald).*	Heat of Formation of Hydroxide less 470 Kj.	Contraction if OH = 12 c.c.
	volts.	kilojoules.	kilojoules.	kilojoules.	cubic cent.
2 K	?	?	518	504	60.2
2 Na	?	?	480	468	33.8
Mg	1.49	288	456	439	12.2
⅔ Al	1.28 ?	253 ?	338	358	. . .
Mn	1.07	207	210	213	3.9
Zn	0.76	147	147	162	. . .
Cd	0.41	71	77	97	6.4
2 Tl	0.36	69	7	6	. . .
Fe	0.34	66	93	101	. . .
Co	0.23	44	71	81	5.0
Ni	0.23	44	67	70	. . .
Sn	0.16	31	14	101	. . .
Pb	0.15	29	2	26	. . .
2 H	0	0	0	102	. . .
⅔ Bi	–0.25	– 48	. . .	7	. . .
Cu	–0.34	– 66	– 66	– 28	– ?
Hg	–0.76	–150	– 83	– 97	– ?
2 Ag	–0.81	–156	–106	–159	– ?

hand, manganese is undoubtedly originally in a very compressed state, its large self-affinity being shown by its exceedingly high melting point; hence the contraction to be observed on forming the hydroxide is not as large as one would otherwise have expected.† Many other irregularities in the approximate parallelism of these four sets of data can be explained on the basis of the principles explained in the present paper and those preceding it, but until more exact figures are at hand, a closer comparison is not worth while.

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* Grundriss, 281 (1899).

† Richards, These Proceedings, **37**, 11 (1901).

In this paper galvanic cells have been used as the most convenient means of measuring change of free energy, and therefore of testing the relation of this quantity to chemical affinity, change of total energy, heat capacity, and contraction of volume. It is clear, however, that all reactions, including such physical ones as vaporization, should indicate the same relationship.

As examples of the effects of changing heat capacity upon free energy, the familiar successes and failures of the law of Guldberg and Waage may be cited. In the most frequent case, when no outside work is done, the concentrations adjust themselves until their combined effect just balances the algebraic sum of the other sources of free energy.* The chief of these, the sum of the chemical affinities $\mathfrak{C}$ concerned, has already been discussed in a previous paper; but to this must be added the free-energy change concerned in change of heat capacity $\mathfrak{P}$. Since there is now no outside work, equation on page 309 becomes

$$0 = RT \ln \frac{c_0 \ldots}{c \ldots} - (\mathfrak{C} - \mathfrak{P}),\dagger$$

or

$$RT \ln \frac{c_0 \ldots}{c \ldots} = \mathfrak{C} - \mathfrak{P};$$

therefore

$$\ln \frac{c_0 \ldots}{c \ldots} = \frac{\mathfrak{C} - \mathfrak{P}}{RT} = \text{“}k.\text{”}$$

Obviously this value will be constant only when the work concerned in the change of the heat capacity and the chemical affinity alter in the same way at the same time. In many cases this simultaneous change actually occurs, especially when the substances concerned are all similar, as in the case of the ester-formation, or the dilution of weak acids. The slight change in the heat capacity in these cases is not enough to cause a serious effect on the result.

The amount of the irregularity is often too small to be observed, but a single example may be given in order to show the real analogy to the case of the galvanic cells. It is well known, from the results of Berthelot and Saint Gilles and others, that the formation of ethyl acetate from equivalent amounts of alcohol and acetic acid is a reaction which attains equilibrium when two-thirds completed. That is to say, the tendency to run in this direction is greater than the tendency to run backwards. The reaction is, however, a cooling one, absorbing perhaps 8 kilojoules per mole. The only available data concerning the heat capacity of the system indicate that there is, however, a considerable gain, perhaps 15 mayers, in this important quantity, as is usually the case when water is formed in a reaction. If it were not for this gain, the reaction would probably have been a warming reaction instead of a cooling one. Thus we have the anomaly of an exothermic reaction (exothermic in the sense that chemical energy is converted into heat) which causes a cooling effect, because the heat capacity is increased in yet greater measure.

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* "The Driving Tendency of Physicochemical Reaction," Richards, Journ. Phys. Chem., **4**, 385 (1900).

† In this case, since $\mathfrak{C} - \mathfrak{P}$ is working in the opposite direction from the other term, its sign will be minus.

With strong electrolytes, the deviations are more marked. Here dilution usually causes a continual decrease in heat capacity. But a decrease in heat capacity seems to involve the expenditure of available energy ($\mathfrak{P}$), therefore a diminution in $\mathfrak{C} - \mathfrak{P}$ and hence also in the "constant" equilibrium ratio $\ln \frac{c_0 \ldots}{c \ldots}$ balanced against it. That is, an increase in the volume causes an increase in the "constant" $\ln \frac{c \ldots}{c_0 \ldots}$. This corresponds qualitatively with the facts, for Rudolphi * found that upon multiplying the normal equilibrium ratio by a direct function of the volume ($\sqrt{v}$) a formula was obtained which represents the facts much more closely. Bancroft † has pointed out the variable nature of the exponent — a variation which would only be expected according to the present analysis.

In the light of these considerations, it is evident that neither "mass-law," vapor pressure, nor freezing point can give correct values for the extent of electrolytic dissociation; and since conductivity is affected by varying migration, it also is unreliable. Hence it would appear that each of the contestants in the recent unfortunately polemical discussion upon this subject is partly right and partly wrong. Nevertheless, although we have not yet data for determining the exact extent of dissociation, it seems to me a great mistake to reject wholly this useful and illuminating hypothesis.

Until more satisfactory data concerning heat capacity are obtained, it is idle to pursue this aspect of the question further, important as it seems to be.

On differentiating the equilibrium equation

$$\ln \frac{c_0^{n_0} \ldots}{c^n \ldots} = \frac{\mathfrak{C} - \mathfrak{P}}{RT}$$

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* Rudolphi, Zeitschr. phys. Chem., **17**, 385 (1895).

† Bancroft, Ibid., **31**, 188 (1899).

the equation of van't Hoff is obtained only when $P = 0$, that is to say, when the heat capacity remains unchanged during the reaction. Evidently, therefore, the equation of van't Hoff, even in its differential form, represents an approximation of only the same order of accuracy as Berthelot's rule, and its deviations from exact fulfilment depend upon the same modifying circumstance. This fact also was inferred by Lewis in the paper already cited on theoretical grounds.

The study of the preceding data shows at once the reason why Berthelot's "rule of maximum work" holds in so many cases. Whenever the change of heat capacity is negligible, the total energy seems to express both the free energy and the attractive affinity. In most cases of simple reactions, the change of heat capacity is very small, and in such cases Berthelot's rule applies. Even in such an intense reaction as that of magnesium upon plumbic oxide the heat capacity changes by only about 6 mayers per gram molecule, which probably could not cause a difference of more than 10 kilojoules between the change of free energy due to affinity and the change of total energy, judging from the figures on page 297. This is a quantity so small in relation to the total heat of reaction (392 kilojoules) that it would not be noticed in an approximate comparison. When concentration effects come into play, in such simple reactions, it is evident that the heat of reaction expresses the affinity more nearly than the total free energy change, although of course it does not express the tendency of the reaction to take place.

The free energy of a reaction is that which can overcome outside obstacles, and hence can enable the change to occur. The heat of reaction seems to be due primarily to affinity, but its magnitude is modified by change of heat capacity. Hence a cooling reaction can occur from two causes: first, because of the loosening of affinities by an overpowering concentration effect working against those affinities, where A is positive in the expression, —

$$A = RT \ln \frac{c_0^{n_0}}{c^n} - A_1,$$

and secondly, because of a sudden increase in heat capacity during the reaction which causes a lowering of temperature without change in the quantity of heat present. These two effects often occur together, because the release from the compression caused by chemical union often involves gain of heat capacity. The former of these cooling causes is strictly endothermic in the sense that heat energy disappears, but the latter is not necessarily so, although it of course always tends to produce a cooling effect. The latter form of reaction may be called "pseudo-endothermic."

Physical reactions involving change of state such as vaporization may be interpreted in a similar fashion. The extension of the argument would throw light upon Trouton's rule,* explaining the approximate regularity and the causes of the deviations, and would give a further insight into the relation between the liquid and the vapor states. For the present, however, this discussion must be postponed. In the near future these and many other aspects of the new kinetic conception will be considered in another paper.

III. SUMMARY.

The conclusions attained in the following paper are summarized below. The first six are generalizations upon fact, the certainty of which is limited only by the moderate number of cases accessible. But one exception has been found, and the probable irreversibility of that reaction annuls its verdict. The last headings contain hypothetical inferences based upon the preceding considerations. It may be that the conclusions recorded below represent merely further approximations, to which exceptions may be found in the future; but the essential agreement of so many facts seems to indicate that at least they are steps in the right direction.

1. In a reaction from which concentration effects have been eliminated, where the total heat capacity remains constant during the reaction, the change of free energy and the heat evolved are equal.

2. When the heat capacity of such a system diminishes during a reaction, the heat evolved is greater than the change of free energy.

3. When the heat capacity of such a system increases during a reaction, the heat evolved is less than the change of free energy.

4. If in addition to these tendencies there is unbalanced osmotic or gas pressure, this pressure adds its share of work algebraically to the change of free energy treated in the three preceding paragraphs, according to the equations

$$A = R\,T \ln \frac{c_0{}^{n_0} \cdots}{c^n \cdots} + \mathfrak{C} - \mathfrak{P} = R\,T \ln \frac{c_0{}^{n_0} \cdots}{c^n \cdots} + U - f(\Delta K).$$

5. Thus in a complex cell containing precipitates the change of free energy, which represents the tendency of the reaction to take place, is by no means a measure of the attractive energy concerned, but has superposed upon this attraction at least two other modifying circumstances.

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* Ostwald, Lehrbuch, I. 354 (1891).

6. The fact that in many reactions the change of heat capacity is small and the concentration effect about balanced affords an explanation for the frequent fulfilment of Berthelot's approximate "rule of maximum work."

7. A distinction is drawn between endothermic and pseudo-endothermic reactions.

8. If the change of total energy could be corrected for the heat energy displaced by change of heat capacity, a measure of the total affinities might be obtained, for into this heat the osmotic tendencies do not enter.

9. If the same contraction in volume as that exhibited on combining could be produced on the uncombined factors in the reaction by outside pressure under similar conditions, the work expended might serve as another measure of the total affinity.

10. In the absence of concentration-effect, the "attracting energy" of a given reaction probably lies between the change of free energy and the change of total energy.

11. It is shown that the hypothesis of compressible atoms affords a conceivable explanation of these facts, and that it is not inconsistent with either of the two laws of energy.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

THE SPEED AND NATURE OF THE REACTION OF BROMINE UPON OXALIC ACID.

By Theodore William Richards and Wilfred Newsome Stull.

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Received June 16, 1902.

While the rapid action of bromine upon neutral oxalates is well known, there is a difference of published opinion as to whether or not oxalic acid itself is affected by this halogen in aqueous solution. Wurtz [*] maintains, without stated authority, that action takes place; while Schönbein [†] denies this contention, giving brief but inadequate experimental evidence.

The further study of the behavior of these two substances is therefore an obvious need. Moreover, the outcome must be of practical value no matter which statement should prove to be true: for if bromine really acts upon oxalic acid, the speed of the reaction could not but give interesting light as to the disposition of the dissolved acid; while if no action takes place, a ready mode of separating bromine from chlorine (which reacts readily) might be afforded.

A few rough experiments sufficed to show that Wurtz was right and Schönbein wrong. The two substances react slowly upon one another at ordinary room temperatures, and rapidly at high temperatures. Schönbein must have been misled by the fact that at 20° no obvious diminution in the intensity of the color of the mixture takes place in many minutes.

The interest of the problem first centers on the question as to whether the bromine attacks the ion C_2O_4'', the ion HC_2O_4' or the undissociated acid. From the fact that the neutral salt is at once oxidized, the first alternative seemed the most likely; but the hypothesis of ions is not yet on so stable a basis as to permit certain inference.

The oxalic acid used in the following work was purified by recrystallization from hydrochloric acid and water. It contained no residual hydrochloric acid. Ten grams of the solid left no residue on evaporation in a platinum dish. The bromine was prepared essentially according to one of the methods of Stas, and was wholly free from all other substances except water.

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[*] A. Wurtz, Dict. de Chem., **2**, 671.

[†] C. F. Schönbein, J. für ppt. Chem. **83**, 484 (1863).

In order to measure out successive, equal portions of bromine for parallel experiments, an almost saturated solution was kept in a covered bottle, and measured out in a glass-stoppered pipette filled by a siphon, so arranged that the top of the pipette communicated with the bottle.* The cork, the only rubber connection in the apparatus, was protected by soft paraffin, which also served when mixed with a little melted rubber † to lubricate the stopcocks. From this pipette successive portions, differing less than 0.1 per cent in bromine-contents from one another, could be delivered at will. A diagram illustrates this description.

The mode of procedure in the quantitative study of the bromine-oxalic reaction was as follows: —

Into a glass-stoppered bottle of about 65 c.c. capacity a quantity of a solution of oxalic acid was introduced, and into this was run a pipetteful of bromine water. The bottle had been previously adjusted by means of glass pearls, so that its remaining volume was just enough to receive the solutions. The stopper was immediately closed and the bottle was immersed in a thermostat whose temperature was kept constant to within one-tenth of a degree. In order to prevent any leakage, the stopper was lubricated with the rubber-paraffin mixture, which serves to seal hermetically the reacting liquids. The rubber-paraffin mixture was not attacked to any appreciable extent. It is necessary to fill the bottles completely, for, if a free space be left above the solution, bromine vapor will pass into this space and thus cause grave uncertainty in the reacting concentration. After a definite time had elapsed, the bottle was taken from the thermostat, the contents were treated with a solution of potassium iodide, and the liberated iodine was titrated with sodium thiosulphate. The thermostat was covered with a black cloth in order to prevent any reaction between the bromine and water, although, as will be noted later in this paper, this precaution was necessary only when working at higher temperatures.

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* When the solution is delivered from the pipette, air must of course flow in to take its place. If this entering air flows through a bulb-tube containing water and a drop of bromine, the atmosphere of the bottle will remain saturated with bromine without loss to the solution. If this bulb-tube is not provided, the air-ingress must be as distant as possible from the air-tube of the pipette, which should almost touch the liquid in the bottle. This latter arrangement was found in our experiments to answer the requirements of the present work, for a single series may be made with sufficient accuracy. After much air has been admitted, the solution will naturally be somewhat weakened. In each series careful determinations of the strength of the solution were made before the experimenting-bottles were filled; again after filling three bottles, and so on until all were filled, when two final determinations were made.

† This mixture, suggested to one of us by W. Ramsay, has proved itself useful in many exigencies. It is made by melting pure rubber, adding equal weights of hard and of soft paraffin, and evaporating off the more volatile constituents. By varying the proportions any desired consistency may be obtained. It may be filtered when hot through fine cloth to remove accidental infused impurities.

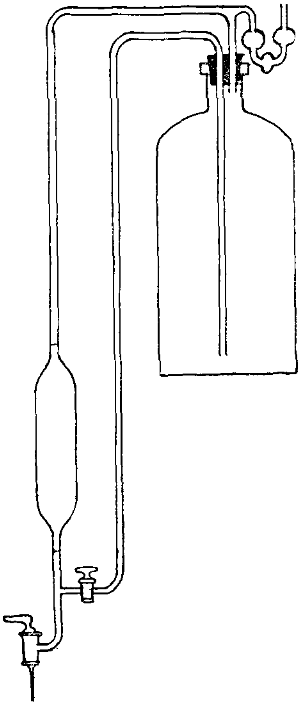

FIGURE 1.

Although there was little probability that the reaction is reversible, yet it was thought advisable to test this point experimentally. For this purpose the rate of reaction of bromine on a solution of oxalic acid in pure water was compared with the rate when bromine acted upon an oxalic solution charged with carbon dioxide. The same quantities of bromine and oxalic acid were present in each case and the time allowed for reaction was one hour. The residual bromine, where carbonic acid was present, was equivalent to 10.29 c.c. of $\frac{n}{10}$ thiosulphate solution. In the absence of carbon dioxide the bromine remaining corresponded to 10.31 c.c. of $\frac{n}{10}$ thiosulphate. The reaction evidently possesses an overwhelming tendency in one direction.

That bromine does not act on water, at ordinary temperature, to any great extent is shown by the following experiment. Three bottles, each containing bromine equivalent to 17.97 c.c. $\frac{n}{10}$ thiosulphate, were tightly stoppered and exposed in the thermostat, at a temperature of 23° C., to the influence of diffused light. At the end of four hours the contents were treated with potassium iodide and the free iodine titrated. The quantities of iodine found corresponded to 17.95 c.c.; 17.96 c.c.; and 17.98 c.c. of $\frac{n}{10}$ thiosulphate. The average 17.96 is essentially equal to the previous value 17.97. For the purposes of this study we may therefore neglect the water-bromine reaction. The probability of an unknown catalytic action called into play in the presence of the oxalic reaction is slight.

I. The Uncorrected Time Curve.

In the first series of experiments a large excess of oxalic acid was taken, in order to have its concentration essentially constant. If no disturbing influences, then, entered into the reaction during its progress, the speed at any moment should be essentially proportional to a simple power of the concentration of the bromine. The actual amounts used were 1.496 grams of oxalic acid and 0.1375 gram of bromine made up to 48.3 cubic centimeters. The temperature was 23°.0.

SERIES I.

No.	Time of Reaction in Minutes.	Weight of Bromine remaining.	Weight of Bromine used.
1	10	.0976	.0399
2	20	.0864	.0511
3	30	.0794	.0581
4	45	.0718	.0657
5	90	.0534	.0841
6	120	.0475	.0900
7	180	.0371	.1004
8	240	.0303	.1072
9	300	.0238	.1137
10	360	.0192	.1183
11	1290	.0013	.1362

Obviously the reaction falls off in speed more rapidly than would have been expected according to any simple interpretation of the results. It was therefore necessary to study the effect of varying, in turn, each condition which might affect the rate of change, in order to find the cause of the anomaly.

II. The Effect of Varying the Concentration of Oxalic Acid.

In the following series of experiments the weight of oxalic acid present varied from 0.0587 gram to 4.112 grams in 63.3 cubic centimeters, while the same amount of bromine (0.1601 gram) was originally present in each case. The time of the reaction in each case was 60 minutes.

SERIES II.

No.	Weight of Oxalic Acid in 63.3 c.c. of Solution.	Weight of Br_2 remaining.	Weight of Br_2 used in an Hour.
12	0.0587	.1003	.0598
13	0.1175	.0909	.0692
14	0.2937	0812	.0789
15	0.5873	.0763	.0838
16	0.8810	.0738	.0863
17	1.1746	.0732	.0869
18	1.4682	.0726	.0875
19	1.7619	.0718	.0883
20	2.0556	.0718	.0883
21	2.3462	.0718	.0883
22	2.5700	.0728	.0873
23	4.1120	.0752	.0849

The most evident feature of these results is the fact that the speed of reaction does not increase indefinitely with increasing concentration, but that above a point corresponding to about three per cent of oxalic acid by weight, the speed decreases. For a considerable space, between concentrations containing 0.7 per cent to 4 per cent of oxalic acid, the speed scarcely changes.

Kohlrausch and Holborn * state that the specific conductivity of a 3.5 per cent solution of oxalic acid is 508 ($10\kappa^4_{18}$), while a 7 per cent solution has a conductivity of 783. The comparison of these data with

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* Leitvermögen der Electrolyte, p. 157 (1898).

the newly presented fact concerning the speed of the bromine-oxalic acid reaction furnishes new evidence that the C_2O_4'' ion is that which is attacked by the bromine. The following reasoning will make this point clear.

The two following equations represent the two possible equilibria existing side by side in the oxalic solution: —

$$(COO)_2'' + 2H^{\cdot} \leftrightarrows (COOH)_2 \qquad (1)$$

$$(COO)_2H' + H^{\cdot} \leftrightarrows (COOH)_2 \qquad (2)$$

If C = concentration of the ion C_2O_4'', C_1 = concentration of the ion C_2O_4H', C_2 that of the hydrogen ion, and C_3 that of the undissociated acid, the law of "mass-action" demands the following equations of equilibrium: —

$$C\,C_2^2 = K C_3 \text{ or } C = \frac{K C_3}{C_2^2} \qquad (3)$$

$$C_1\,C_2 = K C_3 \text{ or } C_1 = \frac{K C_3}{C_2} \qquad (4)$$

The concentration of the singly charged oxalic ion will therefore vary inversely as hydrogen concentration, while the double charged ion, $(COO)_2''$, will vary in concentration inversely as the *square* of the hydrogen concentration. It is fair to conclude from this argument, therefore, that the repressing action due to ionized hydrogen would be vastly more effective in eliminating doubly charged ions from the solution than those with a single charge. Therefore we might expect a maximum concentration of C_2O_4'' at a moderate concentration of oxalic acid; and since such a maximum is found in the action of bromine upon the solution, we may suppose this action to take place upon the group C_2O_4''.*

III. The Effect of Foreign Substances.

It became now a matter of interest to test the conclusions yet further by adding various foreign substances capable of affecting the ionization of the oxalic acid. Accordingly hydrochloric acid and sodic acetate were used as means of respectively increasing and decreasing the concentration of the hydrogen ion. Each of the following solutions had a volume of 58.3 c.c. and contained 1.496 grams of oxalic acid and 0.1375 gram of bromine. Each was allowed to react for two hours, at 23°.0

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* Mr. F. R. Fraprie has lately shown in this laboratory that while alkaline oxalates readily reduce K_2PtCl_6 to K_2PtCl_4, oxalic acid solutions free from alkali effect this reduction only slowly if at all. This is evidently another case where the *ion* of oxalic acid is the active group.

SERIES III.

No.	Foreign Substance added.	Weight of Bromine used in two Hours.
		gram.
24	None	0.0942
25	0.34 gram hydrochloric acid (HCl)	0.0642
26	3.4 grams hydrochloric acid . . .	0.0029
27	1.0 gram sodic acetate *	0.1128
28	Sodic hydroxide to neutralization .	0.1375 (all)

Thus both substances acted in harmony with the hypothesis, the hydrochloric acid retarding the reaction and the sodic acetate hastening it. It is worthy of note that the retarding effect of the acid increases at a faster rate than its concentration, and taking into account the ionized hydrogen already present, it seems quite possible that the relation may be the quadratic one demanded by theory. Another possible disturbing influence will be pointed out in the sequel.

These observations were verified by repeating the experiments with more dilute solutions, but they need not be recorded, since they furnish no further light upon the problem. Sulphuric acid was found to give a result corresponding to hydrochloric acid. (Exps. 29 to 33.)

It became now a matter of interest to determine if the original time-curve might be corrected for the growing concentration of ionized hydrogen caused by the substitution of bromine as an anion instead of the group C_2O_4''. Preliminary calculation led at once to the conclusion that the slight increase in the ionized hydrogen could by no means account for the rapid loss of speed observed in the first series. It seemed therefore possible that hydrobromic acid might possess some specific effect, and preliminary experiments showed that this was indeed the case to a surprising degree. In two hours 0.0008 mole per litre of hydrobromic acid was found to produce as much depressing effect on the total progress of the reaction as was caused by 0.01 mole of hydrochloric acid.

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* It is not impossible that the acetate ion itself may be slightly attacked by bromine, but considerable action is not likely at the low temperature employed.

Clearly, then, the growing presence of hydrobromic acid must be the cause of the abnormal loss of speed in the original time-curve. But in what manner can the substance effect this change?

The next step was to study in great detail the actual effect of hydrobromic acid of varying strengths.

IV. THE EFFECT OF HYDROBROMIC ACID.

The following series give a basis for more certain conclusions concerning the action of hydrobromic acid. Series IV is merely a repetition of Series I with a somewhat more dilute solution of oxalic acid, Series V contained a few centigrams of hydrobromic acid, and Series VI contained about twice as much of this acid. In each the total volume was 59.30 cubic centimeters, containing at the start 1.495 grams of anhydrous oxalic acid and 0.1536 gram of un-ionized bromine.

As before, the temperature was 23°.0.

SERIES IV.

No.	Time of Reaction in Minutes.	Weight of Bromine remaining.	Weight of Bromine used.
34	10	.1091	.0445
35	20	.0970	.0566
36	30	.0899	.0637
37	45	.0801	.0735
38	60	.0720	.0816
39	120	.0528	.1008
40	180	.0394	.1142
41	240	.0315	.1221
42	300	.0257	.1279
43	360	.0205	.1331
44	320	.0179	.1357

SERIES V.

Containing 0.0675 Gram of HBr.

No.	Time of Reaction in Minutes.	Weight of Bromine remaining.	Weight of Bromine used.
45	30	.1249	.0286
46	60	.1080	.0456
47	180	.0741	.0795
48	300	.0536	.1000

SERIES VI.

Containing 0.148 Gram of HBr.

No.	Time of Reaction in Minutes.	Weight of Bromine remaining.	Weight of Bromine used.
49	10	.1476	.0060
50	30	.1410	.0126
51	45	.1360	.0176
52	60	.1315	.0221
53	120	.1161	.0375
54	180	.1039	.0497
55	240	.0944	.0592
56	300	.0846	.0690
57	360	.0779	.0757
58	420	.0729	.0807

The results given in these tables are plotted in Figure 2, the bromine present at any time being drawn as ordinate, and the time as abscissa.

In these experiments the amount of oxalic acid (about 2.5 per cent) by weight was so chosen that the concentration of the reacting oxalic molecular species remained essentially constant during the reaction. In

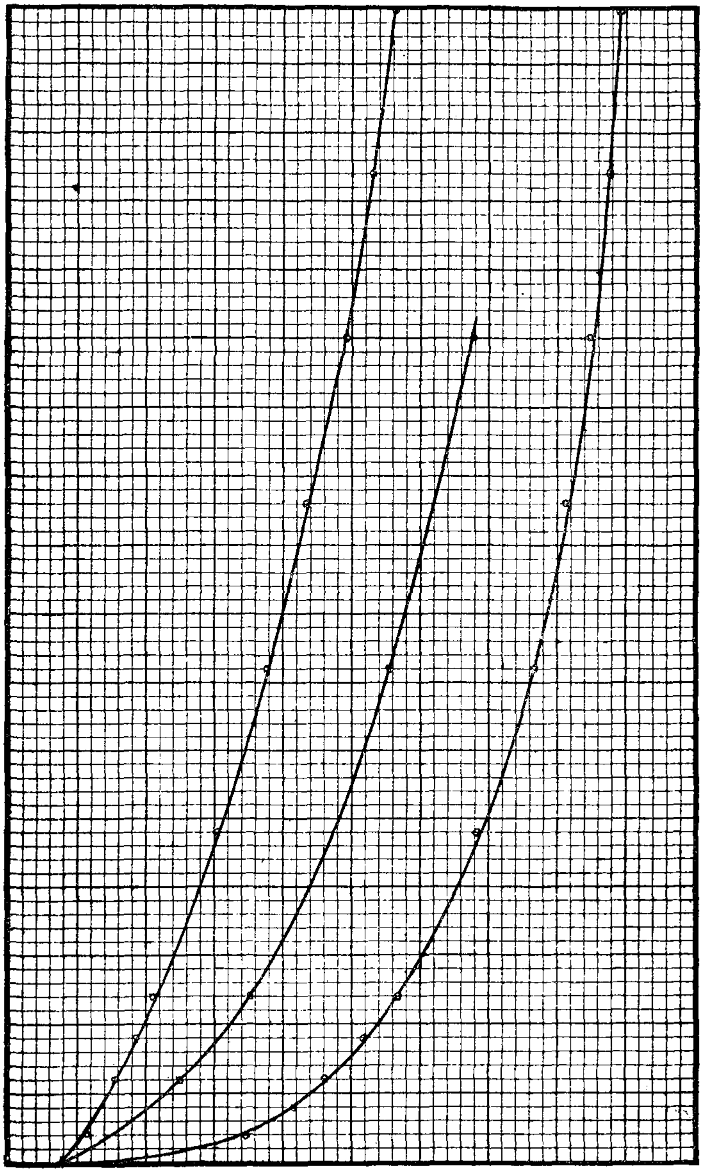

Figure 2. Time in minutes is plotted vertically; the weight of bromine remaining is plotted from right to left. The right-hand curve represents Series IV, the middle curve Series V, and the left-hand curve Series VI.

the second series it had been found that the solution might contain any amount from one per cent to five per cent of oxalic acid without any serious variation in the speed.

Hence the peculiarities exhibited by these curves must be due essentially to the presence of varying hydrobromic acid and bromine. The immensely retarding effect of the hydrobromic acid is at once manifest, without further calculation, and evidently the greater the amount of hydrobromic acid, the less rapid is the change of rate as the reaction progresses.

Thus the downward convexity in Series IV is strongly marked, while in the case of Series VI the curve is but slightly bent. The question at once presents itself, — Would the reaction reduce to a monomolecular one, depending upon the concentration of the bromine alone, if the effect of the hydrobromic acid were eliminated? The results in hand furnish the data for answering this question.

An approximate estimate of the speed of reaction at any point in any one of these curves may be obtained by placing a straight-edge tangent to the carefully drawn curve, and noting the ordinate, or bromine value, which corresponds to a given time. This was done quite independently by each of the authors, and the rates thus found agreed essentially with one another. The averages of several series of such readings are recorded in the following table, together with the amounts of hydrobromic acid and the bromine present in each case. The time interval was always reduced to five minutes. The various points are designated by giving the series and the times from the commencement of the reaction, hence they may all be identified upon the diagram, although of course the actual curves from which the readings were taken were drawn upon a much larger scale. The last column contains a function which would be constant if the speed were directly proportional to the concentration of the bromine and inversely proportional to the concentration of the hydrobromic acid. (See page 332.)

In view of the complicated relations concerning the dissociation of the hydrobromic acid, it is surprising that even an approximately constant function could be found. Nevertheless, it is obvious that the requirement of the function $\frac{ab}{c}$ is nearly fulfilled. In a case as complicated as this the safest method of solution is to eliminate one variable at a time, not attempting to analyze the reaction at once in a single expression. The conditions of the experiment were so planned that such a method of treatment is possible in this case.

COLLATED DATA CONCERNING SPEED OF REACTION.

Designation. Series and Number of Minutes.		(a) Rate = Bromine used in five Minutes. Milligrams.	(b) HBr present. Milligrams.	(c) Bromine present. Milligrams.	$\frac{ab}{c}$
IV	24	4.0	60	94	2.6
IV	37	3.0	71	84	2.6
IV	52	2.4	79	76	2.5
IV	95	1.52	95	58	2.5
IV	105	1.40	98	56	2.5
IV	180	0.83	115	39	2.4
IV	240	8.58	124	32	2.2
IV	360	0.25	134	18	1.9
V	30	3.5	98	124	2.7
V	89	1.7	124	98	2.1
V	123	1.3	134	87	2.0
V	180	1.0	148	74	2.0
VI	52	1.6	168	134	2.0
VI	270	0.76	213	89	1.8

It is reasonable to suppose that the effect of the hydrobromic acid will be eliminated if we compare speeds of reaction in solutions containing equal concentrations of hydrobromic acid. It is true that the cause of the retarding action of this substance may affect its available concentration, but clearly a first approximation may be reached in this way. We should expect to find the residual difference in rate to be due to the bromine alone, and hence be able to discover at once to what type the bromine reaction belongs.

It is easily seen that Series IV contained 124 milligrams of hydrobromic acid after 240 minutes, while Series V contained the same amount after only 89 minutes. Now the amounts of bromine present in these two cases were respectively 32 and 98 milligrams, while the rates were

0.58 and 1.7 respectively. These numbers are so nearly proportional as to indicate beyond question a monomolecular reaction, directly dependent upon the concentration of the bromine alone. The corresponding values of $\frac{ab}{c}$ are respectively 2.2 and 2.1, which of course express the same relation. For further evidence one may compare the values corresponding to 98 to 134 milligrams of hydrobromic acid, in each of which cases a similar equality of the "constant" is to be noted.

The simplest interpretation of this result is to ascribe to hydrobromic acid a catalytic retarding action. This would be rather a classification than an explanation, however, — for no one pretends to understand the exact mechanism of catalytic changes.

In cases like the present one, where some of the side influences are *difficult to interpret, every possible means of determining the order of* a reaction should be employed. The other chief methods which have been proposed are those due to van't Hoff,* who suggested calculating according to the equation of each of the orders so as to find which answers; or else studying the reaction with varying dilutions, and comparing the results; and the "isolation method," which consists in putting in so great an excess of all except one of the reacting substances that it alone changed perceptibly during the reaction. †

The present case is too complex for the first two methods; accordingly the last method was tried, but without notable success. When fifteen or twenty times as much hydrobromic acid is added to the solution as will be formed during the reaction, the speed is so repressed that in order to bring about any appreciable reaction, we were compelled to work at comparatively high temperatures. Even at 50° C. the action was exceedingly slow, and a great part of loss of bromine at this temperature was found to be due to its reaction with water according to the equation $2H_2O + 2Br_2 = O_2 + 4HBr$. It might be argued that we could have obtained the value of the speed of this last reaction, and by subtraction eliminate the error introduced by it into the apparent bromine-oxalic acid reaction. Such a course, however, would be unsafe, inasmuch as we know nothing of the mutual influence of the two reactions. Very probably the speed of reaction resulting from the action of water and oxalic acid on bromine is not equal to the sum of the speed of water on bromine plus the speed of the bromine-oxalic acid reaction. The assumption of such an equality would at all events be hazardous. Since bromine has no appreciable action on water in the dark at ordinary temperatures, this objection has no important weight when offered against the main portion of the work outlined in this paper.

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* Van't Hoff, Vorlesungen, I. p. 193 (1898).

† See Ostwald, Lehrbuch für allg. Chem., **2**, II. 238. Further information concerning these matters may be had from the papers by A. A. Noyes, Zeitschr. phys. Chem., **18**, 118 (1895), **19**, 599 (1896), also T. S. Price, ibid. **27**, 479 (1898), etc.

It becomes now a matter of importance to determine the cause of this remarkable specific action of the hydrobromic acid. That the relation is a complex one goes without saying. As has already been pointed out, the smallness of the effect caused by hydrochloric acid indicates that the depression of the dissociation of the oxalic acid is not the only cause at work. On comparing the effect of different concentrations of hydrobromic acid in the presence of like concentrations of the other substances, it is seen that the retarding effect grows faster than the concentration of the hydrobromic acid. Thus when 88 milligrams of bromine were present, 65 milligrams of hydrobromic acid allowed a rate of 3.3, while 134 depressed the rate to 1.3 (IV, 31, and V, 123). A doubling of the concentration of the hydrobromic acid more than halved the rate.

If this effect is catalytic, and peculiar to this reaction, we should expect to find different effects in other cases. On the other hand, it is possible that the hydrobromic acid may really enter into combination with some of the bromine, and thus withdraw it from the reaction. If hydrobromic acid diminishes the reacting concentration of bromine in this case, it should also produce this effect in all other cases, both chemical and physical. It should, for example, diminish the bromine-vapor tension of bromine water. This conclusion was easily tested, and found to be amply supported by fact. It forms another link in the chain of evidence indicating the existence of polybromides in solution.

When hydrobromic acid is added to an aqueous solution of bromine the color of the latter is lost to a great degree, the solution becoming much paler. Such a color change indicates chemical change. That the vapor tension is much depressed was clearly shown by the following experiment: —

Six gas-washing bottles were arranged in series, and into the first (A) was put a solution of bromine in water. The second and third bottles contained a solution of potassium iodide. Into the fourth (D) a bromine solution containing hydrobromic acid was introduced, and the last two bottles were filled with the iodine solution. The arrangement was as shown below, —

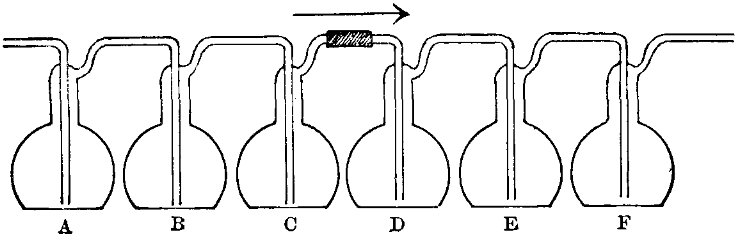

FIGURE 3.

Air was drawn through the solutions, and the liberated iodine in B and E titrated with thiosulphate. Free iodine, equivalent to 22.80 c.c. $\frac{n}{10}$ thiosulphate, was found in bottle B, while the acid solution of bromine, D, had given up to E only the equivalent of 2.50 c.c. of $\frac{n}{10}$ thiosulphate. No iodine was set free in C or F. This experiment was repeated, letting the air first pass through the bromine solution containing hydrobromic acid and then through the aqueous solution of pure bromine. The bromine containing the acid lost bromine equivalent to 2.10 c.c. of thiosulphate $\frac{n}{10}$, while the solution free from acid lost the equivalent of 21.00 c.c. The strength of the hydrobromic acid was about five per cent. It is evident from this that hydrobromic acid in some way combines with bromine. A solution of potassium bromide was found to act in the same way.

Similar results have been attained by Jakovkin * by studying the distribution of bromine between carbon disulphide and solutions of bromides. He found that even chlorides in solution possess a slight, although far less marked tendency to diminish the reacting tendency of bromine, while the effect of bromides is very great.

This effect must be due, of course, to the formation of tribromides in solution, and there is good reason to believe that these are about as highly ionized as the bromides from which they are formed. Hence the retarding action of hydrobromic acid may be supposed to be due chiefly to the tendency of the bromide ion to add bromine, and thus to remove this bromine from the field of action.

The essential details of the action of bromine on oxalic acid may therefore be supposed to take place in accordance with the following scheme, in which the main reaction under investigation is underlined.

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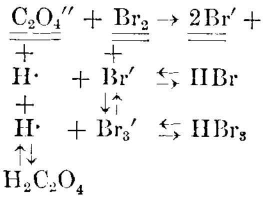
* Zeitschr. phys. Chem. **20**, 19 (1896).



$$\begin{array}{llll}
\underline{\underline{C_2O_4''}} & + \underline{\underline{Br_2}} & \rightarrow \underline{\underline{2Br'}} + \underline{\underline{2CO_2}} & \\
+ & \;\;+ & & \\
H\cdot & + Br' & \leftrightarrows HBr & \\
+ & \;\;\downarrow\uparrow & & \\
H\cdot & + Br_3' & \leftrightarrows HBr_3 & \\
\uparrow\downarrow & & & \\
H_2C_2O_4 & & &
\end{array}$$

But how is it possible to reconcile this complex reaction with the simple mathematical expression $\frac{ab}{c}$ which was found to hold approximately for moderate dilutions? If we represent by fa[*] the concentration of the *active* bromine, by b the total concentration of the hydrobromic acid, and by c the total bromine, the requirement of "mass law" would be at least as complicated as

$$(fa)^n\left[b-\frac{n_1}{n}(c-fa)\right]^{n_1}=k\,(c-fa)^{n_2}.$$

This is on the assumption that the law holds exactly, and that the hydrogen tribromide is ionized to the same extent as the hydrobromic acid. Qualitatively this equation holds true; for an increase in c actually causes an increase in a, as the equation demands. If all the n values are assumed to be unity, it is clear that fa must grow in greater proportion than $c-fa$ with increasing c and constant b. Moreover, c must grow faster than $c-fa$; and it is not at all impossible that for moderate concentrations the relation might be such that $\frac{fa}{c}=k$, — the observed approximate fact.

In view of the many-sided relationships shown by the reaction diagram, incomplete as it is, and the inaccuracy of the "law of mass-action" when applied to electrolytes, it is hardly worth while to attempt to construct any more precise mathematical relationship. That which has been pointed out, taken in connection with facts concerning other reactions and the known existence of polybromides, seems to indicate that it is not necessary to class the action of hydrobromic acid among catalytic phenomena; the reaction indicated on this page seems a more probable explanation.

SUMMARY.

The following points may be repeated as the chief outcome of the present investigation: —

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[*] a signifies the rate of change; the active bromine may be assumed as a function of this rate.

1. Bromine water reacts slowly upon oxalic acid at ordinary room temperatures, and rapidly at higher temperatures.

2. The concentration of the oxalic acid may be varied through wide limits without seriously changing the speed of the reaction.

3. Neutralization or the addition of salts of weaker acids immensely accelerates the reaction.

4. The addition of strong acids retards the reaction.

5. Hydrobromic acid has a retarding effect more than ten times as great as hydrochloric acid.

6. The bromine-vapor pressure of bromine water is likewise greatly diminished by the addition of hydrobromic acid or a bromide.

7. Taking all these facts into consideration, it is shown with the help of the "Law of Mass Action," and the hypothesis of electrolytic dissociation, that the main reaction under consideration is probably the following: $C_2O_4'' + Br_2 \leftrightarrows 2Br' + 2CO_2$; but that this reaction is greatly retarded, as the change progresses, by the conversion of some of the otherwise available bromine into the complex ion Br_3' by union with the bromine ion formed in the main reaction.

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REGULAR SINGULAR POINTS OF A SYSTEM OF HOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS OF THE FIRST ORDER.

By Otto Dunkel.

REGULAR SINGULAR POINTS OF A SYSTEM OF HOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS OF THE FIRST ORDER.*

BY OTTO DUNKEL.

Presented by Maxime Bôcher, May 14, 1902. Received July 10, 1902.

WE will consider in the present paper a system of n differential equations of the form:

$$(1) \qquad \frac{dy_i}{dx} = \sum_{j=1}^{j=n} \left(\frac{\mu_{i,j}}{x} + a_{i,j} \right) y_j \qquad (i = 1, 2, \ldots n),$$

in which the $\mu_{i,j}$'s are constants, and the $a_{i,j}$'s are functions, not necessarily analytic, of the real independent variable x, continuous in the interval:

$$0 < x \leqq b.\dagger$$

We shall require that $|a_{i,j}|$ be integrable up to the point $x = 0$. For the development of certain sets of solutions we shall make the additional requirement that even after certain powers of $\log x$ have been multiplied into $|a_{i,j}|$ the resulting function shall be integrable up to the point $x = 0$; this requirement will be stated more explicitly later.‡

The point $x = 0$ satisfying these conditions may be called a *regular singular point* of the system of equations (1) in conformity with the use of that term by Professor Bôcher in the study of linear differential equations of the second order.* That this terminology is a legitimate extension of that commonly used when the coefficients of the system of differential equations are analytic functions of a complex variable, will be evident if the results of the present paper are compared with the thesis by Sauvage: *Théorie générale des systèmes d'équations différentielles linéaires et homogènes.*†

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* This paper was accepted in June, 1902, by the Faculty of Arts and Sciences of Harvard University in fulfilment of the requirement of a thesis for the degree of Doctor of Philosophy.

† The requirement that the functions $a_{i,j}$ should be continuous in $0 < x \leqq b$ is made only for the sake of simplicity. We might allow them to have a finite number of discontinuities in $0b$ of such a kind that each function $|a_{i,j}|$ can be integrated throughout the interval; and all the following work would hold with very little change.

‡ Cf. p. 367.

Our object is to investigate the nature of the solutions of (1) in the neighborhood of the regular singular point $x = 0$; and for this purpose we shall first reduce the system of equations to a canonical form by means of a linear transformation with constant coefficients of the dependent variable. We shall then apply the method of successive approximations to develop about the point $x = 0$ a system of n linearly independent solutions of the canonical system. By means of the linear transformation we shall return to n linearly independent solutions of the original system; and finally an application will be made to the case of the single homogeneous linear differential equation of the nth order.

§ 1.

A SPECIAL SYSTEM OF EQUATIONS: ITS REDUCTION TO A CANONICAL FORM, AND SOLUTION.‡

Let us first examine the special case of (1) in which the coefficients $a_{i,j}$ are all zero. In this case we have the system of differential equations:

$$(3) \qquad \frac{dy_i}{dx} = \sum_{j=1}^{j=n} \frac{\mu_{i,j}}{x} y_j \qquad (i = 1, 2, \ldots n),$$

where the $\mu_{i,j}$'s are constants.

A solution of this system may be obtained in the following way. Substitute

$$y_i = C_i x^r \qquad C_i = \text{constant}$$

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* Cf. Trans. Am. Math. Soc., Vol. I. Jan. 1900, p. 41. The results of this paper are included as a special case in those we now give. Cf. § 7.

† Paris, 1895. Reprinted from the Annales de la Faculté des Sciences de Toulouse, Vols. VIII. and IX.

‡ The results of this section are not new, being on the one hand only slightly modified forms of Weierstrass's results (cf. the foot-note on p. 345), and on the other hand special cases of the results obtained by Sauvage (cf. the last foot-note.)

in the equations (3), and then determine the constants r and C_i so that the equations are satisfied. We obtain in this way the following system of n homogeneous linear equations for the C's:

$$(4) \qquad \mu_{i,1}\, C_1 + \ldots + (\mu_{i,i} - r)\, C_i + \ldots + \mu_{i,n}\, C_n = 0 \qquad (i = 1, 2, \ldots n).$$

The necessary and sufficient condition that (4) may be satisfied by a set of C's not all zero is that the determinant:

$$(5) \qquad \Delta(r) = \begin{vmatrix} \mu_{1,1} - r & \cdot & \cdot & \cdot & \mu_{1,n} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \mu_{n,1} & \cdot & \cdot & \cdot & \mu_{n,n} - r \end{vmatrix}$$

shall vanish. This determinant equated to zero gives an equation of the nth degree in r, which is called the *characteristic equation* of (3);* the determinant itself we may call the *characteristic determinant* of (3).

If the characteristic equation has n distinct roots $r_1, r_2, \ldots r_n$, we can determine n linearly independent solutions of (3) of the form:

$$(6) \qquad y_{i,j} = C_{i,j}\, x^{r_j} \qquad \begin{pmatrix} i = 1, 2, \ldots n \\ j = 1, 2, \ldots n \end{pmatrix}.$$

If however there is a multiple root, there will be, in general, solutions involving powers of log x. To determine these solutions, we must examine the minors of the characteristic determinant (5), and ascertain if this multiple root is also a root of all the first minors, second minors, etc. For the further study of this case, it will be useful to introduce the conception of the *elementary divisors* of $\Delta(r)$.

Suppose r' is a root of the determinant $\Delta(r)$ such that all the pth minors of $\Delta(r)$ are divisible by $(r - r')^{l_p}$, but no higher power of $r - r'$ divides them all. In the same way $(r - r')^{l_p + 1}$ shall be the highest power of $r - r'$ dividing all the $(p + 1)$th minors. Then the expression:

$$(r - r')^{e_p} \qquad e_p = l_p - l_{p+1}$$

---

* Sauvage: *l. c.* p. 80.

is called by Weierstrass an elementary divisor of the determinant $\Delta(r)$.* It will be convenient to employ a different notation from that used in the definition of an elementary divisor. An elementary divisor of $\Delta(r)$ will be written:

$$(r - r_k)^{e_k},$$

and it is to be noticed that several r's with different subscripts, may be equal, as will be the case when a multiple root furnishes several elementary divisors. We shall always have:

$$(7) \qquad \Delta(r) = \prod_{k=1}^{k=m} (r_k - r)^{e_k} \qquad \sum_{k=1}^{k=m} e_k = n.$$

It can be shown (cf. the next foot-note) that a necessary and sufficient condition that a pair of systems of differential equations:

$$\frac{dy_i}{dx} = \sum_{j=1}^{j=n} \frac{\mu_{i,j}}{x} y_j, \qquad \frac{dz_i}{dx} = \sum_{j=1}^{j=n} \frac{\nu_{i,j}}{x} z_i$$
$$(i = 1, 2, \ldots n)$$

can be transformed the one into the other by means of a transformation:

$$y_i = \sum_{j=1}^{j=n} A_{i,j} z_j \qquad (i = 1, 2, \ldots n),$$

in which the A's are constants whose determinant is not zero, is that the characteristic determinants of the two systems have the same elementary divisors. This theorem enables us to simplify the solution of the system (3); for we can write down a second system of differential equations having the same elementary divisors as (3), as follows:

$$(8) \qquad \frac{dz_{k,l}}{dx} = \frac{1}{x} z_{k,l-1} + \frac{r_k}{x} z_{k,l} \qquad \begin{pmatrix} k = 1, 2, \ldots m \\ l = 1, 2, \ldots e_k \end{pmatrix},$$

where $z_{k,o} = 0.$

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* Cf. Muth: Theorie und Anwendung der Elementartheiler, p. 2.

The characteristic determinant of (8) is:

$$\Delta(r) = \left| \begin{array}{ccc} \begin{array}{|cccc|} r_1 - r & & & \\ 1 & r_1 - r & & \\ & \cdot\;\cdot\;\cdot\;\cdot & & \\ & & 1 & r_1 - r \\ \hline \end{array} & & \\ & \begin{array}{|cccc|} \hline r_2 - r & & & \\ 1 & r_2 - r & & \\ & \cdot\;\cdot\;\cdot\;\cdot & & \\ & & 1 & r_2 - r \\ \hline \end{array} & \\ & & \begin{array}{|cccc|} \hline r_m - r & & & \\ 1 & r_m - r & & \\ & \cdot\;\cdot\;\cdot\;\cdot & & \\ & & 1 & r_m - r \\ \hline \end{array} \end{array} \right| \tag{9}$$

and it will be easily seen that it has the elementary divisors $(r - r_1)^{e_1}$, $(r - r_2)^{e_2}$, . . . $(r - r_m)^{e_m}$. Then, by the theorem above referred to, there exists a set of n^2 constants $A_{i,k,l}$, whose determinant is not zero, such that:

$$y_i = \sum_{k=1}^{k=m} \sum_{l=1}^{l=e_k} A_{i,k,l}\, z_{k,l} \text{*} \qquad (i = 1, 2, \ldots n). \tag{10}$$

The system of differential equations (8) we may speak of as the *canonical system;* and now it is easily seen that:

The canonical system of equations admits e_κ *solutions, corresponding to the elementary divisor* $(r - r_\kappa)^{e_\kappa}$, *of the following simple form:*

$$\begin{array}{lll} z_{k,l} = 0 & k \neq \kappa & \\ z_{\kappa,l} = 0 & l < \lambda & (\lambda = 1, 2, \ldots e_\kappa) \\ z_{\kappa,l} = \dfrac{1}{(l-\lambda)!}\, x^{r_\kappa} (\log x)^{l-\lambda} & \lambda \leqq l \leqq e_\kappa & \end{array} \tag{11}$$

and the n *solutions obtained by giving* κ *the values* 1, 2, . . . m *are linearly independent.*

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* Cf. Weierstrass, Werke, Vol. II. pp. 75, 76. The case considered by Weierstrass is very easily reduced to the one we are considering by the change of independent variable $t = \log x$. This reduction of Weierstrass is also given in Muth's Elementartheiler, pp. 195, 198. On page 198 are a number of references to the use of the theory of elementary divisors in the study of differential equations.

The determinant of these n solutions can be written out in such a way that the elements above the principal diagonal are all zero. Thus the value of the determinant is:

$$x^{\sum\limits_{\kappa=1}^{\kappa=m} e_\kappa r_\kappa},$$

and the n solutions are therefore linearly independent.

On account of the relation (10), each solution (11) of the system (8) will determine a solution of the original system (3). Accordingly we have n solutions of (3) which are linearly independent, for their determinant at any point is equal to the determinant of the A's in (10) multiplied into the determinant of the solutions (11) for the same point, and neither of these determinants is zero.

Suppose now we consider any multiple root of the characteristic determinant (5); for simplicity let us take r_1, and suppose that

$$(12) \qquad r_1 = r_2 = \ldots = r_k,$$

so that

$$(13) \qquad e_1 + e_2 + \ldots + e_k$$

is the multiplicity of the root r_1. Then from (10) and (11) we see that, corresponding to this root, there are k solutions of (3) not involving log x and linearly independent:

$$(14) \qquad y_i^{\kappa, e_\kappa} = A_{i, \kappa, e_\kappa}\, x^{r_1} \qquad \begin{pmatrix} i = 1, 2, \ldots n \\ \kappa = 1, 2, \ldots k \end{pmatrix}.$$

Therefore the constants A_{i, κ, e_κ} are linearly independent solutions of the equations (4) when $r = r_1$, as we readily see by putting the values of y_i^{κ, e_κ} in (3). Now the equations (4), in this case, have only k linearly independent solutions, since there are only k elementary divisors corresponding to r_1, and therefore at least one kth minor in (5) is not zero when $r = r_1$. If, then, we have any other solution of (3) of the form:

$$y_i = C_i x^{r_1}, \qquad (i = 1, 2, \ldots n),$$

the constants C_i must be linear combinations of the k sets of constants A_{i, κ, e_κ}. We have then the following result:

The system of equations (3) *admits* n *linearly independent solutions, such that, corresponding to each elementary divisor* $(r - r_\kappa)^{e_\kappa}$ *of the characteristic determinant, there are* e_κ *solutions:*

$$(15)\quad y_i^{\kappa,\lambda} = x^{r_\kappa} \sum_{l=\lambda}^{l=e_\kappa} \frac{1}{(l-\lambda)!} A_{i,\kappa,l} (\log x)^{l-\lambda} \qquad \begin{pmatrix} i = 1, 2, \ldots n \\ \lambda = 1, 2, \ldots e_\kappa \end{pmatrix}.$$

If r_κ *is a multiple root of the characteristic equation which furnishes* s *elementary divisors with the exponents* $e_\kappa, e_{\kappa+1}, \ldots e_{\kappa+s-1}$, *then the constants:*

$$A_{i,\kappa,e_\kappa},\ A_{i,\kappa+1,e_{\kappa+1}}, \ldots A_{i,\kappa+s-1,e_{\kappa+s-1}}, \qquad (i = 1, 2, \ldots n)$$

are s *linearly independent solutions of the equations* (4) *when* $r = r_\kappa$.

§ 2.

SOLUTION OF THE CANONICAL SYSTEM IN THE GENERAL CASE BY SUCCESSIVE APPROXIMATIONS.

We shall now return to the system of equations (1); and here again we shall make use of the linear transformation (10) to reduce the system to the canonical form:

$$(16)\qquad \frac{d}{dx} z_{k,l} = \frac{1}{x} z_{k,l-1} + \frac{r_k}{x} z_{k,l} + \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j} \ *$$

$$(k = 1, 2, \ldots m) \qquad (l = 1, 2, \ldots e_k).$$

The coefficients $b_{k,l}^{i,j}$ are linear functions with constant coefficients of the coefficients $a_{i,j}$ in (1).

We shall now make use of the method of successive approximations to develop solutions of (16) about the point $x = 0$. It will be convenient to write the equations (16) in the form:

$$(17)\qquad \frac{d}{dx} z_{k,l} - \frac{1}{x} z_{k,l-1} - \frac{r_k}{x} z_{k,l} = \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j}.$$

The first approximation will be indicated by a third subscript 0, and is obtained as a solution of the system of equations resulting from (17) by making the right side zero:

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* This reduction is used by Sauvage in the case of a system of equations with analytic coefficients. L. c., pp. 89, 90.

$$(18) \qquad \frac{d}{dx} z_{k,l,0} - \frac{1}{x} z_{k,l-1,0} - \frac{r_k}{x} z_{k,l,0} = 0.$$

We have seen how to solve this system. Any one of the n solutions that we have obtained may be used as the first approximation. This approximation having been chosen, we insert it in the right side of (17) and obtain the following relations for the first correction:

$$\frac{d}{dx} z_{k,l,1} - \frac{1}{x} z_{k,l-1,1} - \frac{r_k}{x} z_{k,l,1} = \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j,0}.$$

The right side is now a known function of x; and we have, consequently, a system of non-homogeneous linear differential equations to solve for $z_{k,l,1}$. Having determined this first correction, it is inserted in the right side of (17), and the resulting equations are solved for the second correction. This process is repeated again and again, the relation connecting the qth and the $(q+1)$th correction being:

$$(19) \qquad \frac{d}{dx} z_{k,l,q+1} - \frac{1}{x} z_{k,l-1,q+1} - \frac{r_k}{x} z_{k,l,q+1} = \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j,q}.$$

Each equation (19) may be written:

$$x^{r_k} \frac{d}{dx} \left(x^{-r_k} z_{k,l,q+1} \right) - \frac{1}{x} z_{k,l-1,q+1} = \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j,q},$$

whence:

$$(20) \qquad z_{k,l,q+1} = x^{r_k} \left[\int_{c_{k,l}}^{x} x^{-1-r_k} z_{k,l-1,q+1}\, dx + \int_{c_{k,l}}^{x} x^{-r_k} \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j,q}\, dx \right].$$

Now writing out the value of $z_{k,l-1,q+1}$ in the same way, and substituting it in the first integral of (20), we have:

$$z_{k,l,q+1} = x^{r_k} \left[\int_{c_{k,l}}^{x} \frac{1}{x} dx \int_{c_{k,l-1}}^{x} x^{-1-r_k} z_{k,l-2,q+1}\, dx \right.$$

$$\left. + \int_{c_{k,l}}^{x} \frac{1}{x} dx \int_{c_{k,l-1}}^{x} x^{-r_k} \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l-1}^{i,j} z_{i,j,q}\, dx + \int_{c_{k,l}}^{x} x^{-r_k} \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j,q}\, dx \right].$$

Now substitute in this result the value of $z_{k,\,l-2,\,q+1}$; in the result thus obtained the value of $z_{k,\,l-3,\,q+1}$, etc. After a certain number of substitutions we have:

$$(21)\quad z_{k,\,l,\,q+1} = x^{r_k}\left[\int_{c_{k,\,l}}^{x}\frac{1}{x}dx \ldots \int_{c_{k,\,L+1}}^{x} x^{-1-r_k} z_{k,\,L,\,q+1}\,dx \right.$$
$$\left. + \sum_{t=1}^{t=l-L}\int_{c_{k,\,l}}^{x}\frac{1}{x}dx \ldots \int_{c_{k,\,l+1-t}}^{x} x^{-r_k}\sum_{i=1}^{i=m}\sum_{j=1}^{j=e_i} b_{k,\,l+1-t}^{i,\,j}\, z_{i,\,j,\,q}\,dx\right],$$

where $L < l$.

When $L = 0$ we have:

$$(22)\quad z_{k,\,l,\,q+1} = x^{r_k}\sum_{t=1}^{t=l}\int_{c_{k,\,l}}^{x}\frac{1}{x}dx \ldots \int_{c_{k,\,l+1-t}}^{x} x^{-r_k}\sum_{i=1}^{i=m}\sum_{j=1}^{j=e_i} b_{k,\,l+1-t}^{i,\,j}\, z_{i,\,j,\,q}\,dx.$$

The lower limits $c_{k,\,l}$ will be determined later to satisfy several conditions.

We may choose at pleasure any one of the m elementary divisors, say $(r - r_\kappa)^{e_\kappa}$, and then select any one of the corresponding e_κ solutions of (18) for the first approximation. We shall take then for the first approximation $z_{k,\,l,\,0}$, the values given in (11) for a particular λ; the integers κ and λ will remain fixed for the solution we are now developing.

For the development of the solutions corresponding to $(r - r_\kappa)^{e_\kappa}$, we shall make the following further assumption as to the coefficients $b_{k,\,l}^{i,\,j}$. Let us examine all the exponents of the elementary divisors $(r - r_k)^{e_k}$, which are such that $R\,r_k = R\,r_\kappa$, where $R\,r_k$ means "real part of r_k," and pick out one exponent, say e_K, that is as great as any one in this special set of exponents; i. e.,

$$(23)\qquad e_K \geqq e_k, \qquad \text{where } R\,r_k = R r_\kappa.$$

The assumption is that the integrals:

$$\int_0^b |b_{k,\,l}^{i,\,j}|\,|\log x|^{e_K - 1}\,dx \qquad \begin{pmatrix} k = 1,\, 2,\, \ldots\, m \\ l = 1,\, 2,\, \ldots\, e_k \end{pmatrix}$$

converge. If in particular r_κ is a simple root, and no multiple root has the same real part as it, then $e_K - 1 = 0$, and this further restriction drops out. Or it might happen that r_κ is a multiple root, but that all the exponents of the set (23) are unity, and in this case the restriction drops out also.

The lower limits of integration in (22) will be determined as follows:

$$(24)\quad \left\{\begin{array}{lll} \text{I.}\quad R\,r_k > R\,r_\kappa & c_{k,l} = c & \\ \text{II.}\quad R\,r_k = R\,r_\kappa & \left\{\begin{array}{l} c_{k,l} = 0 \\ c_{k,l} = c \end{array}\right. & \begin{array}{l} l \leqq L = e_K - e_\kappa + \lambda \\ l > L \end{array} \\ \text{III.}\quad R\,r_k < R\,r_\kappa & c_{k,l} = 0 & \end{array}\right.$$

where c is a constant not zero, which will be determined more closely later (cf. p. 358). It will be proved in § 3, that even in the cases in which the lower limit is zero, the integrals converge.

When all these conditions have been satisfied, we build from the first approximation and the successive corrections the n infinite series:

$$(25)\qquad z_{k,l} = \sum_{q=0}^{q=\infty} z_{k,l,q} \qquad \begin{pmatrix} k = 1, 2, \ldots m \\ l = 1, 2, \ldots e_k \end{pmatrix},$$

which will be proved in § 4 to converge and to form a solution of the system of equations (16).

It will be convenient to consider in place of the functions $z_{k,l,q}$ certain new functions $\phi_{k,l,q}$, which will be defined by the following formulae:

$$(26)\quad \begin{array}{lll} z_{k,l,q} = x^{r_\kappa}\,\phi_{k,l,q} & \left\{\begin{array}{lr} \text{when}\ \ R\,r_k \neq R\,r_\kappa, & \\ \text{or}\ \ R\,r_k = R\,r_\kappa, & k \neq \kappa \text{ and } l \leqq L, \\ \text{or} & k = \kappa \text{ and } l \leqq \lambda, \end{array}\right. \\ z_{k,l,q} = x^{r_\kappa}\,(\log x)^{l-L}\phi_{k,l,q} & R\,r_k = R\,r_\kappa,\quad k \neq \kappa \text{ and } L < l \leqq e_k, \\ z_{\kappa,l,q} = x^{r_\kappa}\,(\log x)^{l-\lambda}\,\phi_{\kappa,l,q} & \lambda < l \leqq e_\kappa. \end{array}$$

For the case of $q = 0$, i. e., the first approximation, the ϕ's are certain constants (cf. (11)); for all other values of q the formulae (26) define them as continuous functions of x so long as x is not zero; and we shall see later (Proof of Convergence, p. 358 et seq.) that each one approaches zero when x approaches zero. We shall therefore define each $\phi_{k,l,q}$, when $q > 0$, as zero for $x = 0$; and with this definition they will be continuous functions of x in the whole interval $0b$.

These ϕ's can be computed from the following recurrent formulae, which are easily obtained from (21), (22), (24), and (26):

I. $R\,r_k > R\,r_\kappa$.

$$(27)\qquad \phi_{k,l,q+1} = x^{r_k - r_\kappa} \sum_{t=1}^{t=l} \underbrace{\int_c^x \frac{1}{x}\,dx \ldots \int_c^x \frac{1}{x}\,dx}_{t \text{ integrations}} \int_c^x x^{-(r_k - r_\kappa)} \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b^{i,j}_{k,l+1-t}\, \phi_{i,j,q}\, (\log x)^{h_{i,j}}\, dx,$$

where $h_{i,j}$ is a positive integer whose value it will not be necessary to write out, noting, however, that:

$$(28)\qquad h_{i,j} \leqq e_\kappa - \lambda.$$

II. $R\,r_k = R\,r_\kappa$.

a) $k \neq \kappa$, $l \leqq L$.

$$(29)\qquad \phi_{k,l,q+1} = x^{r_k - r_\kappa} \sum_{t=1}^{t=l} \underbrace{\int_0^x \frac{1}{x}\,dx \ldots \int_0^x \frac{1}{x}\,dx}_{t \text{ integrations}} \int_0^x x^{-(r_k - r_\kappa)} \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b^{i,j}_{k,l+1-t}\, \phi_{i,j,q}\, (\log x)^{h_{i,j}}\, dx.$$

b) $k \neq \kappa$, $L < l \leqq e_k$.

$$(30)\qquad \phi_{k,l,q+1} = \frac{x^{r_k - r_\kappa}}{(\log x)^{l-L}} \Bigg[\underbrace{\int_c^x \frac{1}{x}\,dx \ldots \int_c^x \frac{1}{x}\,dx \int_c^x \frac{1}{x}}_{(l-L) \text{ integrations}}\, x^{-(r_k - r_\kappa)}\, \phi_{k,L,q+1}\, dx$$
$$+ \sum_{t=1}^{t=l-L} \underbrace{\int_c^x \frac{1}{x}\,dx \ldots \int_c^x \frac{1}{x}\,dx}_{t \text{ integrations}} \int_c^x x^{-(r_k - r_\kappa)} \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b^{i,j}_{k,l+1-t}\, \phi_{i,j,q}\, (\log x)^{h_{i,j}}\, dx \Bigg].$$

c) $k = \kappa$, $l \leqq \lambda$.

$$(31)\qquad \phi_{\kappa,l,q+1} = \sum_{t=1}^{t=l} \underbrace{\int_0^x \frac{1}{x}\,dx \ldots \int_0^x \frac{1}{x}\,dx}_{t \text{ integrations}} \int_0^x \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b^{i,j}_{\kappa,l+1-t}\, \phi_{i,j,q}\, (\log x)^{h_{i,j}}\, dx.$$

d) $k = \kappa, \quad \lambda < l \leqq L.$

$$(32) \quad \phi_{\kappa, l, q+1} = \frac{1}{(\log x)^{l-\lambda}} \sum_{t=1}^{t=l} \underbrace{\int_0^x \frac{1}{x}\, dx \ldots \int_0^x \frac{1}{x}\, dx}_{t \text{ integrations}} \int_0^x \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b^{i,j}_{\kappa, l+1-t}\, \phi_{i,j,q}\, (\log x)^{h_{i,j}}\, dx.$$

e) $k = \kappa, \quad L < l \leqq e_\kappa.$

$$(33) \quad \phi_{\kappa, l, q+1} = \frac{1}{(\log x)^{l-\lambda}} \Bigg[\underbrace{\int_c^x \frac{1}{x}\, dx \ldots \int_c^x \frac{1}{x}\, dx}_{(l-L) \text{ integrations}} \int_c^x \frac{1}{x}\, \phi_{\kappa, L, q+1}\, (\log x)^{e_K - e_\kappa}\, dx$$

$$+ \sum_{t=1}^{t=l-L} \underbrace{\int_c^x \frac{1}{x}\, dx \ldots \int_c^x \frac{1}{x}\, dx}_{t \text{ integrations}} \int_c^x \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b^{i,j}_{\kappa, l+1-t}\, \phi_{i,j,q}\, (\log x)^{h_{i,j}}\, dx \Bigg].$$

III. $R\, r_k < R\, r_\kappa.$

Here we have the formula (29) again.

In § 4 we shall consider the n series:

$$(34) \qquad \phi_{k,l} = \sum_{q=0}^{q=\infty} \phi_{k,l,q} \qquad \begin{pmatrix} k = 1, 2, \ldots m \\ l = 1, 2, \ldots e_k \end{pmatrix},$$

which are such that if we multiply each by its proper factor $x^{r_\kappa} (\log x)^{h_{k,l}}$, where $h_{k,l}$ is given in (26), we obtain the n series (25). We thus reduce the proof of convergence of (25) to the question of the convergence of (34), and this last question will be settled by reference to certain formulae to be established in the next section.

§ 3.

LEMMAS CONCERNING MULTIPLE INTEGRALS.

We now prove a number of lemmas, which will be useful in the proof of convergence, and which also verify the statement that we have made that the integrals of the last section, in which the lower limit is zero, converge.

LEMMA I. *If b is a function of x, continuous in the interval $0 < x \leqq c$, and such that $|b|\,|\log x|^{t-1}$, (t an integer $\geqq 1$) is in-*

tegrable up to $x = 0$, then b can be integrated t times from $x = 0$ as follows:

$$(35) \qquad f_t(x) = \underbrace{\int_0^x \frac{1}{x}\,dx \ldots \int_0^x \frac{1}{x}\,dx}_{t \text{ integrations}} \int_0^x b\,dx.$$

Lemma II. *If the conditions of Lemma I. hold, then:*

$$(36) \qquad |f_t(x)| = \Bigg|\underbrace{\int_0^x \frac{1}{x}\,dx \ldots \int_0^x \frac{1}{x}\,dx}_{t \text{ integrations}} \int_0^x b\,dx\Bigg| \leqq \int_0^x |b|\,|\log x|^{t-1}\,dx.$$

$$(0 < x \leqq 1).$$

We will prove these lemmas by mathematical induction. They are true when $t = 1$. Let us assume that they are true for a particular value of t, say $t = t_1$.

Let X be any particular value of x in the interval $0 < x \leqq 1$, and choose ϵ at pleasure such that $0 < \epsilon < X$. Then we have:

$$\Bigg|\int_\epsilon^X \frac{1}{x} f_{t_1}(x)\,dx\Bigg| \leqq \int_\epsilon^X \frac{1}{x}|f_{t_1}(x)|\,dx \leqq \int_\epsilon^X \frac{1}{x}\,dx \int_0^x |b|\,|\log x|^{t_1-1}\,dx$$

$$= \log X \int_0^X |b|\,|\log x|^{t_1-1}\,dx - \log\epsilon \int_0^\epsilon |b|\,|\log x|^{t_1-1}\,dx$$

$$- \int_\epsilon^X \log x\,|b|\,|\log x|^{t_1-1}\,dx$$

$$= |\log\epsilon| \int_0^\epsilon |b|\,|\log x|^{t_1-1}\,dx + \int_\epsilon^X |b|\,|\log x|^{t_1}\,dx$$

$$- |\log X| \int_0^X |b|\,|\log x|^{t_1-1}\,dx$$

$$\leqq |\log\epsilon| \int_0^\epsilon |b|\,|\log x|^{t_1-1}\,dx + \int_\epsilon^X |b|\,|\log x|^{t_1}\,dx$$

$$\leqq \int_0^\epsilon |b|\,|\log x|^{t_1}\,dx + \int_\epsilon^X |b|\,|\log x|^{t_1}\,dx = \int_0^X |b|\,|\log x|^{t_1}\,dx.$$

Therefore, when ϵ approaches zero, the integral on the left side of all these inequalities converges, and we have:

$$|f_{t_1+1}(X)| = \left|\int_0^X \frac{1}{x} f_{t_1}(x)\,dx\right| \leqq \int_0^X |b|\,|\log x|^{t_1}\,dx.$$

Thus the two lemmas are proved when $x \leqq 1$; and it is easily seen how to conclude the proof of I. in case $c > 1$.

LEMMA III. *If b is a continuous function of x in the interval $0 < x \leqq c$, and its absolute value is integrable up to $x = 0$, and if:*

$$F_t(x) = \underbrace{\int_0^x \frac{1}{x}\,dx \ldots \int_0^x \frac{1}{x}\,dx}_{t \text{ integrations}} \int_0^x x^r\, b\,dx$$

where r is real and greater than zero, then:

$$(37) \qquad |x^{-r} F_t(x)| \leqq \frac{1}{r^{t-1}} \int_0^x |b|\,dx.$$

When $t = 1$ we have:

$$|x^{-r} F_1(x)| = x^{-r}\left|\int_0^x x^r\, b\,dx\right| \leqq x^{-r} \int_0^x x^r\,|b|\,dx \leqq \int_0^x |b|\,dx,$$

and so in this case III. is true. Assume that it is true for $t = t_1$; then it is also true for $t = t_1 + 1$. For:

$$|x^{-r} F_{t_1+1}(x)| = x^{-r}\left|\int_0^x \frac{1}{x} F_{t_1}(x)\,dx\right| \leqq x^{-r} \int_0^x \frac{1}{x} |F_{t_1}(x)|\,dx$$

$$\leqq x^{-r} \int_0^x \frac{1}{x}\left[\frac{x^r}{r^{t_1-1}} \int_0^x |b|\,dx\right] dx$$

$$\leqq \frac{x^{-r}}{r^{t_1-1}}\left[\int_0^x x^{-1+r}\,dx\right]\left[\int_0^x |b|\,dx\right] = \frac{1}{r^{t_1}} \int_0^x |b|\,dx.$$

Therefore III. is true for all values of t.

LEMMA IV. *If b is a continuous function of x in the interval $0 < x \leqq c$, and if:*

$$g_t(x) = \underbrace{\int_c^x \frac{1}{x}\, dx \ldots \int_c^x \frac{1}{x}\, dx \int_c^x x^{-r}\, b\, dx}_{t \text{ integrations}}$$

where r is real and greater than zero, then:

$$(38) \qquad | x^r g_t(x) | \leqq \frac{1}{r^{t-1}} \int_c^x | b | \, | dx | \qquad (0 < x \leqq c).$$

When $t = 1$ we have:

$$| x^r g_1(x) | \leqq x^r \int_c^x x^{-r} | b | \, | dx | \leqq \int_c^x | b | \, | dx | .$$

Assume that IV. is true for $t = t_1$. Then:

$$| x^r g_{t_1+1}(x) | \leqq x^r \int_c^x \frac{1}{x} | g_{t_1}(x) | \, | dx | \leqq \frac{x^r}{r^{t_1-1}} \int_c^x x^{-1-r} \left[\int_c^x | b | \, | dx | \right] |dx|$$

$$\leqq \frac{x^r}{r^{t_1-1}} \left[\int_c^x x^{-1-r} | dx | \right] \left[\int_c^x | b | \, | dx | \right] = \frac{1}{r^{t_1}} \left[1 - \left(\frac{x}{c} \right)^r \right] \int_c^x | b | \, | dx |$$

$$\leqq \frac{1}{r^{t_1}} \int_c^x | b | \, | dx | .$$

Therefore IV. is true for $t = t_1 + 1$, and the lemma is proved.

LEMMA V. *If b, r, and g_t are defined as in IV. and it is further assumed that the absolute value of b is integrable up to $x = 0$, then:*

$$(39) \qquad \lim_{x=0} x^r g_t(x) = 0.$$

To prove this, let us choose a constant s such that $0 < s < r$. Then:

$$| x^r g_t(x) | = x^s \left| x^{r-s} \int_c^x \frac{1}{x}\, dx \ldots \int_c^x \frac{1}{x}\, dx \int_c^x x^{-(r-s)} x^{-s}\, b\, dx \right|$$

$$\leqq \frac{x^s}{(r-s)^{t-1}} \int_c^x x^{-s} | b | \, | dx | ,$$

as we see from IV. by replacing r by $r - s$ and b by $x^{-s} b$.

If ϵ is a positive number chosen arbitrarily small, we can choose η so near 0 that:

$$\int_0^\eta |b|\,|dx| \leqq \frac{\epsilon}{2},$$

and then $X \leqq \eta$ so that:

$$X^s \int_c^\eta x^{-s}|b|\,|dx| \leqq \frac{\epsilon}{2}.$$

Then:

$$\begin{aligned} x^s\int_c^x x^{-s}|b|\,|dx| &= x^s\int_c^\eta x^{-s}|b|\,|dx| + x^s\int_\eta^x x^{-s}|b|\,|dx| \\ &\leqq X^s\int_c^\eta x^{-s}|b|\,|dx| + \int_\eta^x |b|\,|dx| \qquad (0 < x \leqq X \leqq \eta) \\ &\leqq \frac{\epsilon}{2} + \int_\eta^0 |b|\,|dx| \leqq \epsilon. \end{aligned}$$

Therefore:

$$|x^r g_t(x)| \leqq \frac{\epsilon}{(r-s)^{t-1}} \qquad (0 < x \leqq X)$$

and V. is proved.

LEMMA VI. *If* β *is a function of* x *continuous in the interval* $0 < x \leqq c$, *and*

(40) $$\lim_{x=0} \beta = 0;$$

and if:

$$G_t(x) = \underbrace{\int_c^x \frac{1}{x}dx \ldots \int_c^x \frac{1}{x}\beta\, dx}_{t \text{ integrations}}$$

then:

(41) $$\lim_{x=0} \frac{1}{(\log x)^t}\, G_t(x) = 0.$$

When $t = 0$ it is obvious that (41) is true, for then:

$$\lim_{x=0} G_0(x) = \lim_{x=0} \beta = 0.$$

Assume now that (41) is true when $t = t_1$; then it will also be true for $t = t_1 + 1$. For if ϵ is a positive number chosen arbitrarily small, we can choose η so near 0 that:

$$\left|\frac{1}{(\log x)^{t_1}}\, G_{t_1}(x)\right| \leqq \frac{\epsilon}{2}(t_1+1) \qquad (0 < x \leqq \eta < 1)$$

$$G_{t_1+1}(x) = \int_c^x \frac{1}{x} G_{t_1}(x)\,dx = \int_c^\eta \frac{1}{x} G_{t_1}(x)\,dx + \int_\eta^x \frac{1}{x}\, G_{t_1}(x)\,dx$$

$$\left|\int_\eta^x \frac{1}{x}\, G_{t_1}(x)\,dx\right| \leqq \int_\eta^x \frac{1}{x}\,|\,G_{t_1}(x)\,|\;|dx| \leqq \frac{\epsilon}{2}(t_1+1)\int_\eta^x \frac{|\log x|^{t_1}}{x}\,|dx| \qquad (0 < x \leqq \eta)$$

$$\leqq \frac{\epsilon}{2}(t_1+1)\left|\frac{(\log x)^{t_1+1} - (\log \eta)^{t_1+1}}{t_1+1}\right|$$

$$= \frac{\epsilon}{2}\,|\log x|^{t_1+1}\left[1 - \left(\frac{\log \eta}{\log x}\right)^{t_1+1}\right] \leqq \frac{\epsilon}{2}\,|\log x|^{t_1+1}.$$

Now choose X in the interval $0 < x \leqq \eta$ so that:

$$\left|\frac{1}{(\log x)^{t_1+1}}\int_c^\eta \frac{1}{x}\, G_{t_1}(x)\,dx\right| \leqq \frac{\epsilon}{2} \qquad (0 < x \leqq X).$$

Then:

$$\left|\frac{1}{(\log x)^{t_1+1}}\, G_{t_1+1}(x)\right|$$

$$\leqq \left|\frac{1}{(\log x)^{t_1+1}}\int_c^\eta \frac{1}{x} G_{t_1}(x)\,dx\right| + \left|\frac{1}{(\log x)^{t_1+1}}\int_\eta^x \frac{1}{x} G_{t_1}(x)\,dx\right| \leqq \epsilon,$$

$$(0 < x \leqq X),$$

and:

$$\operatorname*{limit}_{x=0} \frac{1}{(\log x)^{t_1+1}}\, G_{t_1+1}(x) = 0.$$

Therefore VI. is true for all values of t.

LEMMA VII. *If β is a continuous function of x in the interval $0 < x \leqq c \leqq 1$, and such that it is not greater in absolute value than the constant N, and if G_t is defined as in VI, then:*

$$(42) \qquad |\,G_t(x)\,| = \left|\underbrace{\int_c^x \frac{1}{x}\,dx \ldots \int_c^x}_{t \text{ integrations}} \frac{1}{x}\,\beta\,dx\right| \leqq N\,|\log x|^t.$$

For:

$$|\,G_t(x)\,| \leqq \int_c^x \frac{1}{x}\,|dx| \ldots \int_c^x \frac{1}{x} N\,|\,dx\,| \leqq N\int_c^x \frac{1}{x}\,|\,dx\,| \ldots \int_c^x \frac{1}{x}\,|\,dx\,|$$

$$\leqq N\left[\int_c^x \frac{1}{x}\,|\,dx\,|\right]^t \leqq N\,|\log x\,|^t\left[1 - \frac{\log c}{\log x}\right]^t \leqq N\,|\log x\,|^t.$$

§ 4.

Proof of Convergence.

The convergence of the series (34) will now be proved; and in this proof it will also be shown that:

$$(43) \qquad \lim_{x=0} \phi_{k,l,q} = 0 \qquad\qquad q > 0.$$

We shall use in the proof the following functions. A function of x, B, is chosen having the same properties of integrability as $b_{k,l}^{i,j}$, and such that:

$$(44) \qquad B \geqq \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} |b_{k,l}^{i,j}| \qquad \begin{pmatrix} k = 1, 2, \ldots m \\ l = 1, 2, \ldots e_k \end{pmatrix}.$$ [*]

Next consider all the differences $R r_k - R r_\kappa$ which are not zero, and choose a positive constant d such that:

$$0 < d \leqq |R r_k - R r_\kappa|.$$

Then C shall be a positive constant such that:

$$(45) \qquad C \geqq n, \quad C \geqq \sum_{t=1}^{t=n} \frac{1}{d^{t-1}}.$$

We also define:

$$(46) \qquad M(x) = \int_0^x C B \,|\log x|^{e_\kappa - 1} dx.$$

The point c is chosen so that:

$$(47) \qquad 0 < c \leqq b, \quad c \leqq \frac{1}{e} \text{ (where } \log e = 1), \quad M(c) < 1.$$

This is the final determination of c to which we have referred on page 350; and this point c will, from now on, mark one end of the interval for x. Instead of $M(c)$, we shall, for the sake of brevity, write simply M.

The convergence of the series (34) will be proved by showing by mathematical induction that the following inequalities hold for all values of q:

---

[*] We might, for instance, take $B = \sum_{k=1}^{k=m} \sum_{l=1}^{l=e_k} \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} |b_{k,l}^{i,j}|$.

$$(48) \qquad |\phi_{k,l,q}| \leqq M^q \qquad \begin{pmatrix} k = 1, 2, \ldots m \\ l = 1, 2, \ldots e_k \end{pmatrix}.$$

For $q = 0$ they are obviously true. Assuming them true for a special value of q, $q = q_1$, we will consider the cases outlined in (24) in turn.

I. From (27) we have:

$$(49) \quad |\phi_{k,l,q_1+1}| \leqq$$

$$x^{R(r_k - r_\kappa)} \sum_{t=1}^{t=l} \underbrace{\int_c^x \frac{1}{x} |dx| \ldots \int_c^x \frac{1}{x} |dx|}_{t \text{ integrations}} \int_c^x x^{-R(r_k - r_\kappa)} B M^{q_1} |\log x|^{e_\kappa - \lambda} |dx|$$

after replacing

$$|\phi_{i,j,q_1}|, \quad |\log x|^{h_{i,j}}, \quad \sum \sum |b_{k,l}^{i,j}|$$

respectively by the greater values

$$M^{q_1}, \quad |\log x|^{e_\kappa - \lambda}, \quad B$$

(Cf. (28), (30), (32)).

$$|\phi_{k,l,q_1+1}| \leqq M^{q_1} \sum_{t=1}^{t=l} \left[\frac{1}{R(r_k - r_\kappa)}\right]^{t-1} \int_c^x B |\log x|^{e_\kappa - \lambda} |dx| \text{ (Lemma IV.)}$$

$$\leqq M^{q_1} C \int_c^x B |\log x|^{e_\kappa - \lambda} |dx| \leqq M^{q_1} \int_c^x C B |\log x|^{e_\kappa - 1} |dx|$$

$$\leqq M^{q_1} M(x) \leqq M^{q_1 + 1}.$$

From Lemma V. it follows that the limit of the right side of (49) is zero, when x approaches zero; for we have assumed that $B |\log x|^{e_\kappa - 1}$ is integrable up to $x = 0$ (cf. page 358), and therefore $B |\log x|^{e_\kappa - \lambda}$ must also be integrable up to $x = 0$. So (43) is verified for $q = q_1 + 1$.

II. From (29) we have for case (a):

$$(50) \quad |\phi_{k,l,q_1+1}| \leqq \sum_{t=1}^{t=l} \underbrace{\int_0^x \frac{1}{x} dx \ldots \int_0^x \frac{1}{x} dx}_{t \text{ integrations}} \int_0^x B M^{q_1} |\log x|^{e_\kappa - \lambda} dx$$

$$\leqq M^{q_1} \sum_{t=1}^{t=l} \int_0^x B |\log x|^{e_\kappa - \lambda + t - 1} dx \qquad \text{(Lemma II.)}$$

$$\leqq M^{q_1} \sum_{t=1}^{t=l} \int_0^x B |\log x|^{e_\kappa - 1} dx$$

since the greatest value of $e_\kappa - \lambda + t - 1$ is

$$e_\kappa - \lambda + L - 1 = e_\kappa - \lambda + (e_K - e_\kappa + \lambda) - 1 = e_K - 1.$$

$$(51) \qquad |\phi_{k,l,q_1+1}| \leqq M^{q_1} L \int_0^x B\,|\log x|^{e_K - 1}\,dx = M^{q_1} \frac{L}{C} M(x)$$

$$\leqq M^{q_1} M(x) \leqq M^{q_1+1}.$$

It is obvious that the above result holds also for II. (*c*) (31). From (32) we shall obtain in the same way for II. (*d*):

$$(52) \qquad |\log x|^{l-\lambda}\,|\phi_{\kappa,l,q_1+1}| \leqq M^{q_1} L \int_0^x B\,|\log x|^{e_K-1} dx = M^{q_1} \frac{L}{C} M(x)$$

$$|\phi_{\kappa,l,q_1+1}| \leqq \frac{M^{q_1}}{|\log x|^{l-\lambda}} \frac{L}{C} M(x) \leqq M^{q_1} M(x) \leqq M^{q_1+1}.$$

In all three sub-cases (*a*), (*c*), and (*d*), it is easily seen that (43) is true for $q = q_1 + 1$. We have now left of case II. the sub-cases (*b*) and (*e*).

From (30) we have for case (*b*):

$$(53)$$

$$|\phi_{k,l,q_1+1}| \leqq \frac{1}{|\log x|^{l-L}} \Bigg[\underbrace{\int_c^x \frac{1}{x}\,|dx| \ldots \int_c^x \frac{1}{x}\,|dx|}_{(l-L) \text{ integrations}} \int_c^x \frac{1}{x} M^{q_1} \frac{L}{C} M(x)\,|dx|$$

$$+ \sum_{t=1}^{t=l-L} \underbrace{\int_c^x \frac{1}{x}\,|dx| \ldots \int_c^x \frac{1}{x}\,|dx|}_{t \text{ integrations}} \int_c^x B\,M^{q_1}\,|\log x|^{e_\kappa - \lambda}\,|dx| \Bigg].$$

In the first part of the bracket we have replaced $|\phi_{k,L,q_1+1}|$ by $M^{q_1} \frac{L}{C} M(x)$, using inequality (51).

We will consider the two parts of (53) separately.

$$(54)$$

$$\frac{1}{|\log x|^{l-L}} \underbrace{\int_c^x \frac{1}{x}\,|dx| \ldots \int_c^x \frac{1}{x}\,|dx|}_{(l-L) \text{ integrations}} \int_c^x \frac{1}{x} M^{q_1} \frac{L}{C} M(x)\,|dx| \leqq M^{q_1} \frac{L}{C} M(c)$$

$$= M^{q_1+1} \frac{L}{C} \qquad \text{(Lemma VII.)}$$

$$(55)\quad \frac{1}{|\log x|^{l-L}} \sum_{t=1}^{t=l-L} \underbrace{\int_c^x \frac{1}{x}|\,dx\,| \ldots \int_c^x \frac{1}{x}|\,dx\,|}_{t \text{ integrations}} \int_c^x B\, M^{q_1}\,|\log x|^{e_\kappa-\lambda}\,|\,dx$$

$$\leqq \sum_{t=1}^{t=l\ \ L} |\log x|^{t-1}\left[\frac{M^{q_1}}{\log x|^{l-L}} \int_0^c B\,|\log x|^{e_\kappa-\lambda}\,dx\right] \qquad \text{(Lemma VII.)}$$

$$\leqq M^{q_1} \sum_{t=1}^{t=l-L} |\log x|^{t-1-(l-L)} \int_0^c B\,|\log x|^{e_\kappa-\lambda}\,dx$$

$$\leqq M^{q_1}\frac{(l-L)}{|\log x|} \int_0^c B\,|\log x|^{e_K-1}\,dx \leqq M^{q_1+1}\frac{(l-L)}{C}.$$

Therefore :

$$|\,\phi_{k,\,l,\,q_1+1}\,| \leqq M^{q_1+1}\left[\frac{L}{C}+\frac{l-L}{C}\right] = M^{q_1+1}\frac{l}{C} \leqq M^{q_1+1}$$

The limit of the left side of (54) for $x = 0$ is zero by Lemma VI., while the same thing is true of the left side of (55) from the inequalities. Therefore (43) is true in this case for $q = q_1 + 1$.

From (33) we have for case (e):

(56)

$$|\,\phi_{\kappa,\,l,\,q_1+1}\,| \leqq \frac{1}{|\log x|^{l-\lambda}}\left[\underbrace{\int_c^x \frac{1}{x}|\,dx\,| \ldots \int_c^x \frac{1}{x}|\,dx\,|}_{(l-L) \text{ integrations}} \int_c^x \frac{1}{x} M^{q_1}\frac{L}{C}M(x)\,|\,dx\,|\right.$$

$$\left. + \sum_{t=1}^{t=l-L} \underbrace{\int_c^x \frac{1}{x}|\,dx\,| \ldots \int_c^x \frac{1}{x}|\,dx\,|}_{t \text{ integrations}} \int_c^x B\, M^{q_1}\,|\log x|^{e_\kappa-\lambda}\,|\,dx\,|\right].$$

In the first part of the bracket we have used the inequality obtained from (52):

$$|\log x|^{e_K-e_\kappa}\,|\,\phi_{\kappa,\,L,\,q_1+1}| \leqq M^{q_1}\frac{L}{C}M(x).$$

Now the only difference between the inequalities (53) and (56) is in the power of $|\log x|$ outside the brackets; and, since $|\log x|^{l-\lambda} \geqq |\log x|^{l-L}$, all the results that we have obtained from (53) will follow also from (56). Then for all the sub-cases of II., (43) and (48) are true for $q = q_1 + 1$.

III. Here we have:

$$|\phi_{k,l,q_1+1}|$$

$$\leqq x^{-R(r_\kappa - r_k)} \sum_{t=1}^{t=l} \underbrace{\int_0^x \frac{1}{x}\,dx \ldots \int_0^x \frac{1}{x}\,dx}_{t \text{ integrations}} \int_0^x x^{R(r_\kappa - r_k)}\, B\, M^{q_1} |\log x|^{e_\kappa - \lambda}\, dx$$

$$\leqq M^{q_1} \sum_{t=1}^{t=l} \left[\frac{1}{R(r_\kappa - r_k)}\right]^{t-1} \int_0^x B\,|\log x|^{e_\kappa - \lambda}\, dx \qquad \text{(Lemma III.)}$$

$$\leqq M^{q_1} \int_0^x C\,B\,|\log x|^{e_K - 1}\, dx \leqq M^{q_1+1}.$$

It is obvious that (43) is true in this case for $q = q_1 + 1$.

This completes the proof of the inequalities (48).

From (48) we now infer at once *that* $\phi_{k,l,q}$ $(q > 0)$ *approaches zero at the point* $x = 0$.

The inequalities (48) furnish the sufficient condition of Weierstrass for the absolute and uniform convergence of the series (34) in the interval $0 \leqq x \leqq c$. Accordingly the series (25) converges absolutely and uniformly in any sub-interval $0 < \epsilon \leqq x \leqq c$.

It remains now to show that we have a solution in the system of functions $z_{k,l}$. For this purpose let us select any sub-interval $0 < \epsilon \leqq x \leqq c$. Since the point $x = 0$ is excluded, the coefficients $b_{k,l}^{i,j}$ are continuous in this interval; and in it the system of differential equations (16) is satisfied by the functions $z_{k,l}$. For if we multiply the series (25) for $z_{i,j}$ by $b_{k,l}^{i,j}$ we obtain the absolutely and uniformly convergent series:

$$b_{k,l}^{i,j} z_{i,j} = \sum_{q=0}^{q=\infty} b_{k,l}^{i,j} z_{i,j,q}\,.$$

Taking the sum of such series for all values of i and j we have:

$$(57) \qquad \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j} = \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} \sum_{q=0}^{q=\infty} b_{k,l}^{i,j} z_{i,j,q}$$

$$= \sum_{q=0}^{q=\infty} \left[\sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j,q}\right].$$

where the last transformation is valid since we are dealing with absolutely convergent series. Also:

$$\frac{1}{x} z_{k,l-1} = \sum_{q=0}^{q=\infty} \frac{1}{x} z_{k,l-1,q}, \tag{58}$$

$$\frac{r_k}{x} z_{k,l} = \sum_{q=0}^{q=\infty} \frac{r_k}{x} z_{k,l,q}. \tag{59}$$

Adding (57), (58), (59), and changing slightly the summation on the right, we have:

$$\begin{aligned} \frac{1}{x} z_{k,l-1} + \frac{r_k}{x} z_{k,l} + \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j} \\ = \frac{1}{x} z_{k,l-1,0} + \frac{r_k}{x} z_{k,l,0} + \sum_{q=0}^{q=\infty} \Bigg[\frac{1}{x} z_{k,l-1,q+1} + \frac{r_k}{x} z_{k,l,q+1} \\ + \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j,q} \Bigg]. \end{aligned} \tag{60}$$

The series on the right is an absolutely and uniformly convergent series of continuous functions. If we replace the terms by their values given in (18) and (19), we can also write the series in the form:

$$\frac{d}{dx} z_{k,l,0} + \sum_{q=0}^{q=\infty} \frac{d}{dx} z_{k,l,q+1} = \sum_{q=0}^{q=\infty} \frac{d}{dx} z_{k,l,q}, \tag{61}$$

which is the series of derivatives of the terms of (25). From this it follows that, if we differentiate the series for $z_{k,l}$ term by term, we shall obtain an absolutely and uniformly convergent series of continuous functions; and therefore $z_{k,l}$ has a continuous derivative at each point of our sub-interval, which is precisely (61), and this, as we have seen, is the same as the right side of (60). We can therefore write (60) in the form

$$\frac{d}{dx} z_{k,l} = \frac{1}{x} z_{k,l-1} + \frac{r_k}{x} z_{k,l} + \sum_{i=1}^{i=m} \sum_{j=1}^{j=e_i} b_{k,l}^{i,j} z_{i,j};$$

and now giving k and l all possible values we have precisely the system of equations (16). We have, then, the following result:

In any sub-interval of $0\,c$, *not including the point* $x = 0$, *the functions* $z_{k,l}$ *represented by the series* (25) *are continuous in* x, *have continuous* *first derivatives with regard to* x, *and satisfy the canonical system of equations at every point. The terms of the series for* $z_{k,l}$ *are given by the formulae* (26), *in which the functions* $\phi_{k,l,q}$ *are continuous in* x *throughout the whole of the interval* $0\,b$, *and reduce to zero when* $x = 0$, *except when* $q = 0$. *The functions* $\phi_{k,l,0}$ *are constants, zero in all cases but the following:*

$$\phi_{\kappa,l,0} = \frac{1}{(l-\lambda)!} \qquad \lambda \leqq l \leqq e_\kappa .$$

§ 5.

LINEAR INDEPENDENCE OF THE SOLUTIONS OF THE CANONICAL SYSTEM.

We have shown that corresponding to each elementary divisor $(r - r_\kappa)^{e_\kappa}$, there are e_κ solutions of (16) obtained by giving λ the values $1, 2, \ldots e_\kappa$; and for the development of these solutions we have required that $|\, b_{k,l}^{i,j} \,|\; |\log\; x\,|^{e_K - 1}$ shall be integrable up to $x = 0$, where e_K is determined for the root r_κ by the condition (23). We have, then, n solutions which may be written as follows:

$$z_{k,l}^{\kappa,\lambda} = x^{r_\kappa}\,\phi_{k,l}^{\kappa,\lambda} \qquad \left\{\begin{array}{lll} \text{when} & R\,r_k \neq R\,r_\kappa & \\ \text{or} & R\,r_k = R\,r_\kappa , & k \neq \kappa \text{ and } l \leqq L, \\ \text{or} & & k = \kappa \text{ and } l \leqq \lambda, \end{array}\right.$$

$$(62) \quad z_{k,l}^{\kappa,\lambda} = x^{r_\kappa}\,(\log\, x)^{l-L}\,\phi_{k,l}^{\kappa,\lambda} \quad R\,r_k = R\,r_\kappa, \quad k \neq \kappa \text{ and } L < l \leqq e_k ,$$

$$z_{\kappa,l}^{\kappa,\lambda} = x^{r_\kappa}\,(\log\, x)^{l-\lambda}\,\phi_{\kappa,l}^{\kappa,\lambda} \qquad \lambda < l \leqq e_\kappa .$$

where the functions $\phi_{k,l}^{\kappa,\lambda}$ are continuous in x and

$$(63) \quad \begin{array}{ll} \phi_{k,l}^{\kappa,\lambda}\Big|_{x=0} = 0 & \left\{\begin{array}{l} \quad k \neq \kappa \\ \text{or } k = \kappa, \text{ and } l < \lambda, \end{array}\right. \\ \phi_{\kappa,l}^{\kappa,\lambda}\Big|_{x=0} = \dfrac{1}{(l-\lambda)!} & \qquad\qquad \lambda \leqq l \leqq e, \end{array}$$

$$(\kappa = 1, 2, \ldots m) \qquad\qquad (\lambda = 1, 2, \ldots e_\kappa).$$

It is worth while to note three facts in regard to the z's, which will be useful later on:

$$(64) \quad \begin{array}{rl} \text{I.} & z_{k,l}^{\kappa,\lambda} \text{ does not involve } \log x \text{ explicitly whenever } \lambda \geqq l ; \\ \text{II.} & \underset{x=0}{\text{limit}}\; x^{-r_i}\, z_{k,l}^{\kappa,\lambda} = 0 \text{ when } R\,r_\kappa > R\,r_i ; \\ \text{III.} & \underset{x=0}{\text{limit}}\; x^{-r_\kappa}\,(\log\, x)^{-(e_\kappa-\lambda)}\, z_{k,l}^{\kappa,\lambda} = 0 \text{ in all cases except the} \\ & \qquad \text{one, } k = \kappa \text{ and } l = e_\kappa ; \text{ and then the limit is } \dfrac{1}{(e_\kappa - \lambda)!} . \end{array}$$

We will now show that the n solutions (62) are linearly independent. Suppose they were not and that there were n constants $C_{\kappa,\lambda}$ not all zero such that:

$$(65) \qquad \sum_{\kappa=1}^{\kappa=m}\sum_{\lambda=1}^{\lambda=e_\kappa} C_{\kappa,\lambda}\, z_{k,l}^{\kappa,\lambda} = 0 \qquad \begin{pmatrix} k = 1, 2, \ldots m \\ l = 1, 2, \ldots e_k \end{pmatrix}.$$

It will be convenient to suppose that our notation is such that:

$$(66) \qquad R\, r_1 \leqq R\, r_2 \leqq \ldots \leqq R\, r_m .$$

Consider first those equations of (65) for which $R\,r_k = R\,r_1$. We have:

$$(67) \qquad \lim_{x=0} \sum_{\kappa=1}^{\kappa=m}\sum_{\lambda=1}^{\lambda=e_\kappa} C_{\kappa,\lambda}\, x^{-r_k} z_{k,l}^{\kappa,\lambda} = 0 \qquad \begin{matrix} R\,r_k = R\,r_1 \\ (l = 1, 2, \ldots e_k). \end{matrix}$$

Now let $l = 1$ in (67), and consider the limit of each term for any given value, within the range indicated, for k. For each term in which $R\,r_\kappa > R\,r_k = R\,r_1$ the limit is zero by II. We have left, now, only the terms:

$$(68) \qquad C_{\kappa,\lambda}\, x^{-r_k} z_{k,1}^{\kappa,\lambda} \qquad R\,r_\kappa = R\,r_k = R\,r_1 .$$

According to I. no logarithms appear explicitly in (68), and we can write:

$$(69) \qquad C_{\kappa,\lambda}\, x^{-r_k} z_{k,1}^{\kappa,\lambda} = C_{\kappa,\lambda}\, x^{r_\kappa - r_k} \phi_{k,1}^{\kappa,\lambda}.$$

Now by (63) the limit of all such terms is zero except in the one case $\kappa = k$ and $\lambda = 1$, and for this term we have:

$$(70) \qquad \lim_{x=0} C_{k,1}\, x^{-r_k} z_{k,1}^{k,1} = \lim_{x=0} C_{k,1}\, \phi_{k,1}^{k,1} = C_{k,1} .$$

So in the case of $l = 1$ the limit (67) turns out to be $C_{k,1}$ when we evaluate the limit term by term. Now this is impossible unless $C_{k,1} = 0$; and so we must have, writing now κ in place of k:

$$(71) \qquad C_{\kappa,1} = 0 \qquad R\,r_\kappa = R\,r_1 .$$

Now consider in the same way the cases of (67) in which $l = 2$; and choose any one of the values of k indicated. Here again by II. the limit of each term is zero when $R\,r_\kappa > R\,r_k$; and we have left the terms:

$$(72) \qquad C_{\kappa,\lambda}\, x^{-r_k} z_{k,2}^{\kappa,\lambda} \qquad \begin{matrix} R\,r_\kappa = R\,r_k = R\,r_1 \\ \lambda \geqq 2. \end{matrix}$$

Now by I. no logarithms appear explicitly in the terms (72), for the only case in which they could occur would be for $\lambda = 1$, but by (71) such terms do not appear. We can then write each term of (72)

$$73) \qquad C_{\kappa,\lambda}\, x^{-r_k}\, z_{k,2}^{\kappa,\lambda} = C_{\kappa,\lambda}\, x^{r_\kappa - r_k}\, \phi_{k,2}^{\kappa,\lambda}\,;$$

and by (63) the limit of each is zero except in the one case:

$$(74) \qquad \operatorname*{limit}_{x=0}\, C_{k,2}\, x^{-r_k}\, z_{k,2}^{k,2} = \operatorname*{limit}_{x=0}\, C_{k,2}\, \phi_{k,2}^{k,2} = C_{k,2}\,.$$

We have then, reasoning as before, and including the previous result,

$$(75) \qquad C_{\kappa,1} = C_{\kappa,2} = 0 \qquad\qquad R\, r_\kappa = R\, r_1.$$

Now the same reasoning can be applied until we have finally

$$(76) \qquad C_{\kappa,\lambda} = 0\,; \qquad\qquad \begin{matrix} R\, r_\kappa = R\, r_1\,, \\ \lambda \geqq 1 \end{matrix}$$

and there are left in (65) only those solutions for which $R\, r_\kappa > R\, r_1$. Having thus disposed of all the r's in (66) such that $R\, r_\kappa = R\, r_1$, we consider the next set of r's whose real parts are all equal and as small as any other in the new set of r's, and show, in exactly the same way, that the corresponding constants, $C_{\kappa,\lambda}$ are zero. Continuing in this way, we finally reach the result that all the constants in (65) are zero, and that the assumption made as to the dependence of the n solutions leads to a contradiction. *The n solutions are therefore linearly independent.*

§ 6

RETURN FROM THE CANONICAL SYSTEM TO THE ORIGINAL SYSTEM.

We will now return to the original system of equations (1). Each solution of the canonical system (16) determines a solution of the original system (1) on account of the relation (10). We have then the following n solutions of (1):

$$(77) \qquad y_i^{\kappa,\lambda} = \sum_{k=1}^{k=m} \sum_{l=1}^{l=e_k} A_{i,k,l}\, z_{k,l}^{\kappa,\lambda} \qquad \left\{ \begin{matrix} i = 1, 2, \ldots n \\ \kappa = 1, 2, \ldots m \\ \lambda = 1, 2, \ldots e_\kappa \end{matrix} \right\}.$$

Now these n solutions are linearly independent, for the z solutions are linearly independent, and the determinant of the constants $A_{i,k,l}$ is not zero.

Now consider the following limit:

$$(78) \qquad \lim_{x=0} x^{-r_\kappa} (\log x)^{-(e_\kappa - \lambda)} y_i^{\kappa, \lambda}$$
$$= \lim_{x=0} \sum_{k=1}^{k=m} \sum_{l=1}^{l=e_k} A_{i,k,l}\, x^{-r_\kappa} (\log x)^{-(e_\kappa - \lambda)} z_{k,l}^{\kappa,\lambda}.$$

On the right, if we take the limit of each term in the summation, we shall find that it is zero in every case except the one in which $k = \kappa$ and $l = e_\kappa$ (cf. (64) III.). We have, then:

$$(79) \qquad \lim_{x=0} x^{-r_\kappa} (\log x)^{-(e_\kappa - \lambda)} y_i^{\kappa,\lambda} = A_{i,\kappa,e_\kappa} \frac{1}{(e_\kappa - \lambda)!}.$$

So we can write as a set of solutions of (1), corresponding to the elementary divisors $(r - r_\kappa)^{e_\kappa}$:

$$(80) \qquad y_i^{\kappa,\lambda} = x^{r_\kappa} (\log x)^{e_\kappa - \lambda} \psi_i^{\kappa,\lambda} \qquad \begin{pmatrix} i = 1, 2, \ldots n \\ \lambda = 1, 2, \ldots e_\kappa \end{pmatrix},$$

where the functions $\psi_i^{\kappa,\lambda}$ are continuous in the interval $0 \leqq x \leqq c$, and such that:

$$(81) \qquad \psi_i^{\kappa,\lambda} \Big|_{x=0} = \frac{1}{(e_\kappa - \lambda)!} A_{i,\kappa,e_\kappa}.$$

The constants $A_{i,k,l}$ are determined independently of the functions $a_{i,j}$ in (1); and therefore all that we have said on page 346 in regard to certain sets of them as linearly independent solutions of the equations (4) in the special case of $a_{i,j} = 0$ holds equally well here.

In order to obtain solutions corresponding to $(r - r_\kappa)^{e_\kappa}$,* we assumed in the treatment of the canonical system that $|b_{k,l}^{i,j}|\,|\log x|^{e_\kappa - 1}$ was integrable up to $x = 0$. Now, since the coefficients $b_{k,l}^{i,j}$ are linear functions with constant coefficients of the coefficients $a_{i,j}$, it will be sufficient, in order to obtain solutions of (1) corresponding to $(r - r_\kappa)^{e_\kappa}$, to assume that $|a_{i,j}|\,|\log x|^{e_\kappa - 1}$ is integrable up to $x = 0$ for all values of i and j.

Our results may be stated as follows: *If $(r - r_\kappa)^{e_\kappa}$ is an elementary divisor of $\Delta(r)$, and if we consider all the elementary divisors $(r - r_k)^{e_k}$ of $\Delta(r)$ such that $R\, r_k = R\, r_\kappa$, and denote by e_κ that exponent which is as great as any other exponent in this set of elementary divisors, and assume that $|a_{i,j}|\,|\log x|^{e_\kappa - 1}$ is integrable up to $x = 0$; we can develop e_κ solutions of (1):*

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* Cf. p. 349.

$$y_i^{\kappa,\lambda} = x^{r_\kappa}(\log x)^{e_\kappa-\lambda}\,\psi_i^{\kappa,\lambda}, \qquad \begin{pmatrix} i = 1, 2, \ldots n \\ \lambda = 1, 2, \ldots e_\kappa \end{pmatrix},$$

where $\psi_i^{\kappa,\lambda}$ *is continuous in the neighborhood of* $x = 0$ *and*

$$\psi_i^{\kappa,\lambda}\big|_{x=0} = \frac{1}{(e_\kappa - \lambda)\,!}\,A_{i,\kappa,e_\kappa}\,.$$

If the root r_κ *furnishes* s *elementary divisors* :

$$(r - r_\kappa)^{e_\kappa},\ (r - r_\kappa)^{e_\kappa+1},\ \ldots\ldots\ (r - r_\kappa)^{e_\kappa+s-1},$$

then the constants :

$$A_{i,\kappa,e_\kappa},\ A_{i,\kappa+1,e_{\kappa+1}},\ \ldots\ldots\ A_{i,\kappa+s-1,e_{\kappa+s-1}} \qquad (i = 1, 2 \ldots n)$$

are s *linearly independent solutions of the equations* (4) *when* $r = r_\kappa$.

The n *solutions of the differential equations* (1) *that we obtain when* $\kappa = 1, 2, \ldots m$ *are linearly independent.*

§ 7.

THE HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION OF THE *nth* ORDER.

We shall consider homogeneous linear differential equations which can be written as follows:

$$(82)\qquad \frac{d^n y}{dx^n} + \left(\frac{\mu_1}{x} + p_1\right)\frac{d^{n-1}y}{dx^{n-1}} + \left(\frac{\mu_2}{x} + p_2\right)\frac{1}{x}\frac{d^{n-2}y}{dx^{n-2}} + \ldots$$
$$+ \left(\frac{\mu_n}{x} + p_n\right)\frac{1}{x^{n-1}}\,y = 0,$$

in which $\mu_1, \mu_2, \ldots \mu_n$ are constants, and $p_1, p_2, \ldots p_n$ are functions of the real independent variable x, continuous in the interval $0 < x \leqq b$, and such that their absolute values are integrable up to $x = 0$; in short, these p's are to have the same properties as the functions $a_{i,j}$ in (1).

This equation can be replaced by a system of linear differential equations by the following substitutions:

$$(83)\qquad x^i\,\frac{d^i y}{dx^i} = y_{n-i} \qquad (i = 0, 1, \ldots n-1).$$

We thus get the system of differential equations:

$$(84)\quad \begin{aligned} \frac{dy_1}{dx} &= -\left(\frac{\mu_1 - n + 1}{x} + p_1\right)y_1 - \left(\frac{\mu_2}{x} + p_2\right)y_2 - \dots - \left(\frac{\mu_n}{x} + p_n\right)y_n, \\ \frac{dy_i}{dx} &= \frac{1}{x}\, y_{i-1} + \frac{n-i}{x}\, y_i \qquad (i = 2, 3, \dots n). \end{aligned}$$

The characteristic determinant of (84) is:

$$(85)\quad \begin{vmatrix} r - n + 1 + \mu_1 & \mu_2 & \dots\dots & \mu_{n-1} & \mu_n \\ -1 & r - n + 2 & \dots\dots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots\dots & r - 1 & 0 \\ 0 & 0 & \dots\dots & -1 & r \end{vmatrix}.$$

The adjoints of the first line are:

$$(86)\quad r(r-1)\dots(r-n+2);\ r(r-1)\dots(r-n+3);\ \dots\ r(r-1);\ r;\ 1.$$

The characteristic equation is, then:

$$(87)\quad r(r-1)\dots(r-n+1) + \mu_1 r(r-1)\dots(r-n+2) + \dots + \mu_{n-2}\, r(r-1) + \mu_{n-1}\, r + \mu_n = 0.^*$$

There is always one first minor of (85) which is not zero, the adjoint of μ_n; and therefore if r_κ is a multiple root of (87), the only elementary divisor of (85) corresponding to r_κ is $(r - r_\kappa)^{e_\kappa}$, where e_κ is the multiplicity of the root r_κ.

We have seen that, corresponding to the elementary divisor $(r - r_\kappa)^{e_\kappa}$, there are e_κ linearly independent solutions of (84) of the form (80). Here the constants A_{i,κ,e_κ} must satisfy the set of equations:

$$(88)\quad \begin{aligned} (r_\kappa - n + 1 + \mu_1)\, A_{1,\kappa,e_\kappa} + \mu_2\, A_{2,\kappa,e_\kappa} + \dots + \mu_n\, A_{n,\kappa,e_\kappa} &= 0 \\ -A_{i-1,\kappa,e_\kappa} + (r_\kappa - n + i)\, A_{i,\kappa,e_\kappa} &= 0 \\ (i = 2, 3, \dots n). \end{aligned}$$

This system of equations has essentially only one solution, namely:

$$(89)\quad \begin{aligned} A_{i,\kappa,e_\kappa} &= \rho\, r_\kappa (r_\kappa - 1)\dots(r_\kappa - n + 1 + i) \qquad (i = 1, 2, \dots n-1), \\ A_{n,\kappa,e_\kappa} &= \rho, \end{aligned}$$

---

[*] (87) is also called the *indicial equation* of (82) for the point $x = 0$.

where ρ is a constant not zero. We may divide each solution by the corresponding number $\frac{\rho}{(e_\kappa - \lambda)!}$, and the resulting solutions may be written:

$$y_i^{\kappa,\lambda} = x^{r_\kappa} (\log x)^{e_\kappa - \lambda} E_i^{\kappa,\lambda} \qquad \begin{pmatrix} i = 1, 2, \ldots n \\ \lambda = 1, 2, \ldots e_\kappa \end{pmatrix}, \tag{90}$$

where $E_i^{\kappa,\lambda}\big|_{x=0} = r_\kappa (r_\kappa - 1) \ldots (r_\kappa - n + 1 + i)$,

$E_n^{\kappa,\lambda}\big|_{x=0} = 1$.

By means of (83) we can now return to solutions of the equation (82) with the following result:

If r_κ *is a root of the characteristic equation, and* $|p_i|\,|\log x|^{e_\kappa - 1}$ *is integrable up to* $x = 0$, *where* $e_\kappa \geqq e_k$ *for all* k*'s such that* $R\,r_k = R\,r_\kappa$, e_k *being the multiplicity of the root* r_k, *then the equation* (82) *has* e_κ *linearly independent solutions which may be written with their first* $n - 1$ *derivatives*:

$$\begin{aligned} y^{\kappa,\lambda} &= x^{r_\kappa} (\log x)^{e_\kappa - \lambda} E_n^{\kappa,\lambda} \\ \frac{d^i\, y^{\kappa,\lambda}}{d\, x^i} &= x^{r_\kappa - i} (\log x)^{e_\kappa - \lambda} E_{n-i}^{\kappa,\lambda} \end{aligned} \qquad (\lambda = 1, 2, \ldots e_\kappa) \tag{91}$$

where the functions $E_{n-i}^{\kappa,\lambda}$ *are continuous in the neighborhood of* $x = 0$ *and*,

$$\begin{aligned} E_n^{\kappa,\lambda}\big|_{x=0} &= 1, \\ E_{n-i}^{\kappa,\lambda}\big|_{x=0} &= r_\kappa (r_\kappa - 1) \ldots (r_\kappa - i + 1). \end{aligned} \tag{92}$$

Even for the equation of the second order this theorem does not merely give the results of Professor Bôcher's paper above quoted, but goes a step farther, since in the case in which the two roots of the characteristic equation are equal, we require merely that:

$$\int_0^b (\log x)\, |p_1|\, dx, \qquad \int_0^b (\log x)\, |p_2|\, dx$$

converge, while Professor Bôcher's method made it necessary for him to require that

$$\int_0^b (\log x)^2\, |p_1|\, dx, \qquad \int_0^b (\log x)^2\, |p_2|\, dx$$

converge.*

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* L. c. p. 48. The function $x\,q_1$ in this paper is the same as our p_2.

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THE INFLUENCE OF ATMOSPHERES OF NITROGEN AND HYDROGEN ON THE ARC SPECTRA OF IRON, ZINC, MAGNESIUM, AND TIN, COMPARED WITH THE INFLUENCE OF AN ATMOSPHERE OF AMMONIA.

By Royal A. Porter.

Investigations on Light and Heat, made and published wholly or in part with Appropriations from the Rumford Fund.

THE INFLUENCE OF ATMOSPHERES OF NITROGEN AND HYDROGEN ON THE ARC SPECTRA OF IRON, ZINC, MAGNESIUM, AND TIN, COMPARED WITH THE INFLUENCE OF AN ATMOSPHERE OF AMMONIA.

By Royal A. Porter.

Presented by C. R. Cross, May 14, 1902. Received October 8, 1902.

It has been frequently assumed that chemical reactions in the electric arc have a considerable influence on the character of its radiations.* It seems not unreasonable to expect that the oxidation of the electrodes at high temperature in air would tend to increase the intensity over that obtained when the arc is operated in nitrogen alone. This presumed higher temperature might be sufficient to produce atomic vibrations entirely distinct from the vibrations at a lower temperature. If the atmosphere does have any such influence, the effect might be apparent in the spectrum of the arc.

In the case of a hydrogen atmosphere † the most marked effects on the arc spectra of iron, zinc, magnesium, and tin have been found to be a general diminution of intensity and a change of relative intensity among the lines. The lines relatively enhanced by hydrogen are spark lines.

Liveing and Dewar ‡ have noted the effect of atmospheres of hydrogen and nitrogen on a number of lines in the magnesium arc and spark, and of these and other atmospheres on the cyanogen § bands in the carbon arc, but as far as I am aware no extensive study has been made of the influence of nitrogen and ammonia on the arc spectrum of metals. From a study of the effects of hydrogen and nitrogen on certain spark lines of magnesium, Liveing and Dewar [*] were led to remark that "it is possible *that the atmosphere may*, besides the resistance it offers to the discharge, in some degree affect the vibrations of the metallic particles." This conclusion in regard to the spark seems to have been borne out with reference to the arc by the results obtained with a hydrogen [†] atmosphere.

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* Liveing and Dewar, Proc. Roy. Soc., **30**, 161 (1880); **32**, 192 (1881); O. H. Basquin, Astroph. Jour., **14**, 11-12 (1901); A. S. King, Astroph. Jour., **14**, 329-330 (1901).

† H. Crew, Astroph. Jour., **12**, 167 (1900); Liveing and Dewar, Proc. Roy. Soc., **32**, 192, 402-403.

‡ Proc. Roy. Soc., **32**, 189-203 (1881).

§ Ibid., **30**, 152-162.

Nitrogen resembling hydrogen in its inability to combine directly with metals, it seemed reasonable to expect that it would have a very similar effect on the iron, zinc, and tin arcs. As magnesium [‡] combines directly with nitrogen a different effect might be expected in the magnesiun arc.

Method and Apparatus.[§]

In order to eliminate the effect of gases other than the one whose effect was being studied, the enclosed rotating metallic arc,[||] with "chemically pure" zinc, magnesium, and tin electrodes, was used. The iron used, however, was "commercial." The speed of the rotating electrode was approximately eleven hundred revolutions per minute. Excepting in two instances noted later, the arc was operated by a 104 volt alternating circuit.

The spectrum was photographed with a Rowland ten-foot concave grating, four exposures being made on each plate, as shown in the diagram.

Long exposure, arc in air.
Long exposure, arc in gas.
Short exposure, arc in gas.
Short exposure, arc in air.

Near the top of the plate was photographed the spectrum of the arc in air; just below this, was photographed the same arc operated in an atmosphere of the gas being studied; below this, again, was made a short exposure with the arc in the same gas; and at the bottom, a short exposure with the arc in air. The exposures were so timed as to make the intensities of the two inner photographs intermediate between the intensities of the two outer ones. By comparison one can readily determine from such a plate whether any change in intensity is due to length of exposure or to change of atmosphere. This plan of making four exposures on each plate has some additional advantages: (1) it affords a test of uniformity of results; (2) the two short exposures allow an easier comparison of lines so strong as to be ordinarily over-exposed; (3) the second exposure in the gas being made immediately after the first, and without opening the "hood" of the arc, a photograph is obtained of the spectrum of the arc in its final atmosphere; that is, in an atmosphere which includes the gaseous products of the arc, if there be any.

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[*] Proc. Roy. Soc., **32**, 203 (1881).

[†] Astroph. Jour., **12**, 167–175 (1900).

[‡] Liveing and Dewar, Proc. Roy. Soc., **32**, 161.

[§] The funds to meet the expense of this experiment were kindly appropriated by the Committee in charge of the Rumford Fund of the American Academy of Arts and Sciences.

[||] Crew and Tatnall, Phil. Mag., **38**, 379–386 (1894).

The effect of hydrogen was taken from the results published by Professor Crew * and from the original plates obtained by him. These negatives were made under the same conditions as here described, except in a different atmosphere and with a direct current. The change from a direct to an alternating current, however, produces no effect on the spectrum. In the case of the tin arc less dust is produced when a direct current is used, and I have therefore employed such a current in photographing the spectrum of tin in ammonia.

The hydrogen atmosphere was obtained by the electrolysis of acidulated water. The ammonia used was taken from a drum of compressed ammonia gas such as is used in refrigeration. Nitrogen was generated by the reaction of ammonium sulphate and sodium nitrite solutions. Professor J. H. Long kindly suggested an arrangement of the nitrogen generator by which air was excluded from the entire system throughout the work.

Traces of oxygen were removed by pyrogallic acid, while the water vapor was taken out by passage through concentrated sulphuric acid and over phosphorus pentoxide.

Before commencing an exposure a stream of nitrogen was kept flowing through the hood for at least twenty minutes. The same plan was used in filling the hood with ammonia. During an exposure a stream of the gas was kept flowing through the hood, both for the purpose of keeping a fresh supply of the gas about the arc and to drive out the dust which formed.

The spectra of the following four metals have been examined photographically in the region lying between λ 2300 and λ 5300.

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* Astroph. Jour., **12**, 167–175 (1900).

MAGNESIUM.

The ordinary arc lines of magnesium seem to be almost wholly unaffected by substituting nitrogen for air. The heavy spark line at λ 4481, which appears also in the spectrum of the rotating arc, is reduced by nitrogen to about one-fifth its intensity in air. Naturally, the magnesium oxide fluting at λ 5007 is practically blotted out in nitrogen and greatly intensified in oxygen. On the other hand, no new lines make their appearance in nitrogen.

The intensity and reversal of the characteristic line at λ 2852, which are so strongly affected by hydrogen * and ammonia, are unaffected by nitrogen, while the sharp line at λ 4571 is slightly reduced by hydrogen and ammonia, although not changed by nitrogen.

The magnesium-hydrogen fluting beginning at λ 5210, discovered by Liveing and Dewar,† appears in ammonia, as does also the F line of hydrogen. Fairly intense hazy lines at approximately 4580, 4434, 4430, and 4390 also appear in ammonia. These lines are apparently intensified by oxygen also. By nitrogen they are unaffected. A very faint trace of these lines can be seen in air. I have not yet succeeded in identifying them.

TIN.

Of the four metals studied, tin is the one whose spectrum is most modified by changes of atmosphere. In air, nitrogen, and oxygen, the tin arc works well. But in hydrogen and ammonia the arc is very short and is maintained with difficulty.

The intensity of the tin arc in nitrogen is estimated at one-third its intensity in air; while in ammonia its intensity is not more than one-twentieth its intensity in air; and in pure hydrogen it is even less.

Not only is the average intensity of the tin spectrum strongly affected, but also the character and relative intensity of its individual lines. In Kayser and Runge's table of wave-lengths of the tin arc more than forty lines are described as reversed. Many of these reversals show very clearly on my plates. When the arc is surrounded with nitrogen, however, some of these lines appear to be doubly reversed, the rest not reversed at all. On the other hand many of the lines that are reversed in air appear to have their reversal widened by ammonia. These reversals are not affected by an atmosphere of oxygen.

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* Astroph. Jour., **12**, 171 (1900). Liveing and Dewar, Proc. Roy. Soc., **32**, 192.

† Proc. Roy. Soc., **27**, 494–496; **30**, 93–99.

The two very strong spark lines at λ 3351 and λ 3282, appearing also in the spectrum of the rotating arc, have their relative intensity decreased by nitrogen. But by ammonia they are relatively enhanced at least twenty times. In the arc in air, these two lines are barely visible; but in ammonia they become two of the most prominent lines of the spectrum. These two lines are similarly affected by hydrogen The spark line at λ 2368 is also intensified in ammonia but less than the two preceding. The spark lines of wave-lengths 2449, 3471, 3539, 3574, appear plainly in ammonia and hydrogen but not at all in air or nitrogen. They are more enhanced by hydrogen than by ammonia. At λ 3360 and λ 3370 appear two unidentified lines of intensity 6 and 3 (on a scale increasing from 1 to 10), respectively, in ammonia, and not quite so intense in nitrogen, which can scarcely be detected in air and hydrogen. These same lines appear also in the magnesium and zinc arcs in air, nitrogen, and ammonia; but oxygen has the effect of immensely weakening both of them in the tin and magnesium arcs. λ 3370 appears in the tin spark, but λ 3360 does not.

ZINC.

The average intensity of the lines of the zinc arc spectrum is reduced approximately one-half by nitrogen. The width of reversed lines is not affected.

The only zinc lines that suffer a disproportionate reduction by an atmosphere of nitrogen are λ 2558 and λ 2502. These, with λ 5182, are the lines which are enhanced by hydrogen. They are strong spark lines. In ammonia their intensity is two or three times as great as it is in air, although the average intensity of the zinc spectrum is diminished perhaps five times by ammonia.

IRON.

The substitution of nitrogen for air about the iron arc produces very little change in the spectrum. The general intensity is not altered. Compared with the number of lines relatively affected by hydrogen, the number of iron lines affected by nitrogen is small, as has been found to be the case with other metals. Many of the lines that are affected by nitrogen are impurity lines. Of fifty iron lines between λ 3660 and λ 4060 that are markedly enhanced by hydrogen, six are distinctly reduced by nitrogen.

On examination of the region between λ 3600 and λ 4600 I found twenty-six lines that are particularly affected by nitrogen. A few others are slightly changed. Of these twenty-six lines seventeen are from two to ten times as strong in nitrogen as in air. But these seventeen lines were all found to be due to impurities, fifteen belonging to manganese, one to chromium, and one to cobalt. The remaining nine lines are *reduced* by nitrogen to from one-half to one-tenth their intensity in air. Part. if not all, of these lines are iron spark lines.

EFFECT OF EXCLUDING NITROGEN.

The effect of pure nitrogen being so slight, it seemed possible that the presence of so large a percentage of nitrogen in the air about the ordinary arc might account for the smallness of the change. I therefore attempted to determine the effect of nitrogen by a process of exclusion. This was done by substituting for air an atmosphere of commercial oxygen taken from an ordinary stereopticon gas-cylinder. A stream of this oxygen was kept flowing through the hood of the arc.

If the chemical affinity of the electrodes for the atmosphere has any effect on the spectrum, one might certainly expect this effect to be exhibited when such easily oxidizable metals as iron, magnesium, and tin are employed in the atmosphere of oxygen.

Mr. A. S. King[*] found that the metallic lines were intensified by increasing the supply of oxygen about the *carbon* arc. But just the contrary seems to be true with reference to the *metallic* arc. A current as large as ten amperes was tried with chemically pure magnesium electrodes, but the oxygen had little effect on the working of the arc or the appearance of the so-called "flame." To the eyes the "flame," especially with iron electrodes, appears to be less blue and more yellow.

Of eighteen exposures made with the iron, tin, and magnesium arcs in oxygen, all but one show a greater average intensity for the same length of exposure in air than in oxygen. This is not the only respect in which the action of oxygen on the metallic arc resembles that of hydrogen and ammonia. The metallic lines that have been noted as being relatively enhanced or reduced by hydrogen are precisely the ones which are so affected by oxygen. The changes produced by oxygen are not so great as those produced by hydrogen, but they are in the same direction.

SUMMARY.

The results of these experiments may therefore be summarized as follows:

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[*] Astroph. Jour., **14**, 329 (1901).

1. The average intensity of the iron and magnesium arcs is not changed by substituting pure nitrogen for air as an atmosphere. The average intensity of the zinc and tin arcs is reduced two or three times by nitrogen. By hydrogen the average intensities are reduced from five to twenty times as much as by nitrogen. Ammonia apparently does not produce quite so great a reduction as does hydrogen.

2. The relative intensity of many lines depends upon the atmosphere. The lines that are relatively reduced by nitrogen are spark lines. As a rule these lines are relatively enhanced by hydrogen or ammonia.

3. The influence of ammonia on intensities and reversals is intermediate between that of nitrogen and hydrogen, and in general it seems true that *the effect of ammonia is approximately equal to the sum of the effects of its constituents.* This, in fact, is the particular point which I had in mind to determine when I began these experiments.

4. *The influence of oxygen is similar to that of hydrogen.*

5. Nitrogen affects the reversed lines of tin by either destroying the reversal or producing faint double reversals.

These results seem sufficient to show that *the readiness of an atmosphere to form chemical union with the electrodes under ordinary conditions is a very small, probably insignificant, factor in determining the intensity of the arc. The intensity appears to be due to electrical causes rather than to chemical reactions.*

Some experiments have been performed by Professor Basquin* which seem to confirm the theory that the intensification of spark lines in hydrogen is caused by the increased resistance due to the hydrogen atmosphere about the arc. This greater resistance has been attributed to the absence of chemical reaction in the hydrogen arc. Such a change of resistance in the products of the arc may explain the phenomena which occur in hydrogen. But if the resistance of the arc depends on the reactions in it this fact makes it difficult to see how the spark lines can be intensified by an atmosphere of oxygen.

This work was done under the direction of Professor Crew, who, in fact, himself began it and to whom I am indebted for continued advice and assistance.

Northwestern University, Evanston, Illinois.
22 April, 1902.

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* Astroph. Jour., **14**, 14–17 (1901).

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CONTRIBUTIONS FROM THE HARVARD MINERALOGICAL MUSEUM. — XII.

1. *BABINGTONITE FROM SOMERVILLE, MASS.*
2. *BABINGTONITE FROM ATHOL, MASS.*

BY C. PALACHE AND F. R. FRAPRIE.

WITH TWO PLATES.

1. BABINGTONITE FROM SOMERVILLE, MASS.

2. BABINGTONITE FROM ATHOL, MASS.

By C. Palache and F. R. Fraprie.

Presented by J. E. Wolff, October 8, 1902. Received October 8, 1902.

Babingtonite from Somerville, Mass.

Occurrence and Paragenesis.

The occurrence of babingtonite at Somerville has long been recorded, but no adequate description of it seems to have been published. We first find mention of it in a description by Teschemacher * of the minerals of the Charlestown sienite quarry, where it is called "hornblende in the form of oblique rhombic prism with modifications c, k, l; crystals are small and black on white prehnite."

In Alger's Phillip's Mineralogy, 1844, p. 79, it is stated on the authority of Professor Nuttall that babingtonite occurs at Charlestown and that this is the only American locality. No description of the mineral by Nuttall has been found, and Alger probably had the information privately from Nuttall. This statement was copied in Brooke and Miller's Mineralogy (1852), but appears never to have found its way into the American text-books, the only further mention of it being in Dana's System (1892), where, although not referred to in the text, the mineral is listed in the Catalogue of Localities, p. 1059, as occurring at Somerville.

The babingtonite occurs in veins and pockets composed chiefly of prehnite, which traverse a large dyke of diabase.† This dyke is well exposed in an old quarry, recently abandoned, on Granite Street, Somerville, formerly known as the Milk Row quarry, Charlestown, and it is from this quarry that the material here described has all been obtained. The mineral is not abundant, and it was only after several years of collecting in the fresh quarry openings that sufficient material for analysis and adequate crystallographic study was accumulated. In addition to the material now in the Harvard Mineral Cabinet, several fine specimens from the same locality were placed at our disposal by Mr. H. I. Johnson.

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* Proc. Boston Soc. N. H., June, 1839, in Am. J. Sci., XXXVIII., 1840, p. 194.

† For a description of this rock, which is called sienite in all of the above references, see Wadsworth, Proc. Bos. Soc. N. H., XIX. (1879), p. 223, and Jaggar, Am. Geol., XXI. (1898), pp. 203–213.

The minerals commonly associated with the babingtonite are prehnite, quartz, epidote, pyrite, chlorite, feldspar, and calcite. Other minerals reported from the locality are laumontite, stilbite, chabazite, melanolite (known only from here and a doubtful species), and chalcodite.

Prehnite is much the most abundant mineral in the veins, always lining the walls and often forming the whole vein-filling. It varies in color from pure white to quite a deep green, which often fades on exposure to light. It has the characteristic drusy reniform surface of prehnite, the crystals rarely individual. Occasionally the crystals composing the globular or barrel-shaped groups are sufficiently distinct to show that they are tabular parallel to c (001) with edges formed by m (110) and sometimes also a (100). A single specimen showed a number of lath-shaped crystals attached by one end to massive prehnite, on which only the three pinacoids were present. Prehnite also forms fine-grained granular masses of dull white color.

Quartz is sparingly present, generally as small pellucid crystals implanted on prehnite. In one specimen the quartz is in the form of capillary crystals stretching from wall to wall of small cavities in massive prehnite. Most of the crystals are combinations of the common forms m $(10\bar{1}0)$, r $(10\bar{1}1)$, and z $(0\bar{1}11)$ but a few were observed and measured of more complex character on which the following less common forms were found: d $(10\bar{1}2)$, π $(01\bar{1}2)$, G $(13.0.\bar{1}\bar{3}.9)$, s_1 $(\bar{1}\bar{1}21)$, $(9\bar{2}\bar{7}2)$, y $(41\bar{5}1)$. These forms were present as narrow truncations of edges of the common forms. The alternate edges of the unit prism were also truncated by planes of a trigonal prism, apparently etch faces, which gave distinct readings at variable angles not corresponding to any known form of quartz but near $(10.1.\bar{1}\bar{1}.0)$.

Epidote occurs quite commonly in minute yellowish-green needles implanted on prehnite or coating and intimately intergrown with babingtonite and quartz. The crystals are not measurable.

Pyrite is found occasionally in clusters of minute crystals resting on prehnite or babingtonite. The crystals are too minute and too poorly developed to permit of measurement, but they appear to be dominated by the cube.

Chlorite is not common with the babingtonite, but is apt to occur intermixed with the massive form of the prehnite.

Feldspar is found with prehnite and babingtonite in a few specimens, in the form of small imperfect pinkish crystals. They are similar in appearance to the pinkish labradorite of the enclosing diabase and were not further determined. This mineral is found only in certain pockets of oval form entirely enclosed in the diabase. The dyke at this point contains many inclusions of various rocks, and it is probable that these pockets represent inclusions in which the original material has been almost wholly replaced by prehnite and calcite. Wadsworth* found pseudomorphs of prehnite after feldspar here, which fact to some extent strengthens this conclusion, based chiefly upon the form of the pockets.

Calcite occurs in both veins and pockets as the last substance deposited, filling up all the interstices of the other minerals. It is white and glassy, often in large individuals, but never, so far as observed, in developed crystals. Near the surface of the veins the calcite has generally been removed by solution, but away from the zone of weathering it seems always to be present, and all the finer specimens of prehnite and babingtonite were obtained by dissolving away the infilling of calcite with dilute acid.

Of the remaining minerals recorded above from the locality one only, chabazite, has been seen by the authors. It occurs in well-formed twin rhombohedrons, pure white in color, implanted on a specimen of massive prehnite. The specimen was not collected by us and this mineral has not been found at the locality for some years.

The babingtonite is usually in distinct crystals implanted upon, and in rare instances wholly embedded in, prehnite. The crystals are often but slightly attached to the prehnite, and are therefore not infrequently completely bounded by crystal planes. They are generally small, rarely exceeding 2 or 3 mm. in greatest dimension; but a few larger crystals were obtained, the largest measuring 1.5 × 1 ×.4 cm. The complete detached crystal shown in figure 3 is 1 ×.7 ×.2 cm. and is perhaps the largest perfect crystal preserved from the locality. Granular aggregates of babingtonite are also found embedded in prehnite, composed sometimes of fine grains and again of individuals so large that cleavage surfaces of several square centimeters' area may be observed.

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* Loc. cit., p. 223.

The babingtonite is black in color and of a brilliant vitreous lustre. It is, however, very subject to decay, and when the protecting calcite has been removed and the babingtonite exposed to the weather it becomes dull and gradually alters to limonite, of which complete pseudomorphs are frequently found. The decay is most active along the incipient cleavage cracks, and crystals which appear on the surface fresh and sound are often found on being broken to be permeated throughout by limonite films. This character proved to be so universal in the larger crystals that all attempts to prepare plates from them for the study of the optical properties were unsuccessful.

Crystallography.

Twenty of the smaller crystals were measured and the following forms determined, those marked with an asterisk being new to the mineral; c (001), b (010), a (100), h (110), g $(2\bar{1}0)$, f $(3\bar{2}0)$, k *$(1\bar{1}0)$, u *(054), o (011), v *(035), w *(025), s $(0\bar{1}1)$, d (101), x *(305), y *(205), n *$(\bar{1}01)$, p *(111), t *(112), i *$(\bar{1}\bar{1}2)$.

The faces of the prism zone are generally deeply striated parallel to the prism axis, and the basal plane is often curved or faceted. It was found too that even where all the faces were plane there was often a lack of parallelism in opposite faces, especially those of the prism zone, which made the accurate adjustment on the goniometer difficult; and for this reason many of the crystals were measured both by the two-circle method and by adjusting one plane in polar position and determining the interfacial angles of the others to it. Both groups of measurements are presented in the tables which follow, and the wide range of values for many of the angles gives a measure of the irregularities to which the crystals are subject.

The position here adopted for babingtonite is that of Dauber, and his elements have been used for the calculation of the table of angles.*

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* The choice of this position rather than the one taken by Dana and Hintze, which is designed to bring out the similarity in form between babingtonite and the monoclinic pyroxenes, may be justified on several grounds. The dominant habit of the crystals, as may be seen from the figures, is prismatic in the direction of Dauber's prism zone, so that this is the natural position in which to orient them and very much simplifies the adjustment and measurement. In Dana's position this zone is made a pyramid zone. Furthermore Dauber's position gives much simpler symbols for the forms, as may be seen in the accompanying table. The position of Goldschmidt (Winkeltabellen) gives symbols as simple, but, like Dana's,

Dauber's elements were based on a very large number of measurements on crystals of babingtonite from Arendal. His values for many angles show a considerable range, up to a degree or more, on either side of the mean value. Our measurements of interfacial angles, as shown in Table I, show a like range, and the means agree on the whole so closely with Dauber's calculated values that it was deemed best not to attempt to establish new elements for the Somerville mineral. Moreover the co-ordinate angles calculated from them for the two-circle readings agree closely with the measured angles, as may be seen in Table II.

The forms may be characterized as follows.

In the prism zone a (100) and b (010) are always present, generally brilliant and free from striations, but often slightly curved.

h (110) is generally present as a very narrow, brilliant face.

g $(2\bar{1}0)$ and f $(3\bar{2}0)$ are generally both present, most often in oscillatory combination as a broad, gently curved surface striated vertically — a very characteristic feature of crystals from this locality. When f is absent g is less deeply striated, but is rarely perfectly plane. Notwithstanding their striation, both faces give good reflections.

k $(1\bar{1}0)$. A number of faint reflections were observed in the vicinity of the faces f and g, but the only one which was observed more than once and was assignable to a definite form was the new form k, which was found with both faces on one crystal and a single face on another. The faces were narrow and reflection poor, but the form seems assured.

c (001) is always present, generally plane, but sometimes faintly striated parallel to intersection with s and o, or faceted.

The domes o (011), s $(0\bar{1}1)$, and d (101) are always bright faces giving good images. They are all three present on many crystals, s is never lacking, and d is more frequently absent than o. They vary much

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is unsatisfactory in not making the dominant and most easily recognized zone the prism zone.

Dauber.	Dana.	Goldschmidt.	Dauber.	Dana.	Goldschmidt.
c (001)	M $(1\bar{1}0)$	c (010)	n $(\bar{1}01)$	$(\bar{2}21)$	(011)
b (010)	m (110)	b (100)	u (054)	(010)	(540)
a (100)	c (001)	a (001)	v (035)	$(4\bar{1}0)$	(350)
h (110)	h (221)	q (101)	w (025)	$(7\bar{3}0)$	(250)
g $(2\bar{1}0)$	g $(11\bar{1})$	r $(\bar{1}02)$	x (305)	$(10.\bar{10}.3)$	(053)
f $(3\bar{2}0)$	f $(44\bar{3})$	l $(\bar{2}03)$	y (205)	(551)	(052)
k $(1\bar{1}0)$	(221)	$(\bar{1}01)$	p (111)	(401)	(111)
o (011)	a (100)	o (110)	t (112)	$(6\bar{2}1)$	(121)
s $(0\bar{1}1)$	b (010)	s $(1\bar{1}0)$	i $(\bar{1}\bar{1}2)$	$(\bar{2}61)$	$(1\bar{2}1)$
d (101)	d (221)	δ (011)			

TABLE I.

	Angle calculated.	Measured (Mean).	Limits.	No. of Readings.
	° ′	° ′	° ′ ° ′	
ab	67 48	67 54	66 49– 68 40	15
ab′	112 12	112 04	111 13–112 47	13
ac	87 28	87 01	86 44– 87 28	6
ac′	92 32	92 45	92 18– 93 15	5
af	59 36	59 26	59 03– 59 48	4
af′	120 24	120 33	120 12–120 57	4
ag	47 33	47 15	46 59– 47 22	4
ag′	132 27	132 42	132 38–132 50	3
ah	43 05	43 37	43 12– 44 30	8
ah′	136 55	136 20	135 45–136 49	6
fg	12 00	12 58	11 42– 13 51	6
f′h	77 18	76 43	75 49– 77 32	6
g′h	89 18	89 04	88 23– 89 34	6
bc	92 36	91 48	91 26– 92 20	8
bc′	87 24	87 43	87 17– 88 26	6
bf	52 36	52 37	51 55– 53 14	6
bf′	127 24	127 07	126 46–128 01	5
bg	64 39	64 48	64 26– 64 59	4
bg′	115 21	115 12	115 04–115 35	6
bh	24 43	24 15	23 01– 25 07	11
bh′	155 17	155 43	155 16–156 43	11
bo	47 23	46 46	46 25– 47 18	6
bo′	132 37	132 42		1
bs	44 40	44 48	44 20– 45 17	2
bs′	135 20	134 53	134 03–135 58	6
cg	85 27	84 59	84 40– 85 18	5
cg′	94 33	94 37	94 08– 95 13	5
co	45 13	45 10	44 19– 46 13	11
co′	134 47	134 43	134 33–134 53	2
cs	42 44	43 08	42 33– 44 00	6
cs′	137 16	136 51	136 13–137 33	3
cd	29 55	29 38	29 16– 29 58	9

in size from being dominant terminating planes to narrow truncations of the edges between c and faces of the prism zone.

The remaining forms observed are all new and vary much in quality. Of the domes, n $(\bar{1}01)$ is well established, having been observed on six

TABLE II.

Elements.

$p_0=0.59098$	$\lambda=92° 36'$	$a=1 635$	$a_0=1.8257$	$\alpha=86°09'$	$x_0=0.0662$	$d=0.0803$
$q_0=0.9662$	$\mu=87° 28'$	$b=1.$	$b_0=1.1167$	$\beta=93°48'$	$y_0=\bar{0}.0454$	$\delta=124° 24'$
$r_0=1.$	$\nu=67° 48'$	$c=0.8955$	$c_0=1.$	$\gamma=112° 22'$	$h=0.9968$	

	Calculated.		Measured (Mean).		Limits.	Limits.	No.	Quality.
	ϕ	ρ	ϕ	ρ	ϕ	ρ		
	° ′	° ′	° ′	° ′	° ′ ° ′	° ′ ° ′		
c 001	124 24	4 36	119 37	4 53½	116 02–123 34	4 12– 5 00	8	very good
b 010	00 00	90 00	00 00	90 00	. . .	. . .	20	“ “
a 100	67 48	90 00	67 55	90 00	66 49– 68 47	. . .	28	“ “
h 110	24 42	90 00	24 16	90 00	23 01– 25 07	. . .	22	“ “
g $2\bar{1}0$	115 24	90 00	115 12	90 00	115 04–115 35	. . .	10	“ “
f 320	127 24	90 00	127 16	90 00	126 44–128 05	. . .	11	“ “
k $\bar{1}10$	143 38	90 00	143 50	90 00	143 27–144 13	. . .	3	poor
u 054	3 16	49 26	3 26	49 31	3 24– 3 28	48 42–50 20	2	fair
o 011	4 07	42 48	4 06	43 00	3 47– 4 20	42 34–43 45	8	very good
v 035	7 04	28 22	7 08	29 07	. . .	. . .	1	“ poor
w 025	10 59	19 13	11 03	19 25	10 45– 11 22	18 54–19 57	3	poor
s 011	176 15	45 29	175 51	45 29	175 23–176 17	45 03–45 44	10	very good
d 101	73 49	32 39	73 37	32 45	73 02– 73 56	32 40–32 54	8	“ “
x 305	77 20	22 04	76 56	22 00	. . .	. . .	1	fair
y 205	81 14	16 08	80 05	16 50	. . .	. . .	1	“
n $\bar{1}01$	–119 12	28 55	–119 08	28 44	118 20–119 31	28 33–28 52	6	“
p 111	28 11	52 29	28 19	52 27	28 15– 28 27	52 17–52 48	2	very good
t 112	31 44	32 56	31 20	33 17	. . .	. . .	1	poor
i $\bar{1}\bar{1}2$	–162 04	34 01	–165 03	34 37	. . .	. . .	1	“

crystals with small but plane faces giving angles agreeing closely with calculated positions.*

x (305) and y (205) were each observed but once as narrow faces between d and c, giving fair reflections and moderately close to calculated positions. They need confirmation.

u (054) was observed on two crystals as a narrow face between o and

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* In the figure of babingtonite in Dana (System, 1892, p. 382) a plane lettered n is shown corresponding to this form ($\bar{1}01$), which would be the pyramid ($\bar{2}21$) in Dana's position. No mention of this form is made in the text, nor is it found in the figure in Des Cloiseaux (Atlas, Pl. XII. Fig. 66), which is assigned as the source of Dana's drawing. Hintze (Mineralogie, II. p. 1169, Fig. 422) seems to have reproduced Dana's figure without correcting this apparent draughtsman's error.

b. It may be regarded as assured, although the values for the two observations differ rather widely.

w (025) was found on three crystals as a narrow face between o and c and may be regarded as established.

v (035) was observed but once as a faint line face between o and c. Its position is fairly good, but it needs confirmation.

Of the pyramids, p (111) was measured on two crystals and observed on several others. It is always a distinct face giving brilliant reflections, and is well established by the close agreement of measured and calculated angles.

t (112) and i $(\bar{1}\bar{1}2)$ were each measured but once as narrow truncations of the edges between d and o and s and n respectively. The reflections were poor and the angular positions are not very close to calculated values, but their positions in the zones and the fact that each was observed several times, although with faces too narrow to give measurable reflections, seem to assure both of these forms.

The drawings, each presenting a plan and a clinographic projection of a crystal, illustrate the various habits of the Somerville babingtonite and especially the occurrence of the new forms. Figure 3 is a simple combination particularly characteristic of the larger crystals. Figures 4, 5, and 6 are as nearly as possible in the proportions of the crystals on which the various new forms were found. Figure 6, with the omission of the planes u, x, and w, would reproduce very well the appearance of the majority of the smaller crystals. Figure 7 is an ideal combination of all the forms here described, and is represented as terminated below by the perfect basal cleavage, which is very commonly developed. The gnomonic projection, Figure 1, also brings out well the relations of the forms.

Chemical Analysis.

The material for analysis was obtained from a single large pocket of prehnite in which the babingtonite was embedded in fine granular form. The whole mass was roughly hand-picked, and the babingtonite crushed and passed through a coarse sieve, washed to remove dust, and treated with dilute acid to remove traces of calcite present. It was quite free from limonite. It was then separated with an electro-magnet, which readily attracted the babingtonite, leaving behind the prehnite and almost all of the epidote, of which a very small quantity was present. After a second treatment under the magnet with a slightly less powerful field, the material was placed in methylene iodide at its maximum density, 3.34, in which the larger portion of the babingtonite slowly settled, leaving floating all fragments with adhering prehnite. About 3 grammes was thus obtained, which under the microscope showed no appreciable impurity.

The analysis, by Fraprie, was made according to the method of Hillebrand for silicate analysis, except that the manganese was precipitated with bromine water.

	Analysis.	Recalculated to 100% omitting H_2O.	Molecular Ratio.		
SiO_2	52.25	52.66	0.8777	0.8800	8.86
TiO_2	0.18	0.18	0.0023		
Al_2O_3	5.27	5.31	0.0521	0.0993	1.00
Fe_2O_3	7.49	7.55	0.0472		
FeO	11.05	11.14	0.1547	0.5630	5.67
MnO	1.94	1.96	0.0276		
CaO	20.36	20.52	0.3664		
MgO	0.46	0.46	0.0115		
$(K,Na)_2O$	0.22	0.22	0.0028		
Loss (H_2O)	0.29	. . .			
Total	99.51	100.00			

The analysis shows a babingtonite somewhat lower in manganese and much higher in aluminum than the average, but otherwise of normal character. It may be expressed by the formula,

$$(Ca,Fe,Mn)\,SiO_3 \cdot (Fe,Al)_2\,(SiO_3)_3$$

with the two molecules present in the proportion of about $5\frac{3}{4} : 1$.

2. Babingtonite from Athol, Mass.

Babingtonite was first reported from Athol by Shepard * in 1849, and the locality was cited for the mineral in his Mineralogy, 1857, p. 183.

J. D. Dana was the first to examine the crystals critically, and his measurements, made on very small crystals, were not conclusive, as they agreed but poorly with the angles of babingtonite from Arendal. His figure and measurements are to be found in his Mineralogy, 1854, p. 368, and the question of the identity of this mineral has remained unsettled. In Dana's System, 1892, reference is simply made to the description cited above, the matter being still left in doubt.

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* A. J. Sci., VIII. (1849), p. 275.

Through the kindness of Professor B. K. Emerson we were given the opportunity of studying a suite of specimens of the Athol babingtonite belonging to the Amherst Mineral Cabinet, and the result of our examination is to establish Shepard's original determination as correct. According to Professor Emerson the locality is no longer accessible, and a very small amount of the mineral is believed to be extant.

The material studied consists of a large matrix specimen and a number of small crystals, all apparently detached from the larger mass. This is a fragment of chloritic gneiss bounded on two surfaces by what were clearly veins. These faces are coated with a thin drusy layer of dark green epidote, upon which, on one side, the babingtonite is implanted. It is in brilliant shining black crystals, most of the faces striated and curved or warped, the largest crystal not exceeding .3 cm. in diameter. Only by using the very smallest crystals, on which the curvature of the faces was least marked, could satisfactory measurements be obtained.

With the babingtonite are a number of sharp cubes of perfectly fresh pyrite. These were deposited before the babingtonite, as is shown by a narrow vein of the latter mineral traversing one of the pyrite crystals. There are also several minute rosettes of prehnite implanted on the babingtonite and a small amount of a platy zeolite which from incomplete tests appears to be apophyllite.

Six crystals were measured and the following forms determined: a (100), b (010), c (001), h (110), g $(2\bar{1}0)$, f $(3\bar{2}0)$, s $(0\bar{1}1)$, o (011), d (101), and w (025).

The following table shows some of the measured angles, together with the calculated values and Dana's corresponding approximate values.

	Measured (Mean).		No. of Read's.	Calculated.		Dana.	
	°	′		°	′	°	′
ab	67	35	8	67	48	69	30
bg	115	20	6	115	24	. . .	
af	59	38	5	59	36	59	30
cd	29	59	3	29	55	26	40
cs	42	28	2	42	44	44	20
co	44	54	1	45	13	44	30
ca	86	52	2	87	28	85	00
cb	87	36	2	87	24	89	00–90° 00′

The forms h, o, and w were each observed but once. The others are generally all present and form combinations of the habit shown in Figure 2. This figure is essentially the same as that given by Dana,* except that in his drawing o is present and g is lacking. The following table gives the identification of Dana's, Hintze's, and Dauber's letters. The crystal figured by Dana was correctly oriented and redrawn and the symbol for f first given by Schrauf in Atlas der Krystallformen, Pl. XXX. Figure 2.

Dana.	Hintze.	Dauber.
I'	M	a
I	m	b
$i\text{–}\breve{3}$	z	f
$\frac{1}{2}'$	r	d
1	g	o
–1	p	s
0	c	c

Material for an analysis of the Athol babingtonite was not available, but the above crystallographic evidence seems amply sufficient to establish its identity.

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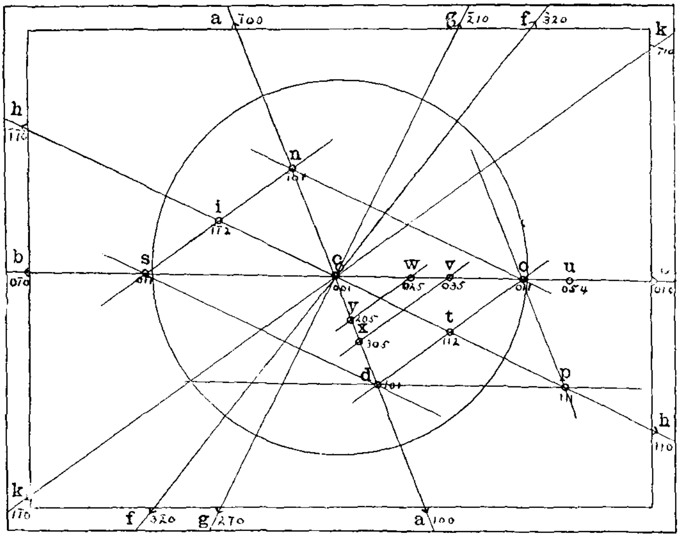
[*] Loc. cit. This figure is copied in Hintze, Mineralogie, II. p. 1169, Fig. 425.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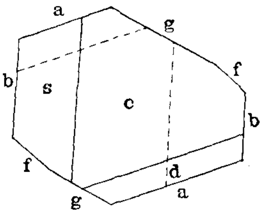
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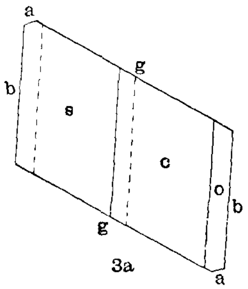

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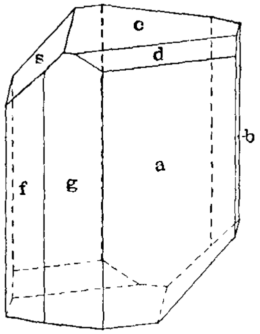

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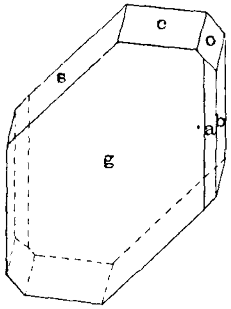

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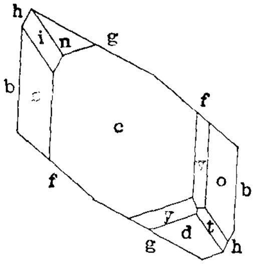
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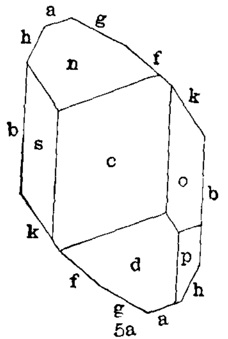
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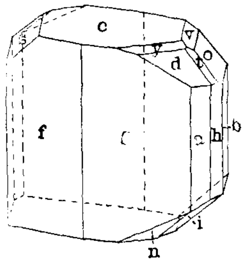
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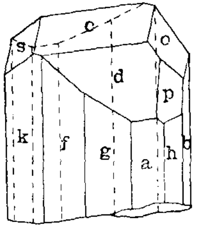
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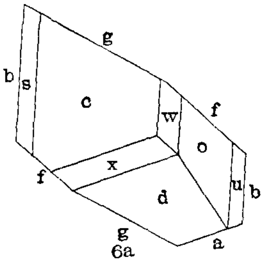
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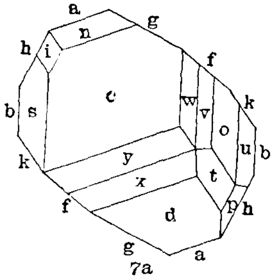
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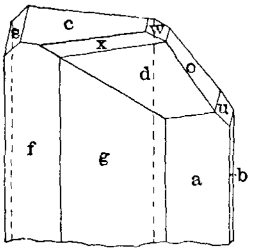
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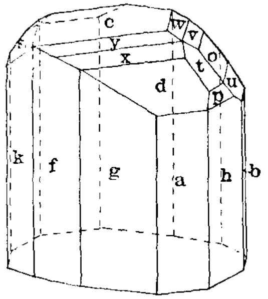
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Proceedings of the American Academy of Arts and Sciences.

VOL. XXXVIII. NO. 12. — DECEMBER, 1902.

ON THE THERMAL DEVELOPMENT OF THE SPARK SPECTRUM OF CARBON.

BY HENRY CREW AND JOHN C. BAKER.

INVESTIGATIONS ON LIGHT AND HEAT, MADE AND PUBLISHED WHOLLY OR IN PART WITH APPROPRIATIONS FROM THE RUMFORD FUND.

ON THE THERMAL DEVELOPMENT OF THE SPARK SPECTRUM OF CARBON.

BY HENRY CREW AND JOHN C. BAKER.

Presented by Charles R. Cross, October 8, 1902. Received October 8, 1902.

DURING the course of some experiments which Professor Basquin was making on the production of arc and spark spectra from the same electrodes, it was observed by one of us, standing at the eyepiece of his spectroscope, that the lines of the spark spectrum made their appearance gradually, and not suddenly, beginning at the instant at which the direct current feeding the arc was cut off and the high-voltage current producing the spark was switched on.

It was evident at once that the appearance of these lines in deliberate succession was due, primarily at least, to the gradual cooling of the electrodes and of the region between them. But we were uncertain whether, after all, the effect was not merely a physiological one, the lines first observed being the stronger lines, and those observed later being the weaker lines. We accordingly set about making a series of photographs which should show the spark spectrum at each successive instant beginning at the time at which the arc current is interrupted.

At first our attempt was to employ metallic spark-electrodes for the spectrum and to use the carbon arc to heat the spark gap. And in order to retard the development of the spark as much as possible, the carbon electrodes were enclosed between two saucer-shaped clay scorifiers as shown in Figure 1. The terminals of the spark circuit were introduced into this cell at right angles to those of the arc. Various other forms of cells, hollowed limes from the stereopticon, clay pipes, fire brick,

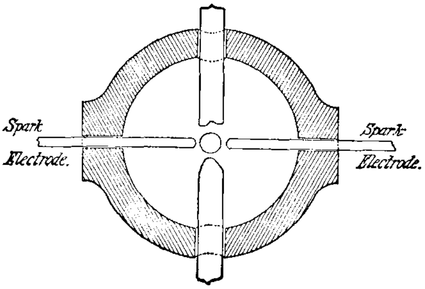

FIGURE 1.

etc., were tried. But in each case, as soon as the region inside got hot enough to affect the character of the spark and render it quiet, we found (as, indeed, ought to have been anticipated) that the walls of the vessel became conducting.

We tried next to get a gradual variation of temperature by moving the spark gap slowly from the centre to the edge of an ordinary carbon arc, knowing that, at the centre of the arc, the spark was quiet and non-luminous, while just outside the arc it became noisy and brilliant. But in carrying the spark electrodes from one of these positions to the other, we encountered a peculiar discontinuity, i. e. a position at which the spark *instantly* changed character.

When the spark was passed through the "horsetail" above the horizontal arc at a distance of from ½ to 2 centimeters from the arc, the quiet discharge mentioned above was still obtained and a spectrum of feeble intensity could be observed. When, however, the terminals were removed slightly farther above the arc, a point was reached at which the discharge instantly assumed the ragged character of the ordinary cold spark; and when the spark was then moved back toward the arc it did not resume its quiet character, but blew the "horsetail" away, and in most cases put out the arc. It did not seem possible to obtain any intermediate stages. The instability was very marked. The spark was liable at any time to break down into the ragged character, and when it had once done so it retained that character until the circuit was broken.

Apparatus and Method.

Accordingly we had recourse to soft-cored carbons worked in air, *using the same electrodes for both arc and spark;* in other words, we used the hot region between the poles of an ordinary carbon arc as the heated medium in which to study the slowly developing spark.

The next step consisted in isolating the particular phase of the development which we wished to examine.

This was accomplished by means of a device (designed with the generous aid of Professor Basquin) which performs automatically the following cycle of operations: —

1. Closes the arc circuit and lights the arc, thus heating the carbon electrodes and the region between them to a very high temperature.
2. After an interval of a few seconds, sufficient for the carbons to become thoroughly heated, interrupts the arc circuit.
3. After an interval which is less than one-tenth of a second, closes the spark circuit.

4. After a variable (but definite and measurable) interval of time, opens a shutter in front of the slit of the spectroscope and exposes the plate during any desired length of time, generally between $\frac{1}{2}$ second and 1 second.

5. Interrupts the spark circuit.

1. Again closes the arc circuit; and so on, as before.

The arc was operated with 15 amperes showing 40 volts between the electrodes; while the spark was produced by a large induction coil of the type devised by Rowland in 1887 and described in Kayser's Handbuch der Spectroscopie, p. 183. This induction coil, or step-up transformer, was operated on a 104-volt alternating circuit, of frequency 120, with a primary current of 20 amperes. In parallel with the spark gap was placed a capacity of $\frac{1}{50}$ microfarad. The arrangement of the circuit is shown in Figure 2 where s_1 and s_2 are each double pole mercury switches so fixed that *one can be closed only after the other is opened.* s_1 is kept closed by a spring until an electromagnet begins to close s_2 by rocking a light beam of which its armature is a part. The question of changing from arc to spark circuit is then merely a question of closing the battery circuit which actuates this electromagnet. This battery circuit is closed and opened by a continuously rotating switch (shown at the left in Figure 3) which is driven at the uniform rate of 10 R. P. M. by a small electric motor. This rate of rotation is maintained constant by means of a pair of cone pulleys and a heavy flywheel.

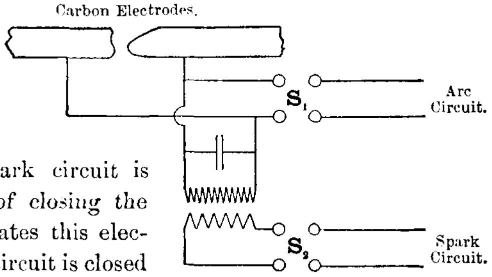

FIGURE 2.

This same rotating switch, or commutator, by means of the sliding contact marked "2" in Figures 3 and 5, opens the shutter in front of the slit of the spectroscope at any phase of the spark desired and holds the shutter open for a small but definite period of time varying usually from $\frac{1}{2}$ second to 1 second. On this same rotating commutator shaft is a stud (D, Figure 3) which, immediately after the arc circuit is closed, pushes a carbon rod into the arc gap for an instant and thus "lights" the arc.

By clamping the sector P (Figure 5) to the rotating commutator in successive angular positions about its axis, one is enabled to open the slit for the successive phases of the spark which he may wish to photograph; not only so, but he can repeat any phase as many times as he likes, and thus obtain a strong composite in cases where a single exposure would produce no visible effect.

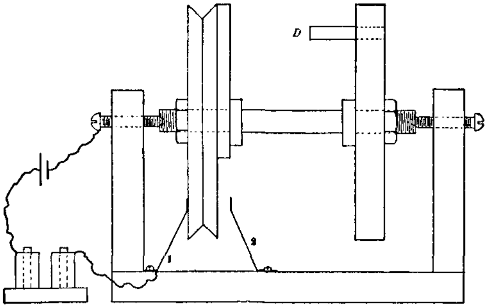

FIGURE 3.

In this manner we have photographed the spark spectrum of carbon, with a ten-foot concave grating, in nine different phases,* which may be roughly described as follows: —

1. Exposure begins $\frac{1}{6}$ second after breaking arc and lasts $\frac{1}{2}$ second. Here the carbon poles are still white hot and the spark is practically silent when compared with the noise which the cold spark makes. In this stage the luminosity is so exceedingly feeble that, with a slit of the same width as in the rest of the series, six to ten hours (i. e. about 5000 exposures) are required to get a fair negative.

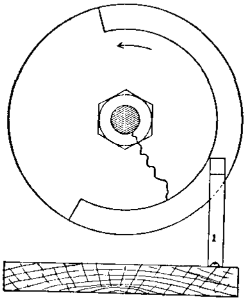

FIGURE 4.

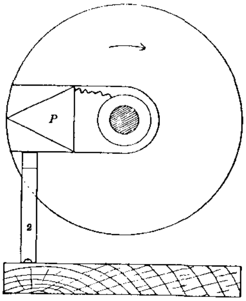

FIGURE 5.

2. Exposure begins $\frac{1}{4}$ second after breaking of arc and lasts for 1 second. The middle of the exposure, therefore, occurs $\frac{3}{4}$ second after the beginning of the spark. Here, again, the image of the spark on the slit of the spectroscope is quite invisible during the entire exposure.

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* The purpose of this experiment, it will be observed, is therefore fundamentally different from that in which Sir Norman Lockyer examined the spark spectra of salts volatilized in flames and which he described in Proc. Roy. Soc., **30**, pp. 22–31 (1879).

3. Exposure begins ½ second after breaking of arc, and lasts for 1 second; middle of exposure one second after beginning of spark. Here the image of the spark is barely visible just before the slit is covered. The spark is distinctly louder than in the preceding phases.

4. Middle of exposure 1¼ seconds after beginning of spark.

5. Middle of exposure 1¾ seconds after beginning of spark.

6. Middle of exposure 2⅝ seconds after starting spark.

7. Middle of exposure 5¼ seconds after starting spark. Here the electrodes begin to show merely red, instead of white, hot.

8. Middle of exposure 7⅞ seconds after beginning: spark distinctly noisy.

9. The last photograph in the series was taken at twelve seconds after the beginning of the spark, the duration of the exposure being, as in the preceding cases, one second. Even at this late stage a distinct *crescendo* is still noticeable in the noise of the spark.

The enormous increase of brilliancy from the hot spark to the cold may be judged from the fact that in order to make the cyanogen band at λ 3883 of uniform intensity the exposure time for the first of the series was eight hours and for the last of the series twenty minutes.

RESULTS.

As in the case of the Swan spectrum and the carbon arc, so also in the case of the carbon spark, the flutings are, of course, the dominant features of the entire spectrum. The first question, therefore, which naturally arises, in the development of the spark, is concerning the order and the relative intensity in which these cyanogen bands make their appearance. A second question might be asked concerning the stage at which the air lines make their appearance. A third query is when and how do the numerous metallic impurities present themselves? Our photographs permit at least partial answers to these three questions for the region lying between λ 4500 and λ 3000. The phenomenon is one which cannot be accurately observed by the eye, and the exposure times are so long as to render photographing in the visible region well-nigh impracticable.

I. CARBON FLUTINGS AND LINES.

The cyanogen bands at λλ 4216, 3883, and 3590 all make their appearance on the first photograph of the series. Their relative intensity is practically the same as in the case of the spark between cold electrodes, which, for the sake of brevity, we shall hereafter call the "cold spark." In view of this fact we have employed these three bands as standards of intensity; and have called any two spectra of "equal intensity" when these three bands were of equal intensities on the respective negatives. Each member of the series was, in this way, made of practically the same intensity.

As to the carbon lines, very few appear in this region. The line at λ 4556.3 does not appear in the hot spark, i. e. in the earliest phase of the series described above. The broad hazy line at λ 4267.5, which Eder and Valenta* call the "chief carbon line," disappears completely on introducing inductance into the circuit of the cold spark. And it does not appear at all in the hot spark. These two facts raise the question as to whether this line is due to carbon. The line at λ 3361 persists in the hot spark; but it also appears in the aluminium spark and, greatly enhanced, in the copper spark when there is no capacity in the circuit. As to the remaining lines which Eder and Valenta describe in this region λλ 3920.8, 3877.0, and 3848.0, they are weak, and we have not been able to identify them to our satisfaction.

II. AIR LINES AND FLUTINGS.

Not one of the ordinary air lines appears on any photograph whose phase is earlier than $\frac{3}{4}$ second. On the plate whose phase is $\frac{3}{4}$ second appear only the very heaviest of the air lines, viz., λλ 4630: 4447: 3995: 3433: 3330. Indeed the elimination of air lines is so complete in these earlier phases that non-appearance in the hot spark might be used as one criterion for air lines, analogous to the inductance test discovered by Schuster and Hemsalech.

As to nitrogen *flutings* which appear in spark spectra, when the electrodes are close together or when inductance is placed in series with the condenser, the case is very different — quite reversed, indeed — from that of ordinary air lines. The nitrogen flutings with heads at λλ 3371.1 and 3158.7 respectively come out very strong in the earliest phase; at $\frac{3}{4}$ second they begin to weaken; after 3 seconds, only a trace of them is left.

The nitrogen flutings of wave-length longer than 3371 do not appear in the spark under the conditions in which we are working, namely, a 3-millimeter spark gap in series with a condenser of $\frac{1}{50}$ microfarad capacity; no inductance.

We have not found any description of these nitrogen bands *as they*

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* Eder and Valenta, Denksch. K. Akad. Wien, **60**, 249 (1893).

appear in the spark spectra of elements in air at barometric pressure. At first we took the band at λ 3371.1 to be a hitherto undescribed carbon band; and it was only through an excellent suggestion from Professor Hale that we discovered our mistake. He advised us to try the spark *without capacity.* On trying this experiment, we found the band at λ 3371.1 strongly present in the spectra of aluminium, zinc, and other metals in air; but when the spark was worked in atmospheres of oxygen or coal gas, these flutings all disappeared save the merest trace of the strongest two.

The cold carbon spark (unlike that of metals) without capacity shows these bands only with extreme faintness; and the condensed carbon spark does not show them at all; but *carbon when white hot shows them strongly*, as indicated above.

In this connection, the question may be raised whether the band described by Professor Hutchins [*] does not belong to this nitrogen group. For we have found in the spark spectrum of aluminium a band, with its edge at λ 3914.41, which shows a weak line alternating with a strong one exactly as in Hutchins's photograph. But on examining this spark in a current of oxygen, not the slightest trace of the band was found. Since it is found in metals, but not in the carbon spark, and since it disappears when nitrogen disappears, it seems to us more probably due to nitrogen [†] than to carbon.

What is apparently the same band may be seen very distinctly on McClean's map of the spark spectrum of copper; and again a similar fluting has been found by Deslandres at the negative electrode of a spectrum tube filled with nitrogen. For Deslandres' drawing see *Comptes rendus*, 9 Aug., 1886. This is probably also the same band which is marked *very strong* at λ 3914.4 in Hemsalech's* table of nitrogen bands. The fact that Hutchins is able to intensify the band he describes by making and breaking the arc circuit would seem to indicate that it appears in the arc spectrum primarily in consequence of high electromotive force.

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[*] Hutchins, Astrophysical Journal, **15**, 310 (1902).

[†] Mr. F. J. Truby has measured the first 14 lines of this fluting, which form a group lying between the edge and the heavy impurity line at λ 3905.74. His values are as follows:

3914.41, head
3913.89
3913.35
3912.62, strong
3912.17, weak
3911.70, strong
3911 17, weak
3910.61, strong
3909.95, weak
3909.30, strong
3908.52, weak
3907.80, strong
3906.88, weak
3906.16, strong
3905.74, impurity

There are possibly two other weak lines near the head which Mr. Truby's definition does not permit him to measure.

III. Metallic Impurities.

The only electrodes which we have employed are the unplated, cored carbons sold by A. T. Thompson, 25 Bromfield St., Boston, for use in projection lanterns. Their size is 7½ × ½ inches and they are marked "imported." The metallic impurities which present themselves are practically only aluminium, calcium, copper, iron, and potassium. Possibly others might be detected by very long exposure or by study of portions of the spectrum other than that to which we have limited ourselves, namely, λ 4500–λ 3200. The strongest lines in this region of the hot spark spectrum are two at λ 4047.338 and λ 4044.294 belonging to the principal series of potassium. They are faintly represented in the carbon arc; but *no trace of them can be found in the ordinary, or "cold," carbon spark.* Is it not rather surprising to find on a spark spectrum plate that the strongest lines are due not only to an impurity, but to an impurity which is introduced apparently by the condition of high temperature in the medium? For, so far as the energy delivered by the spark itself is concerned, this would seem to be enormously greater in the loud and brilliant cold spark than in the quiet and invisible hot spark. We use the expression "high temperature" in this connection only with great hesitation, and then only with reference to the medium after the heating current has been cut off. *But this potassium pair persists very distinctly for five seconds after the heating (arc) current has been interrupted.* Accordingly we find it difficult to imagine any electrical effect, other than heat, which would persist for this length of time, especially, as the electrodes were placed always horizontally so that strong convection currents were sweeping out anything in the nature of electrolytic products.

It seems not improbable that these effects of the hot spark are brought about through an increased conductivity — and hence a lowered electromotive force — between the poles of the spark gap. So that, in the series described above, the earlier phases partake of the character of the arc, while the later phases represent the spark. *If this be true, the nine members of this series constitute nine different steps between the arc and the spark spectra.*

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* Hemsalech, Recherches expérimentales sur les spectres, etc., p. 126 (Paris, 1901).

A similar diminution of E. M. F. between the hot poles is indicated by the work of Schenck,* who finds that, with hot poles, the "Mg. spark line at λ 4481 shrinks down close to the electrodes, while the arc triplet at λ 5170 does not." And this view is rendered all the more probable by a fact noted by Basquin,† viz. that an auxiliary cold spark gap, in series with the hot spark gap, suffices to render the spark lines immediately visible.

The general effect of the hot spark upon metallic impurities may perhaps be most clearly described in the following three statements:

1. Some *new* impurities are introduced, e. g. Potassium λλ 4047.34, 4044.29, 3447.49, 3446.49. This is analogous to the introduction of the nitrogen fluting at λ 3371 above mentioned.

2. Among lines *due to a single element* some may be diminished while others are enhanced in intensity. Thus the calcium pair at λλ 3968.6 and 3933.8, and also the calcium pair at λλ 3179.4 and 3159.0, are immensely diminished, while the calcium line at λ 4226.9 is so greatly enhanced by the hot spark that, after the potassium pair, it becomes the strongest in the entire region studied. It is perhaps worth noting that all of the lines belonging to any one of Kayser and Runge's series are similarly affected. It would be interesting to know just how this behavior of potassium and calcium is explained in terms of the dissociation hypothesis.

3. The lines of some elements are affected either not at all, or very slightly, by the hot spark. This class is illustrated by the omnipresent copper pair at λλ 3274 and 3247; also by the aluminium pair between Fraunhofer's H and K; and by the great majority of the iron lines.

In general, it may be noted that there is nothing in the nature of a sudden change anywhere in the series. Indeed the growth of the air lines and the diminution of certain impurity lines is so gradual and definite that one might use their relative intensities to determine the phase at which any particular photograph was taken. The triplet formed by the potassium pair at λλ 4047 and 4044, together with the strong iron line between them, serve to illustrate this principle and also to point out an exception to the rule that the iron lines are generally unaffected by the hot spark. For curiously enough *this iron line increases* in intensity as the spark-gap (the medium) cools down, while, as noted above, *the*

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* Schenk, Astroph. Jour., **14**, 131 (1901). † Basquin, Ibid., **14**, 15 (1901).

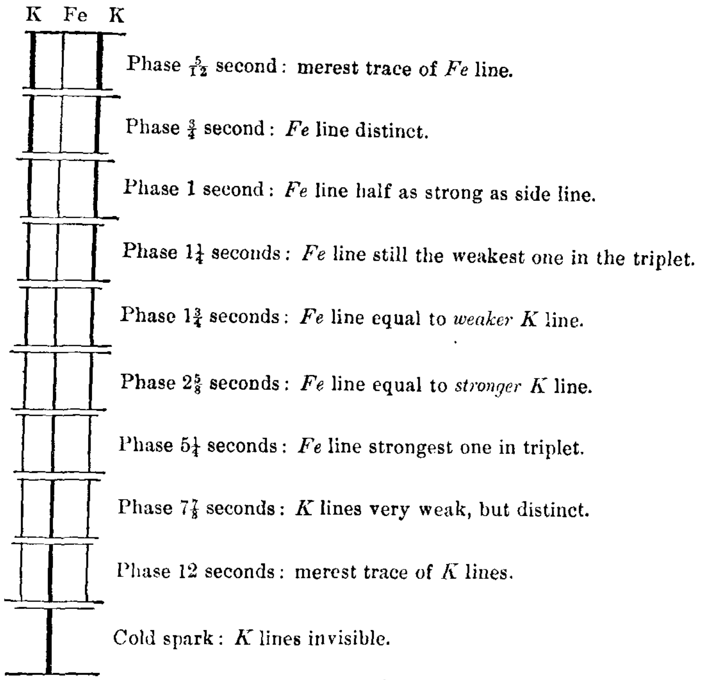
K Fe K
Phase $\frac{5}{12}$ second: merest trace of *Fe* line.
Phase $\frac{3}{4}$ second: *Fe* line distinct.
Phase 1 second: *Fe* line half as strong as side line.
Phase $1\frac{1}{4}$ seconds: *Fe* line still the weakest one in the triplet.
Phase $1\frac{3}{4}$ seconds: *Fe* line equal to *weaker* *K* line.
Phase $2\frac{5}{8}$ seconds: *Fe* line equal to *stronger* *K* line.
Phase $5\frac{1}{4}$ seconds: *Fe* line strongest one in triplet.
Phase $7\frac{7}{8}$ seconds: *K* lines very weak, but distinct.
Phase 12 seconds: merest trace of *K* lines.
Cold spark: *K* lines invisible.

FIGURE 6.

potassium pair diminishes with the temperature. In this comparison the three cyanogen bands are taken as the standard of intensity, and have essentially the same density on each plate. The triplet thus assumes the successive appearances shown in the accompanying figure. If we had measured the temperature of the region between the carbon poles at each of these nine phases, we could have identified with certainty each of these temperatures from the appearance (relative intensity) of the triplet. It is not to be forgotten that the temperature here referred to is *not* the much-talked-of and little-understood "temperature of the spark;" nor is it any temperature peculiar to certain "streaks" as perhaps is the case in the Geissler tube discharge. The temperature here referred to is that of the medium at the instant in which the shutter of the spectrograph is opened. The appearance of this triplet is then a criterion for a temperature which may be measured directly with a thermo-electric couple of sufficiently fine wire; it is a function of the phase, and not of the duration, of the exposure.

NORTHWESTERN UNIVERSITY, EVANSTON, ILLINOIS,
July 19, 1902.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

THE UNIVERSALLY EXACT APPLICATION OF FARADAY'S LAW.

By Theodore William Richards and Wilfred Newsome Stull.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

THE UNIVERSALLY EXACT APPLICATION OF FARADAY'S LAW.*

BY THEODORE WILLIAM RICHARDS AND WILFRED NEWSOME STULL.

Presented by T. W. Richards, November 12, 1902. Received October 9, 1902.

RECENTLY it has been shown † in this Laboratory that Faraday's Law holds with great precision at ordinary temperatures in aqueous solutions, for the two metals silver and copper. The long series of experiments, with probable errors of less than one in twenty thousand, yielded a ratio of the electrochemical equivalents equalling the ratio of the chemical atomic weights, within this small range of error.

While this was indeed but the comparison of a single pair of elements, it seems, nevertheless, safe to conclude that when side-reactions are wholly eliminated all other cases of comparison in aqueous solution would yield similar results. The experiments in question do not in the least serve to show that the law is independent of the nature of the solvent or the temperature, however. In attempting to probe into the meaning of this fundamental law, definite knowledge concerning these conditions is necessary; hence, new experiments at high temperature and with different solvents seemed highly desirable.

It will be remembered that approximate experiments at high temperatures and with other solvents have already been made by several experimenters; ‡ and there could have been no doubt as to the *approximate* fulfilment of the law under widely varying conditions. But none of these series of experiments yielded results trustworthy within one part in a thousand, and most of them showed deviations of at least one per cent; hence the possibility of slight deviations remained open.

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* A preliminary announcement of this work was made at the meeting of the Am. Asso. Adv. Sci. at Pittsburg in July, 1902, and an abstract was published in "Science" (N. S.) **16**, 284 (1902).

† Richards, Collins, and Heimrod, These Proceedings, **35**, 123 (1899); Richards and Heimrod, ibid., **37**, 415 (1902); also Zeitschr. phys. Chem., **32**, 321 (1900), **41**, 302 (1902).

‡ Faraday; see Ostwald, Electrochemie, 520; Kahlenberg, J. Phys. Chem., **4**, 349 (1900); Merrill, Phys. Rev., **10**, 169 (1900); Helfenstein, Z. anorg. Chem., **23**, 255 (1900).

In the experiments which are described below, a single metal, silver, was deposited in two successive, widely different cells by the same current. The conditions were so chosen as to make possible the elimination or determination of all conceivable side-reactions.

One of the cells consisted simply in the porous cup voltameter, or coulometer, which has been shown to give consistent and accurate results.* The other was a precisely similar arrangement of apparatus filled with a solution of argentic nitrate in fused potassic and sodic nitrates, maintained at about 250°.

This mixture of nitrates has a double advantage over pure argentic nitrate as an electrolyte. In the first place, the comparatively small concentration of silver diminishes the danger of accidental reduction, and in the next place, the foreign solvent enables one to determine the amount of mother liquor included in the crystals. The solution-tensions of the alkali metals are so high as to prohibit the electrolytic deposition of a weighable trace of either in the presence of an excess of dissolved silver.

It is true that this plan of experiment introduced two variables at once into the problem, a proceeding logically incomplete. If a difference were found between the two deposits of silver, further experiments would be needed in order to determine if this difference were due to the change of solvent or to the change of temperature. Such further experiments would have been made if they had seemed necessary; but the chances against accidental coincidence seemed so great as to eliminate any serious danger from the interpretation given below. This is fortunate, for it is not easy to find a single solvent capable of yielding satisfactory results over so large a range of temperature.

Although all three nitrates were prepared in a state of great purity by known methods, it was found very difficult to obtain fused solutions free from a slight precipitate of reduced silver. Hence, in order to avoid danger of error from this source, the fused mixture was filtered by its own gravity through an asbestos mat in a Gooch crucible, the whole apparatus being heated to 250° in a clean air-bath. Thus may be obtained a limpid and colorless fused mixture, which was used immediately for the electrolysis, being maintained at the high temperature with the help of a suitably arranged alcohol lamp. Great care was exercised to prevent the possibility of electrical leakage, the connections being made by carefully insulated air lines.

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* Richards and Heimrod, loc. cit.

When a first crude experiment had shown the feasibility of the arrangement, three more careful preliminary experiments were made. After the completion of each allotted time, the current of 0.2 ampere was broken, the two porous cups removed, and the two electrolytes decanted. Each cathode-crucible was thoroughly rinsed and allowed to stand for fifteen to eighteen hours in water. Two more washings with water, two with alcohol, and a thorough drying at 160° C. completed the treatment. In those cases when a trace of metallic silver was found in the decanted liquid, this trace was carefully collected on a Gooch crucible, thoroughly washed, and weighed. It was noted that the uppermost crystals deposited at the higher temperature were somewhat larger than the others, and also that no "anode dust" formed at 250°. The first of these facts may be ascribed to local action, due to slight thermal inconstancy * while the second is probably due to the rapid adjustment of equilibrium at the high temperature. Thus the chemical explanation given by Richards and Heimrod † of the appearance of the "anode dust" in aqueous solutions is supported; for if the anode dust were due solely to the mechanical detaching of bits of silver, there is no obvious reason why this process should not occur at high temperatures as well as at low temperatures.

Since all crystals formed from solutions contain included mother liquor, concealed in inaccessible cells, ‡ the silver which had been deposited from the molten solution was now examined for included sodic and potassic nitrates by dissolving it in pure nitric acid, precipitating the silver by hydric sulphide, evaporating the filtrate, dissolving and filtering the residue, drying once more, and finally weighing. Hydric sulphide was used in order that platinum vessels might be employed throughout. The mother liquor included in the crystals deposited from aqueous solution was assumed to be 0.016 per cent, the average of many determinations. §

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* Gladstone and Tribe (Phil. Mag.), (5) **11**, 508.

† These Proceedings, **37**, 428 (1902).

‡ The effect of such inclusion is particularly obvious in the work of Merrill already quoted.

§ Richards and Heimrod, These Proceedings, **37**, 441.

PRELIMINARY COMPARISON.

	Silver deposited at 250°.			Silver deposited at 20°.		Difference.	
	Observed.	Impurity.	Corrected.	Observed.	Corrected.	mg.	per cent.
	grams.	m.g.	grams.	grams.	grams.		
2	0.5761	0.4	0.5757	0.5758	0.5757	0.0	0.00
3	0.5942	0.2	0.5940	0.5739	0.5738	0.2	0.03
4	1.1729	1.1	1.1718	1.1720	1.1718	0.0	0.00
			Average				0.01

According to these preliminary results the varying conditions seemed to affect the weight of silver by only one part in ten thousand, the higher temperature yielding this slight excess of weight.

It became now a matter of interest to determine if this slight difference might be yet further reduced by the employment of yet greater care. Accordingly three much more accurate experiments were made, in a large dark-room which was for the time devoted wholly to this work. No injurious gases were allowed in the vicinity, and dust was as much as possible excluded. The current of 0.2 ampere was open for nearly two hours in each experiment. The impurity included in the crystals was more carefully determined in every case, the deposit from the aqueous solution being ignited at a dull red heat, while the alkali nitrate residues were converted wholly into sulphate, and freed from a last trace of silver by means of hydrochloric acid. The final dried residues of mixed sulphates weighed respectively 0.30, 0.54, and 0.32 milligrams. Since the nitrates of sodium and potassium were present in molecular proportions, and since the hot mother liquor contained ten per cent of argentic nitrate, these values correspond to corrections of 0.39, 0.69, and 0.42 milligrams respectively — corrections which are applied below.

The mother liquor included in the silver formed in the cool cell must also have contained dissolved argentic nitrate to the extent of about 0.02 milligrams in each case, an unimportant amount which is nevertheless subtracted below for the sake of completeness.

All weighings were made by substitution on a very precise balance with weights carefully corrected; and in general all the precautions needed in work upon atomic weights were observed.

The final figures thus obtained are as follows : —

FINAL COMPARISON.

Silver deposited at 250°.		Silver deposited at 20°.	Difference.	
Observed.	Corrected.	Corrected.	mg.	per cent.
grams.	grams.	grams.		
1.14958	1.14919	1.14916	0.03	0.003
1.12264	1.12195	1.12185	0.10	0.009
1.10242	1.10200	1.10198	0.02	0.002
Average				0.005

The largest difference is thus one tenth of a milligram, an amount not much greater than the possible experimental error, while the average difference has sunk to one in twenty thousand. It may be noted that of the six experiments, preliminary and final, four show no significant difference at all. The decreasing average difference with increasing care suggests that the slightly larger weights of silver obtained from the fused salt might be due to reduction of silver by organic dust, which it is difficult wholly to exclude. But even without this explanation, the results are close enough for present purposes, because an accuracy greater than one part in twenty thousand is rarely attained in other physicochemical measurement at the present time.

SUMMARY.

In this paper it is shown that a galvanic current deposits essentially the same amount of silver from a solution of argentic nitrate in other nitrates at 250° as it does from an aqueous solution at 20°, within 0.005 per cent. Taken in connection with the previous work of Richards, Collins, and Heimrod, this result shows that Faraday's law is not a mere approximation, but is rather to be ranked among the most precise and general of the laws of nature.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

AN APPARATUS FOR THE MEASUREMENT OF THE EXPANSION OF GASES BY HEAT UNDER CONSTANT PRESSURE.

BY THEODORE WILLIAM RICHARDS AND KENNETH LAMARTINE MARK.

INVESTIGATIONS ON LIGHT AND HEAT, MADE AND PUBLISHED WHOLLY OR IN PART WITH APPROPRIATIONS FROM THE RUMFORD FUND.

CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

AN APPARATUS FOR THE MEASUREMENT OF THE EXPANSION OF GASES BY HEAT UNDER CONSTANT PRESSURE.

By Theodore William Richards and Kenneth Lamartine Mark.

Presented November 12, 1902. Received October 22, 1902.

Most of the work which has been done upon the relation to the law of Gay-Lussac or Dalton * has involved the measurement of changing tension in constant volume. The results are neither numerous nor comprehensive enough to justify their use in connection with the scanty data concerning Boyle's law, for the construction of the isobars or isopiestic lines representing the temperature-volume relation.

Since in some respects this temperature-volume curve is less difficult to interpret than the temperature-pressure curve, and since our knowledge of the fundamental tendencies underlying the slight but weighty irregularities of aeriform material cannot be complete without it, the present investigation was undertaken with the object of directly measuring the expansion of gases under constant pressure.

It is needless to point out that the results when complete may throw important light on the new theory of compressible atoms; indeed this hope really caused the beginning of the research.

In planning an investigation of this kind it is highly important to study the degree of definition required by the various experimental observations. The coefficient of expansion of hydrogen is not far from 0.00366 at 0°; and if the next decimal place of this value is to be determined accurately, the temperature, when the range is between 0° and 100° C., must be known to within 0.03° C., the pressure under which the gas is measured to within 0.02 mm. of mercury, and the volume to within one part in four thousand.

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* Or Charles. See Klassiker der exakt. Wiss., **44**, 14 (1894).

Volumes, together with the correction for the expansion of the glass, can easily be measured with the required accuracy. The temperatures, also, of melting ice and of steam, are certainly known to within 0.03° C.; but in the pieces of apparatus previously used, not all of the gas which was being measured has been at the temperature of the bath, except in some constant volume experiments. In the apparatus of Regnault,* the bulb which contained the gas to be measured was connected by a capillary tube to a graduated cylinder, where the expansion was read by the rise and fall of the mercury which closed the lower end of this cylinder. Some doubts may be entertained as to the accuracy of the correction to be applied for the temperature of the gas in the capillary tube, and also as to the perfect equality of temperature throughout the water bath which surrounded the graduated cylinder.

With the exception of Magnus † (who used an apparatus, modelled after Gay-Lussac's, which proved very unsatisfactory because of leakage), other investigators, as Mendeléeff, ‡ Andrews, § Kuenen and Randall, ‖ and Callendar, ** have employed methods similar to that of Regnault. Callendar, instead of correcting for the amount of gas outside the bath, introduced a system of automatic compensating tubes.

The attainment of the required accuracy in the reading of pressures is a far more difficult matter than the problem offered by constancy of temperature. Regnault states his belief that the height of the barometer can be read only to one-tenth of a millimeter. Chappuis, †† however, has devised an apparatus, open to the atmosphere, for measurement of increased tension in constant volume, by which he makes record of pressures to the thousandth of a millimeter. Of course, this extra care would have been needless if the barometer had been at fault. Callendar ‡‡ has overcome the difficulty by using a constant artificial atmosphere, attained by keeping a large gas reservoir packed in ice, and by using sulphuric acid instead of mercury in the manometer. The very recent work of Travers §§ also attained great uniformity of condition. Unfortunately none of these investigators have made any extended determinations of the expansion of a gas under constant pressure, and Regnault's observations furnish still the only considerable data concerning this condition.

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* Mémoires de l'Académie des Sciences, **21**, 112–120 (1847).

† Poggendorf Annalen, **55**, 1–27 (1842); **57**, 177–199 (1842).

‡ Abstract in B., **8**, 1681 (1875); B., **9**, 1311 (1876); B., **10**, 81 (1877).

§ Philosophical Transactions, **166**, 421–449 (1876).

‖ Proceedings of the Royal Society, **59**, 60–65 (1895).

** Proceedings of the Royal Society, **50**, 247–251 (1891).

†† Travaux et Mémoires du Bureau Internat. des Poids et Mesures, **16** (1888).

‡‡ Proceedings of the Royal Society, **50**, 247–251 (1891).

§§ Abstract in Proc. Roy. Soc., **70**, 484 (1902); also Chem. News, **86**, 61.

THE APPARATUS.

The apparatus about to be described has several advantages over others. In the first place all the gas is within the constant bath. Again, a smaller range of temperature is used, thus involving less uncertainty on account of the changing of coefficient of expansion with the temperature. The two fixed points used were the melting point of ice, 0°, and the transition temperature of sodic sulphate, 32.383°.* Lastly, the pressure exerted by the gas can be certainly read to within the hundredth of a millimeter of mercury and always under the same conditions, which are entirely independent of atmospheric pressure.

The last object is attained by comparing the pressure in a special reservoir (which is not open to the atmosphere and which is protected from changes in temperature) with the pressure of the gas under investigation, and then reading the pressure within this reservoir with the required degree of accuracy by means of the admirable barometer devised by Lord Rayleigh.† In this way the pressure can always be read under the same conditions and entirely independently of the accidental state of the atmosphere. It must, of course, be proved that the means of comparing the pressure of the investigated gas and the pressure within the reservoir introduces no new source of error.

The arrangement which has been devised is as follows: The gas to be experimented upon is contained in the bulb *A* (Figure 1), which is firmly held in place in the bath *B*. In adjusting the apparatus at the lower of the two temperatures (0°) the mercury is raised by means of the levelling bulb *V*, which is carried by a finely threaded screw, until electrical contact with the platinum point *a* is established. The establishment of this contact between the platinum point and the mercury is shown by the movement of a galvanometer needle. Preliminary experiments proved that by this method a smaller change in the level of the mercury could be detected than could be noted by observation through the telescope; that is, a change less than the hundredth of a millimeter.

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* Am. J. Sci., **6**, 201; Zeit. phys. Chem., **26**, 690 (1898). A more accurate study of this point has been made recently, and very trustworthy data, giving its true value in terms of the international standard, have been obtained. This investigation, by T. W. Richards and R. C. Wells, will be published in this Volume.

† Lord Rayleigh, Proc. Roy. Soc., **53**, 135–138 (1893).

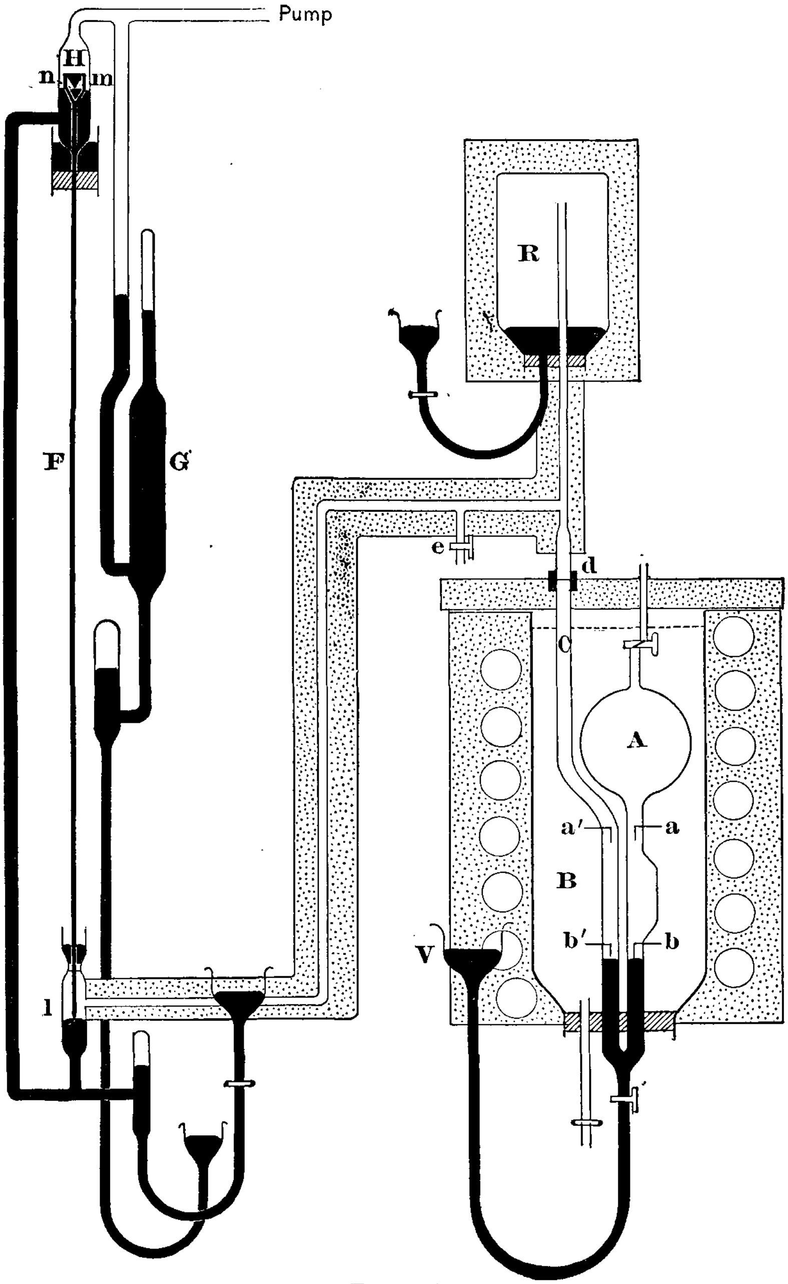
Pump
H
n
m
F
G'
R
e
d
C
A
a'
a
B
b'
b
V
l

Figure 1.

The electromotive force and strength of the indicating current should be very small, in order to avoid the occurrence of sparks. This point *a* fixes both the volume and the level of the mercury.

After the position of contact with *a* has been fixed, the pressure in the opposite arm *C* is adjusted by changes in the amount of mercury in the insulated reservoir *R*, so that contact is just made with the second point *a′* also. The reservoir *R* is the outside reservoir referred to above, and consists of a glass bottle of one and one-half litres capacity, packed in cotton and resting inside a wooden box. The time between the final adjustment of pressure within this reservoir and the reading of this pressure is so short (from one to two minutes), that with this amount of packing, perceptible change in pressure due to change in temperature does not take place. The pressure of the gas in *A* is the same as the pressure in *C* and in the connecting reservoir *R*, plus or minus the difference in height between the points *a* and *a′*. Before proceeding, the adjustment at these two points is made again several times, in order that each may be certainly established.

The pressure in *C* and in *R* is now measured with great accuracy by means of the Lord Rayleigh pressure gauge, shown in the left part of the figure. Here contact is made with the lower point *l* of the iron rod *F*. The observation is made through a magnifying glass permanently fixed in position. The space *H* is exhausted by a Sprengel pump and the vacuum obtained is measured by a McLeod gauge *G*. The pressure in the tube *C* and reservoir *R* is thus measured by the height of the mercury meniscus *m* above the point *l*. This is easily determined by measuring through a telescope the distance between point *n* and the meniscus *m*, since the distance from the point *l* to the point *n* of the iron rod *F* can be directly measured. This reading completes the performance at the lower temperature.

The ice in the bath *B* is then replaced by about ten kilograms of sodic sulphate at its transition temperature, and a similar setting is made, using the points *b* and *b′* instead of *a* and *a′*. Since the tubes are essentially alike at *a*, *a′*, *b*, and *b′*, and since the volumes are so arranged that there is very little change in pressure upon change in temperature of the bath, the conditions of measurement of pressure are always the same, and the measurements may be made very accurately. This uniformity of conditions eliminates constant errors, such as those due to capillarity and a possible refraction of the glass.

MEASUREMENTS.

The first measurement to be undertaken was the determination of the required volumes. The bulb *A* and side tube *C*, before being placed in the bath, but while held in an upright position, were filled with mercury. The end of the side tube was then closed firmly with a pinch-cock at *d*, and the mercury filling the space between the stop-cock at the top of the bulb and the point *a*, and that between the point *a* and the point *b* were in turn run out and separately weighed. Allowance was made for the weight of mercury held by the stop-cock. The temperature was noted, and the volume calculated from the tables of the specific gravity of mercury given by Landolt and Börnstein. In correcting for the expansion of the glass, the value 0.000025 for the coefficient of expansion of glass was assumed as accurate enough for preliminary experiments. In conclusive work this value should of course be determined.

Temperature Centigrade.	Weight of mercury in large bulb.	Weight of mercury in expansion bulb.	Volume of large bulb at 0° C.	Volume of expansion bulb at 32.5° C.
	grams.	grams.	c.c.	c.c.
20.0°	3709.7	441.90	273.72	32.631
20.0°	3710.5	442.02	273.77	32.640
20.0°	3709.5	441.93	273.70	32.633
		Average . . .	273.73 ± 0.01	32.635 ± 0.002

The "probable errors," obtained by the well known method of least squares, show that the average values are sufficiently accurate.

The large bulb must have been 0.222 c.c. larger at 32.4° than at 0°. Thus, since the volume of the large bulb at 0° is 273.73 c.c., that of the total, as far as the point *b*, at 32°.4, is 306.59 c.c.

In an ideally perfect apparatus of this type the distances between the platinum points *a* and *b* on the one side and the points *a′* and *b′* on the other should be exactly the same. Since, however, it is not possible to seal these platinum points into the glass tubes with the requisite accuracy of position, it is necessary actually to measure the distances between the two sets of points and apply the corrections thus obtained to the successive observed readings of the pressures in the reservoir *R*.

In measuring the difference between the distances *a* to *b* and *a′* to *b′*, the bulb, before being mounted in its final position, was inclined very slightly, at such an angle that the points *b* and *b′* were at exactly the same level, as shown by their simultaneous electrical contact on slowly raising the mercury in the tubes. The difference in level between the points *a* and *a′* was then observed by means of the micrometer-telescope of an excellent Geneva cathetometer. The divisions of the micrometer had previously been standardized by reference to various parts of an accurate Geneva brass meter-stick placed at the same distance from the telescope as was the bulb. For the sake of greater simplicity in calculations, this distance (79 cm.) between the scale and the object lens of the telescope was always adopted whenever the telescope was used. Five determinations on different parts of the scale gave an average value of one division of the micrometer screw dial as equal to 0.01248 mm. In order to verify the measurement of the platinum points, the *upper* pair were afterwards levelled, and the heights of the lower pair were observed through the telescope. The following nine determinations showed the distance between the points in the side tube to be greater than that between those in the bulb by 0.90 mm, 0.89 mm., 0.86 mm., 0.88 mm., 0.89 mm., 0.88 mm., 0.87 mm., 0.86 mm., 0.87 mm.; average 0.88 mm.

The bulb was then placed into its position in the bath, joined at *d* by a rubber connector to the rest of the apparatus, and firmly clamped to a massive horizontal iron rod. This part of the apparatus is not shown in the diagram. It is absolutely essential that this clamping be as rigid as possible, since a change in the position of the bulb between one set of readings and another would be fatal to the accuracy of the experiment. The ends of the horizontal rod were clamped to two firm iron supports a meter apart. In order to cause a change in the observed relative heights of the platinum points great enough to affect the result of the experiment, one end of the rod would have to be raised five tenths of a millimeter above the other end. In our apparatus, the unequal coefficients of expansion of the supports caused as a matter of fact a slight displacement of this kind, which will be obviated in further work. An empirical evaluation of this effect showed it to amount to 0.025 mm. (at the points) per degree; but this correction was often eliminated, since the observations were made as much as possible at constant temperature, when the bulb was unalterable in position.

The bath consisted of a large inverted glass bottle, from which the bottom had been cut off. It was surrounded by cotton, then by a coil of lead pipe through which warm water was passed when the bath was filled with sodic sulphate, and then again by cotton. The whole was enclosed in a wooden box, which served as a support and which was securely screwed to the immovable desk upon which all the apparatus stood.

In order to dry the bulb, it was exhausted five times by means of a Sprengel mercury-pump, while the bath was filled with water at 70° C. The air which entered was dried by passing through sulphuric acid and over phosphoric oxide. The hot water of the bath was next replaced by a mixture of hydrous and anhydrous sodic sulphate, which was kept at its transition temperature by the passage of warm water through the lead-pipe coil surrounding the bath. The temperature of the sodic sulphate bath was verified by very accurate thermometers.*

It was found necessary to fill the bulb at the higher temperature, because that mixture of rubber and paraffine, used as a lubricant for the stop-cock, which was of the proper viscosity at 32° C., became so stiff at 0° C. that the stop-cock could not be turned at that temperature without leakage.

The exhausted bulb was now filled with hydrogen, which had been made electrolytically from hydrochloric acid with a zinc amalgam anode.† The hydrogen was purified by passing through strong sodic hydroxide and over phosphoric oxide. After filling the bulb, the hydrogen was allowed to stream through it for half an hour, passing out at the stop-cock *e*. The enclosing within the bulb an amount of hydrogen suitable for the experiment is a somewhat delicate operation. In the first place the connection between the bulb and the side tube *C* was closed by mercury. By regulating the amount of mercury in the reservoir *R*, a pressure was obtained in the tube *C* which caused the mercury in the Lord Rayleigh barometer to stand within a millimeter of the upper iron point *n* when the lower point *l* was just in contact. The pressure in the bulb itself was next made approximately equal to that in *C*, as shown by simultaneous contacts at the points *b* and *b′*. This was accomplished by adding hydrogen to or removing it from the bulb by means of another small bulb with mercury inlet attached to the hydrogen supply system. The stop-cock at the top of the bulb was then closed.

A more accurate adjustment of the pressure in the reservoir *R* was now made, so that the contacts of the mercury with the platinum points *b* and *b′* were perfectly simultaneous; the contact at the lower point *l* of the iron rod of the gauge was made with great care; and finally the distance between the meniscus *m* and the point *n* was read through the telescope. It was found possible to be certain of this very important reading within 0.01 mm. when the illumination was properly arranged. The pressure in the upper part *H* of the gauge was immediately taken by means of the McLeod gauge *G* — the reading of which multiplies the real pressure by fifty — and also the temperature of the Lord Rayleigh barometer was observed by means of two thermometers placed one at its top and one at its base. It is necessary that this temperature be known to within the tenth of one degree, in order to correct for the unequal expansion of the mercury column and the iron rod. To prevent sudden changes of temperature the barometer was incased in a wooden air-jacket, the lower iron point being observed through a glass window. When the room containing the apparatus varied rapidly in temperature, the jacket was not always adequate for its purpose, hence the observations were usually made when the room was at constant temperature. After the first setting, which involved the admission of the suitable amount of hydrogen into the bulb, subsequent determinations consisted merely in adjusting the pressure in the side tube so as to make simultaneous contacts at the platinum points, and in making the readings on the Lord Rayleigh barometer. This process took about five minutes and was repeated at intervals during the day. When the temperature of the room remained nearly constant, these readings were very uniform, showing that the setting of the instrument could be accomplished with sufficient accuracy. For example, three successive readings in a typical case were 1.30 mm., 1.31 mm., and 1.30 mm.

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* The Baudin thermometers No. 9389 and No. 9390, used for the determination of the transition temperature of sodic sulphate by Richards and Churchill (American Journal of Science, **6**, 201 (1898), also Zeitschr. phys. Chem., **26**, 690, were employed for this purpose. Their relation to the international scale has since been verified by Richards and R. C. Wells with the greatest care.

† Cooke and Richards, These Proceedings, **23**, 168 (1887).

The readings at the lower temperature were made in every particular in the same manner as those just described. The purity of the ice in the bath was tested by the temperature of melting, recorded by the same thermometers as were used in the sodic sulphate, and also by the analysis of the water coming from the melted ice. A complete observation, then, consisted of one series of readings at high temperature, and a similar one at low temperature. It will be noticed that the essential part of the experiment consisted in the observation of slight changes in pressure, measured in such a way that any error tends to eliminate itself. These changes were so small that Boyle's law may safely be used in referring them to changing volume.

In calculating the coefficient of expansion, let α equal the expansion per degree centigrade of unit volume at 0°C. Let V equal the volume at 0° C; v the increase in volume; t the difference in temperature between the two baths; P the original pressure; p the small increase of pressure in the gauge on the reading at higher temperature; and K the constant value to be subtracted from p due to the greater distance apart of the platinum points in the side tube over the distance apart of those in the bulb.

$$\text{Then } a = \frac{(P + p - K)\ (V + v) - PV}{PVt}$$

It will be seen that it is not necessary to know the numerical value of P very accurately, while p must be known as precisely as possible. The values V, v, and K are constant, and have already been given as $V = 273.73$ c.c., $v = 32.86$ c.c., $K = 0.88$ mm. The value $t°$ would also be a constant, but for the fact that the sodic sulphate in the bath was not always perfectly pure. A correction for the exposed stem of the thermometer was made in taking this temperature, and the temperature was then reduced to the international hydrogen scale.

The results of the experiments on hydrogen appear in the following table. The first few determinations are omitted, since the manipulation of the apparatus had not at that time become perfected.

HYDROGEN.

Number.	P.	p.	$t°$.	a.
		mm.	°C.	
1.	744.5	–0.16	32.38	0.003659
2.	742.7	–0.40	32.30	0.003656
3.	742.7	–0.19	32.38	0.003657
4.	742.7	–0.15	32.38	0.003659
5.	742.7	–0.17	32.38	0.003659
6.	742.7	–0.20	32.37	0.003658
7.	742.7	–0.17	32.37	0.003660
8.	742.7	–0.29	32.32	0.003660
9.	743.9	–0.29	32.32	0.003660
	Average			0.003659

Since the temperature of the sodic sulphate is referred to the international hydrogen scale, — that is, to the increase in tension of hydrogen at constant volume, — this series of results is merely a comparison between the thermal coefficient of hydrogen at constant volume and that at constant pressure.

A mixture of ninety-eight per cent of nitrogen and two per cent of hydrogen had the following coefficient of expansion:

IMPURE NITROGEN.

Number.	*P.*	*p.*	*t.*	*a.*
		mm.	° C.	
1.	745.2	−0.16	32.37	0.003660
2.	745.2	−0.16	32.37	0.003660
3.	745.2	−0.24	32.34	0.003660
4.	745.2	−0.21	32.34	0.003661
Average				0.003660

Carbon dioxide made by the action of hydrochloric acid on marble, and purified by passing through acid sodic carbonate solution, sulphuric acid and over phosphoric oxide, gave the following results:

CARBON DIOXIDE.

Number.	*P.*	*p.*	*t.*	*a.*
		mm.	° C.	
1.	738.0	+1.50	32.34	0.003741
2.	738.0	+1.26	32.35	0.003729
3.	738.0	+1.16	32.35	0.003724
4.	738.0	+1.30	32.35	0.003730
5.	738.0	+1.20	32.35	0.003726
Average of last 4 . . .				0.003727

The comparison of these last two tables with that concerning hydrogen is instructive and valuable even if a small undetected constant error is concealed in them all, — for such an error must be the same in each, and thus eliminate itself during comparison. Evidently the coefficient of expansion of nitrogen under constant pressure must be very nearly the same as that of hydrogen, while that of carbon dioxide is far greater. The newly found value 0.003727 for carbon dioxide is larger than the value found by Regnault (0.00371); but it must be remembered that the latter is merely an average value over a wide range of temperature, while the former involves only the change from 0° to 32.38°, where carbon dioxide is rather a vapor than a gas. The highest value found in the first experiment is probably to be attributed to the adsorption of the gas, which seems to have varied consistently thereafter. This subject will be treated in greater detail in subsequent communications.

We are much indebted to the Rumford Fund of the American Academy for valuable pecuniary assistance in this investigation.

SUMMARY.

The desirability of new determinations of the coefficient of expansion by heat under constant pressure for various gases is pointed out. An apparatus is described which possesses the advantages of having all the gas at known temperature, of affording the means of measuring pressure to within one-hundredth of a millimeter of mercury, of using a small temperature-interval, and of eliminating many possible constant errors by making the observations of pressure always under similar conditions.

A table of results of experiments on the expansion of hydrogen under constant pressure, as compared to its previously determined tension in constant volume, shows the possibility of attaining an accuracy equal to that desired, namely, one part in thirty-six hundred. Two other sets of measurements are appended which show that nitrogen expands at essentially the same rate as hydrogen, while carbon dioxide exhibits a very great deviation. It is proposed to make a large number of experiments on different gases at different pressures with the aid of this apparatus.

It may also be worthy of note that for the first time sodic sulphate has been used on a large scale for maintaining a constant temperature.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

THE TRANSITION TEMPERATURE OF SODIC SULPHATE REFERRED ANEW TO THE INTERNATIONAL STANDARD.

By Theodore William Richards and Roger Clark Wells.

Investigations on Light and Heat, made and published wholly or in part with Appropriations from the Rumford Fund.

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THE TRANSITION TEMPERATURE OF SODIC SULPHATE REFERRED ANEW TO THE INTERNATIONAL STANDARD.

By Theodore William Richards and Roger Clark Wells.

Presented November 12 Received October 22, 1902.

In a previous paper * it was shown that the transition temperature of sodic sulphate serves as a very well defined fixed point in thermometry. The precautions to be observed in its determination were there stated and a preliminary value was given, referring the transition temperature to the international standard. This value was found by means of two accurately standardized Tonnelot thermometers, Nos. 11142 and 11143, kindly loaned by the Jefferson Physical Laboratory; but in order to obtain more certain knowledge of a point so important, obviously more instruments should be used. Furthermore, there was involved an uncertain correction for the column of mercury in the stem, which, owing to the long divisions of the scale, necessarily projected a considerable distance into the air. From final determinations such uncertainty must be eliminated.

This present paper describes a number of determinations wholly independent, although in excellent confirmation of the older results. These new determinations, in the first place, obviate the uncertain correction for the projecting column. Secondly, by means of three new Baudin thermometers acquired by this Laboratory and recently standardized by the International Bureau of Weights and Measures, at Sèvres, the transition temperature is very accurately referred to the international standard.

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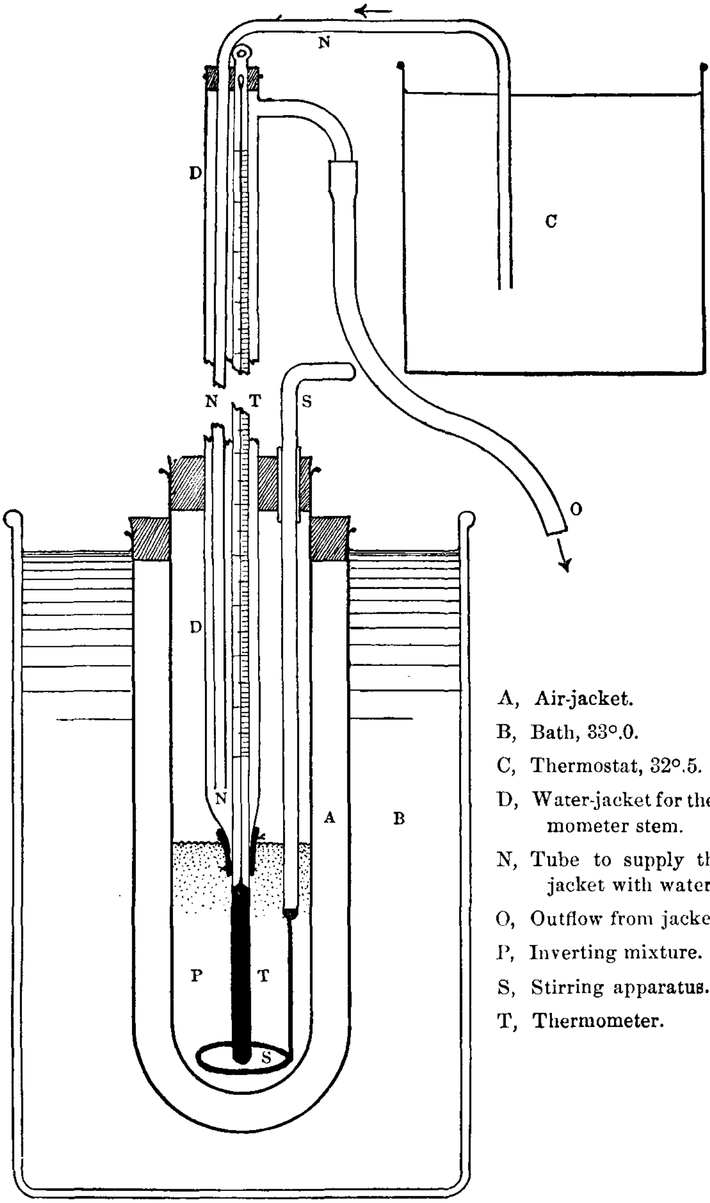
* Am. J. Sci., [4], **6**, 201; Zeit. phys. Chem., **26**, 690 (1898).

The general plan of the apparatus used in this work was similar to that described in the previous papers, but there were some modifications in detail. The essential parts of the arrangement are shown in the accompanying figure.

The sodic sulphate was placed in a large test tube in a Beckman freezing-point apparatus. It was found absolutely necessary to surround this test tube by an air-jacket in order to prevent a too sudden inflow of heat, which results in the rapid transition of the hydrated salt and consequent superheating. The outside bath was easily maintained about half a degree above the transition temperature by means of a small platinum resistance coil in the water. The galvanic current used to supply this energy was regulated by an external rheostat. With this arrangement the transition temperature was easily maintained many hours at a time.

As is well known, the true reading of a mercury thermometer is obtained only when the whole stem is the same temperature as the bulb. It was evidently inconvenient to surround the whole stem with sodic sulphate, so the following device was employed. A long tube, of clear glass and even bore, of a diameter about three times as great as the thermometer, was used as a jacket to enclose the thermometer. Through the jacket was kept running a stream of pure water, maintained at a definite temperature by a large thermostat. The water, which entered through a long thin tube parallel with the thermometer, rose through the jacket and flowed out at a side neck at the top. The thermometer bulb emerged from the jacket below through a rubber connection, which was carefully wired with fine platinum wire. In use the jacket almost touched the surface of the sodic sulphate. The diagram facilitates the understanding of the arrangement.

Since the cubic dilation of mercury is 0.000182 and that of hard glass is about 0.000022, the apparent dilation of mercury in glass is about 0.00016. A projecting column thirty-six degrees in length, subjected to a change in temperature of 0°.2 C., will therefore record a change of 0°.001 C. in its reading. Since, if possible, we did not wish the error to exceed this amount, the thermostat was not allowed to vary in constancy more than 0°.2 C., and the thermometer used to record its temperature was itself standardized with sodic sulphate. It should be remarked that in most cases an auxiliary thermometer would be a sufficient guide to the correction for the projecting column since this great degree of accuracy is not often sought. The jacket was used in this particular case in order to make certain of the temperature of the stem throughout its



A, Air-jacket.

B, Bath, 33°.0.

C, Thermostat, 32°.5.

D, Water-jacket for thermometer stem.

N, Tube to supply this jacket with water.

O, Outflow from jacket.

P, Inverting mixture.

S, Stirring apparatus.

T, Thermometer.

whole length.* To prevent the introduction of an error, on the other hand, from refraction in the glass of the jacket as well as of the thermometer, readings were made from various sides, and the average taken; this error of refraction was of barely perceptible magnitude, and was surely corrected by the diversity of our readings. Great care was taken to place the thermometer in a vertical position.

In all, five thermometers were used. The oldest Baudin thermometer No. 9389 and the Tonnelot No. 11143 have been described in the former paper. The latter was standardized in June, 1894. The new Baudin thermometers were especially constructed for this transition-temperature work, each one including the temperature 32°.38. They are numbered 15200, 15275, and 15276, and were standardized in June, 1900. Their bulbs are no larger in diameter than the stem.

No. 15200 has no enlargement of the capillary throughout its entire length, and the scale runs from −5° to 104°. Each degree is about six millimeters in length.

No. 15275 has an enlargement between +2° and 30°, being intended to give readings from −11° to +2° and from 30° to 112°. Its degrees are about as long as those of No. 15200. After its use in the present research, it will serve as a means of verifying thermometers to be used for determining the freezing and boiling points of solutions.

No. 15276, lastly, has one enlargement between 38° and 65°, another between 67° and 98°; it is intended to give very accurate readings between −2 and 38°, its degrees being each eight millimeters long.

On account of the diversity in the forms of these thermometers, each had to be calibrated by the Bureau in a different fashion, and especial confidence may therefore be placed in the average of their readings.

In the first place it seemed desirable to determine if sodic sulphate made in various ways would yield the same value for the transition point.

The first sample was made from commercially "pure" Glauber's salt. A filtered solution was allowed to crystallize; this recrystallization was found to raise the transition temperature only a few hundredths of a degree. After five recrystallizations, readings of its transition temperature were taken on the old sensitive but only approximately standardized Baudin thermometer, No. 9389, which alone was used in the preliminary comparisons of the various preparations.

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* After the experiments were finished, it was found that in spite of the rapid circulation of water through the jacket, its temperature was about 0°.4 lower than that in the thermostat. Hence a correction of 0°.002 must be added to the final result.

The second sample of sodic sulphate was made by neutralizing with sulphuric acid a quantity of sodic bicarbonate, which had been well washed with cold water. The product thus obtained was recrystallized many times, and readings taken as recorded below.

Sample III was later made from the filtrates of the various recrystallizations, and sample IV was made from a mixture of I and II when they became too small in amount to recrystallize separately.

It was convenient in this preliminary work to take successive readings of the transition temperature of several samples of salt, and then to take the reading of the thermometer in melting ice. In all cases except the first two, the lower fixed point was determined immediately after the upper one. Since commercial ice was found to contain impurity enough to change its melting point by several thousandths of a degree, pure ice was made for the zero observations. The purest water, twice redistilled, was frozen in platinum. After using some ice thus prepared it was found that 100 grams of the resulting water gave, on evaporation, a residue of only 0.0006 gram, the effect of which on the freezing point would be negligible in this work even if it had all consisted of sodic chloride.

The results of these measurements, recorded in terms of the hydrogen scale, were as follows:

	Sample.			Temperature of Transition.
(1)	I,	5th	recrystallization	32.376°
(2)	I,	6th	"	32.372°
(3)	II,	6th	"	32.376°
(4)	II,	7th	"	32.384°
(5)	II,	8th	"	32.380°
(6)	II,	8th	"	32.380°
(7)	III,	1st	"	32.376°
(8)	III,	2d	"	32.383°
(9)	III,	3d	"	32.376°
(10)	IV,	1st	"	32.389°
(11)	IV,	2d	"	32.376°
	Average			32.379° ± 0.001

In the previous communication already quoted, this same thermometer is reported to have given as the transition of sodic sulphate the value 32.482 in mercurial degrees, or 32°.378 on the hydrogen basis. Thus it is clear that the newly made sodic sulphate is identical with that prepared in 1898; and since the most serious deviation to be observed above is seen in the two determinations of the purest sample (No. IV) it is evident that the slight irregularities are to be ascribed rather to the difficulties of thermometry than to any fault of the preparations.

The preliminary values with this one thermometer having thus proved that the salt made by different methods gives identical results within the limit of error of the experiment, readings were taken with the more accurate thermometers.

In the final work, all the precautions mentioned by Guillaume to be observed in using thermometers were adhered to as closely as possible.*

Inasmuch as it was not convenient to make large quantities of pure ice where it was desired to make only one reading at a time, we preferred to make our ice readings in a tube similar to that used for the sodic sulphate; but the tube was always well filled with the pure ice and the requisite volume of purest cold water and then surrounded with a large jar of ordinary ice. The thermometer, in a vertical position, was read with an accurate Geneva cathetometer.

The purest sodic sulphate, as obtained by the methods given above, was used. It was found that a slight efflorescence was amply sufficient to supply the anhydrous phase of the sodic sulphate. The average specific gravity of the inverting mixture was found to be about 1.5.

Besides the barometric pressure, the depth of immersion of the thermometer in the solution was always noted in order to correct for external pressure.

The complete record of an observation is given below in order to show the manner of applying the corrections: † —

No. 15, March 7, 1901.

Baudin, 15200.

Corrected Barometer	766.3 mm.	766.3 mm.
59 mm. pressure of salt	6.6	
" " of water		4.3
	773.9 mm.	770.6 mm.

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* Guillaume, Traité pratique de la Thermométrie de Précision (1889).

† See "Notice sur les Thermomètres à mercure," Paris, Gauthier-Villars et Fils, 1896. This paper accompanies the thermometers.

Reading in salt	32.486°	in ice	.054°
Correction, calibration	—.002		.000
" exterior pressure	—.002		—.001
" interior "	+.039		+.009
Corrected reading in ice	—.062		.062°
	32.459°		
Correction, fundamental interval	+.023		
	32.482°		
Correction, hydrogen scale	—.104		
	32.378°		

Eleven such observations were made, with the help of four very accurately standardized thermometers of the finest possible workmanship. Since the salt was essentially the same in each case, these are grouped below according to the instruments. The question no longer concerns the accuracy of the temperature, but it concerns rather the accuracy of the several thermometers. If they can be shown to be in substantial agreement, the certainty of the reference to the international standard will be correspondingly assured.

FINAL DETERMINATIONS.

(HYDROGEN STANDARD.)

Baudin 15200	(12)	32.379°	
	(13)	32.380°	
	(14)	32.377°	
	(15)	32.378°	Average, 32.379°
Baudin 15275	(16)	32.385°	
	(17)	32.386°	Average, 32.385°
Baudin 15276	(18)	32.378°	
	(19)	32.376°	Average, 32.377°
Tonnelot 11143	(20)	32.380°	
	(21)	32.381°	
	(22)	32.386°	Average, 32.382°
Total Average			32.381°
Correction (footnote, p. 434)			+0.002
Final result			32.383°

It will be seen that the maximum variation of any minor average from the total average is only 0°.004. The Tonnelot thermometer, standardized by the Bureau Internationale des Poids et Mesures in 1894, gives essentially the same result as the average of the three new thermometers — a result which indicates in the clearest manner the excellent quality of the work of the Bureau. The final result, 32°.383, is only 0°.004 higher than the value found by one of us in 1898 — a difference about equal to the average deviation of the older experiments from their mean. If each experiment of the recent series is taken as of equal weight, the "probable error" — of the total average is 0°.0007; while if each minor average is considered as an observation, the "probable error" becomes 0°.001. These very small figures indicate that further multiplication of observations is needless. The transition temperature of sodic sulphate has been determined in terms of the international standard to within the limit of accuracy of the standard itself.

Before closing this report, it is perhaps worth while to emphasize in comparison the possible uncertainties of the three fixed points in the thermometric scale, — 0°, 32.383°, and 100°.

The freezing point of water (an equilibrium of two phases) is dependent upon three essential conditions: first, the pressure; secondly, the purity of the water; thirdly, the homogeneity of the crystal form of the ice.

It is well known both theoretically and practically, that pressure lowers the melting point of ice, because this substance is more voluminous than water. The temperature chosen for the international standard of thermometry is not the true triple point of the substance, but a point 0°.007 lower, — the freezing point under atmospheric pressure. Ordinary variations in this pressure cause an effect on the temperature smaller than the necessary errors of observation; hence the effect of changing atmospheric pressure may be disregarded, as well as the effect of the pressure of a few centimeters of liquid above the thermometer bulb.

The second possible cause of deviation, impurity in the water, causing a lower freezing point because of its osmotic pressure, is easily avoided. Water is more easily purified than almost any other substance, and even commercial ice often approximates the required purity within a few thousandths of a degree. Attention should be called, however, to the danger of dissolving volatile impurities from the atmosphere. This danger is much less when the interstices between the pieces of ice are wholly filled with pure water; and it is obviously well to add water which has been boiled in a platinum or silver flask and rapidly cooled, rather than to wait for the ice to supply the needed liquid by melting. Marek * has shown that pure ice after being rinsed and drained may indicate a freezing point as much as 0°.016 too low; and six carefully conducted experiments of our own indicated almost as great an error from this source (0°.008). It is almost needless to state that in the final experiments above the ice was properly mixed with the purest water.

The third cause of irregularity in the melting point of ice is not so frequently considered. Nichols † has shown that the specific gravity of ice may vary from 0.9161 to 0.9180, according to circumstances; and it is hardly conceivable that such varieties could give identical melting points. It may be that after a skeleton of freely forming crystals has been built, the interstices between the crystals are filled with a less compact structure.‡ Tyndall's well known experiment on the "flowers of ice" seems to support this conclusion. Marek has shown that even the strains produced in cracking the ice may cause serious errors, unless much water is present. In our own experiments, as already stated, the ice was made by freezing the purest boiled water in a platinum dish, and was wholly clear and devoid of apparent crystalline structure. The pressure under which the ice was formed was essentially the atmospheric pressure, the layer of liquid being shallow, and the ice was easily broken.

The transition temperature of sodic sulphate, a quadruple point, compares favorably with this long established fixed point, in every respect. The volume changes so little during the transition that even the pressure of an atmosphere produces no perceptible effect on the temperature,§ the purity of the substance is quite as easily obtained as that of water, and the certainty of definite crystalline form and consequent homogeneity is probably greater, because the habit of growth causes many independent freely growing crystals, rather than a commingled heterogeneous mass. The somewhat lower "latent heat of melting" is not a serious drawback if reasonable precautions are taken to prevent the inflow of heat from outside.

The third constant point, 100°, involves pressure and the purity of substance, like the others; but since no solid enters into the question, the doubt as to homogeneity is eliminated. To neutralize this advantage, two disadvantages appear; first the danger of superheating the vapor;

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* Guillaume, Traité, p. 118.

† Nichols, Phys. Rev., **8**, 21 (1899); Z. phys. Chem., **36**, 240.

‡ See also Pernet, Guillaume's Traité, p. 118.

§ Dr. G. N. Lewis has found in this Laboratory that the volume change is not over 0.5 per cent.

and secondly the necessity of knowing the pressure with the greater precision, since the tenth of a millimeter changes the boiling point by 0°.004. Hence, without a barometer of the best construction, and many precautions, one can hardly hope to attain a result accurate to within 0.01°.

The causes of error in the three fixed points just mentioned are inherent in the nature of the equilibria, and to these must be added those dependent upon the method of thermometry. The behavior of the liquid mercury, usually used to indicate temperature, leaves little to be desired; but as is well known, the glass receptacle is far less satisfactory. Its sluggish expansion and contraction may easily introduce errors of a degree or more at high temperatures, and the effect of this cause of error becomes the more important as the temperature-interval increases. Even at 100° serious mistakes may thus result. Hence it is important to choose as a standard some temperature as near as possible to the temperature read. In this respect 32°.383 is far superior to either of the usually accepted standards, because most temperature measurements lie in the neighborhood of 20°.

The careful study of the transition temperatures of other salts has been continued, and while it has led to results which may be of subsidiary importance, no salt has been found which seems to be as generally useful as sodic sulphate. Hence we unhesitatingly recommend this substance as providing one of the most convenient means known for attaining a precisely definite temperature. It has even been used in this Laboratory with success on a large scale in a bath containing over ten kilograms; the description of this undertaking will be found in the preceding paper.

The writers are much indebted to the Rumford Fund of the American Academy for pecuniary assistance in this investigation.

SUMMARY.

The present paper contains the record of experiments which verify in a satisfactory manner the earlier estimate of the transition temperature of sodic sulphate. Its exact place on the international hydrogen scale is found to be 32°.383 ± 0.001, by means of four thermometers standardized by the Bureau International des Poids et Mesures. It is recommended as being at least as trustworthy as the other two fixed points used for defining temperature, and is possibly the most generally useful of the three.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY
OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF CAESIUM.

By Theodore William Richards and Ebenezer Henry Archibald.

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I. Introduction.

Several considerations led to the renewed investigation of the atomic weight of caesium. Forming as it does the highest known member of an important and well-marked series, this metal might furnish data for the discovery of the long-sought mathematical relationship between the atomic weights; moreover, its study might also lead to the discovery of a yet higher member filling the supposedly vacant place in the classified system. In case both of these somewhat illusory hopes remained unfulfilled, there would yet be obtained data concerning an important constant of nature whose value possessed previously an undesirably large possible error. Moreover, the highly electropositive nature of caesium and its univalence combined to promise the probability of definite results from several compounds, which might throw light upon the possible variability of atomic weights. All these inducements caused us to accept with alacrity the exceedingly kind offer from Professor Horace L. Wells of a large quantity of very pure caesium material, with a view to determining its atomic weight with precision.

II. Review of Earlier Determinations.

Since the discovery of caesium by Bunsen and Kirchhoff in 1860 only four investigations concerning its atomic weight have been made; two by Bunsen * himself, one by Johnson and Allen, † and one by Godeffroy. ‡

In 1861 Bunsen published the results of his first determination. The material used in this investigation was recovered from 150 tons of water taken from the mineral spring at Durkheim. The amount of caesium obtained from this large quantity of water was only about twelve grams, and the difficulty in purifying so small a portion probably accounts for the inaccuracy of the results. Bunsen purified his material by recrystallizing the chlorplatinate, which is not nearly as soluble as the corresponding rubidium salt. The first value found by Bunsen for the atomic weight of caesium was 132.35, but this number was soon afterwards rejected even by himself, since his salt undoubtedly contained traces of rubidium.

A year later, Johnson and Allen, of the Sheffield Scientific School, New Haven, having discovered a much richer source of caesium, began their work on the atomic weight of this element. From the mineral lepidolite found in Hebron, Maine, which contained about three per cent of caesic oxide, they prepared a mixture of lithic, sodic, potassic, rubidic, and caesic chlorides. On treating this mixture with chlorplatinic acid the rubidium and caesium were thrown down as the corresponding chlorplatinates. To separate the rubidium from the caesium, the chlorides were converted through the carbonates into the bitartrates. The acid tartrate of rubidium being only one-eighth as soluble as the corresponding caesic salt, successive fractional recrystallization left the mother liquor nearly free from rubidium. A portion of the salt thus obtained was treated with chlorplatinic acid in quantity sufficient for complete precipitation, and the caesic chlorplatinate thrown down was washed, and reduced in a stream of hydrogen.

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* Bunsen, Zeit. anal. Chem., **1**, 137 (1862); Pogg. Annal. **119**, 1 (1862).

† Johnson and Allen, Am. Jour. Sci. and Arts (2), **35**, 94.

‡ Godeffroy, Annal. der Chem., **181**, 176 (1876).

For the purpose of finding the atomic weight of caesium, they made four determinations of the amount of chlorine in the chloride prepared as above. Silver nitrate was used to precipitate the chlorine, and the precipitated argentic chloride was collected on filter paper. Their highest result was 133.15 and their lowest 132.89, the average being 133.04, if oxygen is taken as 16.000 and the correction to vacuum is applied.

A few months after the appearance of Johnson and Allen's paper, Bunsen published the results of his second investigation. His method of purification was now not unlike that used by the other investigators, the most important difference consisting in the fact that several recrystallizations of chlorplatinate were made, and each successive product was analyzed. The method of analysis also resembled that of Johnson and Allen. The three final results were 132.949, 133.04, and 132.98, averaging 132.99.

In 1865 Redtenbacher * pointed out that there is a greater difference in solubility between the alums of caesium and rubidium than between the chlorplatinates; that therefore these metals might be separated more completely through the former salts than through the latter. Godeffroy made use of this fact in preparing his pure caesium chloride. When he had obtained caesium alum entirely free from rubidium, the alum was dissolved in hot water and the aluminum precipitated with ammonia. After filtering, the solution was evaporated to dryness in a platinum dish, and heated to drive off ammonic sulphate. The residue was then dissolved in water, and baric chloride was added to precipitate the remaining sulphuric acid. After filtering off the baric sulphate the solution was treated repeatedly with ammonia and ammonic carbonate in order to eliminate the last traces of aluminium and barium. The caesium chloride obtained by this treatment was fused in a platinum dish to drive off any trace of ammonium salts that might have been present. On dissolving this fused mass in water, the solution was found to be alkaline, but the crystals deposited by evaporation were not hygroscopic. Godeffroy weighed the chlorine in this salt as argentic chloride, as usual, obtaining four results, which ranged from 132.50 to 132.71, averaging 132.57.

This low result is probably due to the admixture of sodium salts coming from the utensils employed in the complicated manipulation of the material. The salt used by the earlier experimenters was undoubtedly much purer; their errors probably lay in the analysis, and had the

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* Redtenbacher, Wiener. Acad. Anzeiger f., 1865, 39.

opposite tendency. Either the loss of chlorine from the caesic chloride during its ignition, or the solution of argentic chloride by the water used in washing would tend to increase the apparent atomic weight. Hence one might have expected the atomic weight of caesium to lie between 133.0 and 132.6; probably nearer the former than the latter value. The value usually selected by Clarke and others, 132.9, has been in accord with this conclusion, and the present investigation shows it to have been surprisingly accurate.

III. THE ANALYSIS OF CAESIC CHLORIDE.

(a) *The Preparation of Materials.*

Caesic chloride is a colorless salt, crystallizing in anhydrous cubes which are very soluble in water. The aqueous vapor tension of the saturated solution is somewhat greater than that of the average air of American steam-heated laboratories, hence the crystals are deliquescent only on damp days. The German ascription of deliquescence to the salt is due evidently to the moister conditions prevalent in that country. The addition of alcohol to its solution precipitates much of the dissolved salt; and since it has no great tendency to form an acid chloride in solution, hydrochloric acid also precipitates it. Caesic chloride fuses into a limpid, colorless liquid at about 600°, losing a trace of its chlorine thereby if moisture is present and hydrochloric acid absent. On the other hand, the salt shows no trace of alkalinity after fusion in perfectly dry air, especially if it has been recrystallized from a hydrochloric acid solution. In the latter case, acid must always be present in the microscopic inclusions wherever water is, and the trace of hydrolysis on fusion is effectually prevented. Its properties thus fit it admirably for accurate work.

Wells * has shown that caesium may be separated from rubidium and other alkali metals by making use of the difference in solubilities of their trihalide salts. Of these the dichloriodide ($CsCl_2I$) offers peculiar advantages; for this salt not only is from eight to ten times less soluble than the corresponding rubidium salt, but also will crystallize below 70° in a rhombohedral form, while the other metals give the corresponding salts in the orthorhombic form only. By keeping the temperature below this point, crystals of the caesium salt may be obtained which are not likely to carry down rubidium, because of this helpful heteromorphy.

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* Wells, Am. J. Sci. (3), **43**, 17 (1901); Chem. News, **84**, 2184, Oct 4 (1901).

The recrystallization may be carried out very easily without much decomposition by cooling from moderately dilute hydrochloric acid (sp. gr. 1.06). The crystals should then be dried at room temperature; but after they have become almost dry they can be heated to 75° or 80° without much decomposition. In order to test the efficiency of this method, we experimented upon a mixture of 150 grams of lithium, sodium, potassium, rubidium, and caesium chlorides. From this mixture were obtained, after six recrystallizations as dichloriodide, about eight grams of caesic chloride, spectroscopically free from all the other metals, while only traces of caesium remained in the last few residual mother liquors. The greater part of the caesium used in this research had been prepared by Professor Wells in this way.* To test its purity as regards rubidium, the mother liquor from one recrystallization had been evaporated down to an extremely small bulk by systematic recrystallization, and was finally tested in the spectroscope. Not the slightest trace of rubidium could be detected. In all four successive recrystallizations as dichloriodide were made from about a kilogram of the salt, the crystals being well washed each time, and nothing was used over from any mother liquor. The product, weighing about 280 grams, was received from Professor Wells in this state, and our results show that it had reached a remarkable degree of purity.

To obtain the normal chloride from this pure caesium dichloriodide, the latter was placed in a porcelain crucible, and heated at from 90° to 100° in an electric oven, — a temperature far below the melting point of this salt (about 240° C.). The iodine and extra chlorine are slowly driven off, leaving the chloride beautifully white and very porous.

To eliminate any possible trace of iodine which might remain, one might dissolve the salt in a very small amount of warm water, and precipitate with pure alcohol. A better method, however, seemed to be to precipitate it by saturating the solution with hydrochloric acid. The caesic chloride was therefore transferred to a platinum dish, and dissolved in the least possible amount of warm water; hydrochloric acid gas from pure boiling concentrated aqueous hydrochloric acid was passed into the solution by means of a platinum tube; the mixture was allowed to cool, and the mother liquor was poured off into another platinum basin, the precipitate being washed with a little pure concentrated hydrochloric acid. This salt was again dissolved in water, precipitated, washed as before, and partly dried by suction, in a platinum Gooch crucible. After further drying for some time the salt was ready for use. The portion thus prepared was used as Sample I. As will be seen, the atomic weight of caesium found from this sample was 132.886.

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* The source of this material was pollucite, found at Mt. Mica, Paris, Maine.

A second sample was prepared from the mother liquor of the first by evaporating further in platinum vessels and precipitating, as before, with hydrochloric acid gas. The product thus obtained was again dissolved and again precipitated, and after being washed and dried gave Sample II, which yielded the atomic weight 132.883. It was essentially similar to Sample I, and is classed with it below.

The temperature of the caesic chloride solution containing an excess of hydrochloric acid in contact with platinum, must not be allowed to rise above 60° or 70°, otherwise the platinum will be attacked, and some caesium chlorplatinate precipitated, — a mishap which causes much trouble. This difficulty was probably due to the presence of a residual trace of undecomposed dichloriodide. The careful analyst well knows that platinum, which he is forced to use, forms by no means the inert receptacle which he desires. This fact was apparent more than once during the present research, and it is not impossible that both Samples I and II contained traces of platinum from this source, accounting for the slightly high atomic weight. Subsequently this error was yet more vigilantly guarded against, and the later specimens were as free as possible from platinum.

A third sample of caesic chloride was prepared from a somewhat impure caesium alum. In the first place the sulphuric acid was eliminated by means of baric chloride; the dichloriodide was obtained from the filtrate by adding to it an excess of aqua regia, and subsequently adding somewhat more than the calculated amount of iodine. The trihaloid salt separated out, upon cooling, in beautiful rhombohedral crystals. The iodine used in this preparation had been carefully resublimed. The dichloriodide was recrystallized eight times, no trace of rubidium being found by the spectroscope after the third recrystallization, although the original alum had contained much of this impurity. The normal caesic chloride was prepared from this product by heating in the electric oven at 95° for a much longer time than before, until long after all traces of iodine had disappeared; and great care was taken not to allow the solution of caesic chloride to become at all hot when precipitating this salt with hydrochloric acid gas in the platinum vessel. The atomic weight obtained from this third sample was 132.873. A trace of thallium was found spectroscopically in the extreme mother liquors.*

In the search, which proved fruitless, for an unknown heavier metal like caesium, a fourth sample of material was prepared. Since the insolubility of the dichloriodides seems to increase with the atomic weight of the basic element, the unknown element would probably be found in the extreme fraction of crystals. Hence, the most likely way of concentrating a trace of such an element would be to subject a large quantity of the caesium dichloriodide to systematic recrystallization, until a very small amount of substance remained.

Such a process was carried out, starting with 150 grams of the dichloriodide belonging to the sample used in the first analysis. This was systematically fractioned by crystallization twenty-five times, when only about one gram of the substance remained. The normal chloride was prepared from this, and its spectrum was carefully examined. No difference could be distinguished between the spectrum given by this portion and that given by any of the other preceding samples, or the extreme mother liquor. The last four extreme crops of crystals, averaging twenty-three crystallizations, were now combined, and enough caesium chloride was obtained from these for several analyses. The value of the atomic weight obtained from this fourth sample was 132.878, or about the mean of the preceding values.

This result agrees with the verdict given by the spectroscope, showing that recrystallization has in no way affected the purity of the caesium chloride. It must be concluded from this that the most efficient means known to us at present are not capable of isolating any unknown element from this material; and further, that the occurrence of such an element in the original mineral is unlikely. Wells has independently come to the same conclusion. Having gained considerable experience in the technique of all parts of the determination, it now seemed advisable to make a series of determinations in which the highest possible degree of accuracy would be attained at every step.

To this end a fifth portion of caesium chloride was prepared from the already pure chloriodide of Wells. The method of preparation was identical with that employed before, except that in every step greater care was taken, and in driving off the iodine and extra chlorine the salt was heated for two or three days in the electric oven after all apparent traces of iodine had disappeared.

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* This experience agrees with that of Wells, kindly communicated to us in a private communication. It is not impossible that thallium really represents the recurrence of the alkaline characteristics in the periodic system, modified by conflict of overlapping tendencies belonging to other groups.

This treatment left a salt which gave a perfectly colorless solution when dissolved in a small amount of water. Even the mother liquor that remained after precipitating the chloride with hydrochloric acid gas showed no trace of the yellow color due to platinum or iodine. The salt was precipitated twice successively with hydrochloric acid, each time being washed with a concentrated aqueous solution of the same acid. The analysis of this last sample, as will be seen, was conducted with all known precautions, but yielded, nevertheless, a result 132.877, essentially identical with the last.

Silver. Since the ratio of silver to caesic chloride was to be determined, as well as the ratio of argentic chloride to caesic chloride, the purity of the silver to be used must be unquestioned. The method of obtaining the pure metal was so nearly similar to that previously used in this Laboratory that further detailing is scarcely necessary. During the electrolytic part of the purification it is well to wrap the anode in a washed filter, as the "anode dust" is then prevented from mingling with and possibly contaminating the pure silver. The sparkling crystals were thoroughly washed, and fused on sugar charcoal, before the blow-pipe, and cooled in a reducing flame. The melted globule was clear, and free from any film; after cooling it showed no traces of having contained absorbed oxygen. The button was now scrubbed with pure water and clean sand, and cut into small pieces with a clean steel chisel. These pieces were then digested for a short time with dilute nitric acid and washed with ammonia water, in order to remove any traces of iron. To shield them from injurious gases they were kept under distilled water in a small flask. This silver was used in all the analyses, excepting in Nos. 11, 12, 13, 23, 24, and 25. For these analyses the silver was fused in a vacuum on a boat of lime.*

The hydrochloric and nitric acids used were carefully redistilled, the first and last portions of the distillate being rejected. The nitric acid, of course, showed no trace of opalescence with excess of silver nitrate after suitable dilution.† The phosphoric oxide was resublimed in an oxidizing environment. The water used was redistilled, first with alkaline permanganate, and subsequently with a trace of acid potassic sulphate, using in the second case a pure block-tin condenser, fitted to the neck of a Jena flask. It was kept in Jena flasks, and used as soon after distilling as possible. Portions were always tested with the nephelometer * for the presence of chlorine.

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* For further information concerning the preparation and purity of silver, see Proc. Am. Acad., **29**, 64 (1893); **32**, 62 (1896); **33**, 121 (1897), etc.; Z. anorg. Chem., **6**, 98 (1893); **13**, 90 (1896); **16**, 371 (1897).

† Proc. Am. Acad., **29**, 74 (1893).

(b) *Method of Analysis.*

Having now obtained pure material, it was essential to free the chloride from all traces of moisture without loss of a trace of chlorine.

It was soon found that this condition is assured when the substance is fused in a current of nitrogen and bottled in an atmosphere of dry air. For this purpose the apparatus which has been employed in this Laboratory to such advantage in determining other atomic weights is suitable and convenient, — namely, the "bottling apparatus," devised first for preparing and weighing pure magnesic chloride. † This device enables the substance to be ignited in a boat contained in a tube of the hardest glass or porcelain in the purest of dry nitrogen; and the final transference of the boat to the weighing bottle filled with dry air takes place without an instant's exposure to the outside atmosphere. The platinum boat, which had been freed from superficial iron in other investigations, was boiled in several different portions of nitric acid, and finally hydrochloric acid. Afterwards it was scrubbed with clean round sand, washed, ignited, and cooled in its own weighing bottle. Another weighing bottle, containing an appropriate amount of metal, of the same weight and volume as the first, was used as tare. The platinum boat was now weighed in its bottle, the weighing being done by substitution, and all precautions used in recent work done in this Laboratory were strictly followed. ‡ This weight of tube and boat scarcely changed throughout

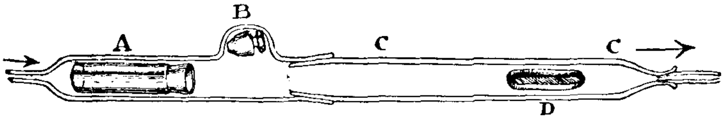

FIGURE 1.—BOTTLING APPARATUS, HORIZONTAL SECTION.
A = weighing bottle. B = stopper of bottle. C C = hard glass tube.
D = Platinum boat containing fused caesic chloride.

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* Richards, Proc. Am. Acad., **30**, 385 (1894); Z. anorg. Chem., **8**, 269 (1894).

† Richards and Parker, These Proceedings, **32**, 55 (1896); Z. anorg. Chem., **13**, 81 (1896).

‡ Richards, Proc. Am. Acad., **26**, 240 (1891); **28**, 1 (1893); **29**, 55; **30**, 369; Z. anorg. Chem., **1**, 150 (1892); **6**, 441 (1893), etc.

the whole series of investigations, and during a single experiment no appreciable variation in weight could be detected.

The boat was now filled with caesic chloride, and placed within the drying apparatus in the position indicated in the diagram, which illustrates the "bottling apparatus," but not the elaborate purifiers of air and nitrogen. The weighing bottle belonging to the platinum boat is placed in the extreme left of the tube A, while its stopper rests in the crevice B, and pure nitrogen thoroughly dried by means of phosphoric oxide was allowed to traverse the apparatus. After about three-quarters of an hour the platinum boat was gradually warmed by means of a large "fish-tail" spreader on the Bunsen burner, so that the whole of the boat may be equally heated. The caesic chloride soon fused, and was kept in a state of tranquil fusion until it was reasonably certain that every trace of residual water must have been expelled. The temperature was now gradually lowered, and the boat was finally allowed to cool to the temperature of the room. A current of air dried by phosphoric oxide was then turned on by opening the appropriate stop-cock, and was allowed to flow for about an hour before the bottling, so as to sweep out all the nitrogen. While the dry air was still flowing, a glass rod was inserted at the right, and the boat was pushed back into its weighing bottle. The stopper was now rolled out of its hiding-place, and was pushed by means of the glass rod firmly into the neck of the weighing bottle. After the tube A had been removed, the weighing bottle was partly removed by means of a hooked wire, and transferred by the aid of a pair of tongs to the desiccator. After suitable delay, the tube, boat, and substance were weighed in the same manner as before.

Since argentic chloride is essentially insoluble in solutions containing either silver or chloride, the chlorine in this pure, dry caesic chloride was precipitated by means of an excess of argentic nitrate. Since, moreover, it was desired to determine the weight of silver needed to effect the precipitation, this argentic nitrate was prepared from a carefully weighed portion of pure silver, and the excess in the filtrate also was weighed as chloride. It is not necessary to describe again the many precautions necessary to carry out, in a strictly quantitative fashion, the solution, mixing, and filtration of these materials, because previous communications from this Laboratory have discussed them in adequate detail. Due care was taken concerning the use of nonactinic light, the collection of the fragments of asbestos often lost from the Gooch crucibles, the traces of water retained by argentic chloride even at 150°, and all the other small causes of error. The most serious cause of difficulty is the dilemma introduced by two conflicting tendencies of argentic chloride, — namely, its solubility and its tendency to absorb argentic nitrate. When it is washed with very dilute argentic nitrate it does not dissolve, but it retains some of the dissolved nitrate; on the other hand, when repeatedly shaken with pure water it retains no impurity, but dissolves to an appreciable extent. In analyses 1 to 10 and 14 to 22, below, the chief precipitate was thoroughly washed by shaking with a solution containing a very small known amount of ionized silver, and the residual liquid moistening the Gooch filter was displaced by a few cubic centimetres of pure water. This reduces each error to a minimum, and almost balances their slight residual effect; but the fused argentic chloride thus obtained is not perfectly white and pearly. Therefore, in the final determinations (11 to 13, and 23 to 25) a safer but more troublesome procedure was adopted. After the precipitate had been well shaken in its glass-stoppered Erlenmeyer flask with several portions of wash water, containing a known amount of silver nitrate, and rinsed with about 50 c.c. of pure water, it was again shaken violently with five separate portions of water (in all about 500 c.c.) which contained no silver nitrate. This treatment was found to remove all traces of adsorbed silver nitrate. These wash waters were kept by themselves, and the amount of dissolved silver chloride which they contained was estimated by means of the nephelometer, this correction being added to the observed weight of silver chloride. Argentic chloride thus washed fuses into a beautiful translucent pearly white mass.

The balance used in this research was the one which has been used in the investigations of many other atomic weights.* It is short armed, and sensitive to about one-fiftieth of a milligram, with the largest load used in this work. The weights were of brass, gold plated. They were very carefully standardized, according to the method suggested by one of us,† and were used in no other work.

All weighings were reduced to the vacuum standard. For this purpose the specific gravity of caesic chloride was determined. 1.0436 and 0.8877 grams of the salt were found to displace, respectively, 0.2313 and 0.1965 gram of benzol at 20°. The specific gravity of the benzol referred to water at 4° was 0.880, hence the two values for caesic chlo-

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* Richards, Proc. Am. Acad., **26**, 242 (1891).

† Richards, Jour. Am. Chem. Soc., **22**, 144; Z. Phys. Chem., **33**, 605 (1900).

The Comparison of Caesic and Argentic Chloride.

$Ag = 107.930$; $Cl = 35.455$.

No. of Analysis.	Sample of CsCl.	Weight of CsCl in Vacuum.	Weight of AgCl in Vacuum.	Ratio AgCl : CsCl = 100.000 : x.	Atomic Weight of Caesium.
1	I	3.83054	3.26240	117.415	132.901
2	I	3.95120	3.36532	117.409	132.892
3	I	2.27237	1.93555	117.402	132.882
4	II	3.02935	2.58003	117.415	132.901
5	II	3.19774	2.72382	117.399	132.878
			Average (a) . . .	117.408	132.891
6	III	2.35068	2.00253	117.386	132.858
7	III	2.06245	1.75678	117.399	132.878
8	III	2.56372	2.18358	117.409	132.892
			Average (b) . . .	117.398	132.876
9	IV	2.01881	1.71972	117.392	132.868
10	IV	1.77391	1.51093	117.405	132.886
			Average (c) . . .	117.399	132.877
11	V	3.08160	2.62484	117.401	132.881
12	V	3.13117	2.66720	117.395	132.872
13	V	5.06656	4.31570	117.398	132.876
			Average (d) . . .	117.398	132.876

ride were 3.970 and 3.975. The average, 3.972, agrees sufficiently well with the value 3.992, found by Setterberg.* Therefore, for every gram of caesic chloride, as weighed in air, 0.00016 gram has been added to correct the observed weight.

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* Oefvers. Stockh. Acad. Forh., **39**, No, 6, 23 (1882).

THE COMPARISON OF CAESIC CHLORIDE AND SILVER.

Ag = 107.930; Cl = 35.455.

No. of Analysis.	Sample of CsCl.	Weight of CsCl in Vacuum.	Weight of Silver in Vacuum.	Ratio Ag : CsCl = 100.000 : x.	Atomic Weight of Caesium.
14	I	3.83054	2.45600	155.967	132.880
15	I	3.95120	2.53351	155.958	132.871
16	I	2.27237	1.45686	155.977	132.891
17	II	3.02935	1.94244	155.956	132.868
18	II	3.19774	2.05023	155.970	132.883
			Average (e) . . .	155.966	132.878
19	III	2.35068	1.50720	155.963	132.876
20	III	2.06245	1.32251	155.950	132.862
			Average (f) . . .	155.957	132.869
21	IV	2.01881	1.29434	155.972	132.886
22	IV	1.77391	1.13743	155.958	132.871
			Average (g) . . .	155.965	132.878
23	V	3.08160	1.97590	155.959	132.872
24	V	3.13117	2.00760	155.966	132.879
25	V	5.06656	3.24850	155.966	132.879
			Average (h) . . .	155.964	132.877

(c) *Results of Analysis of Caesic Chloride.*

The eight average values when collated give the following mean value :

From	(a)	Analysis	1 to 5	132.891
"	(b)	"	6 to 8	132.876
"	(c)	"	9, 10	132.877
"	(d)	"	11 to 13	132.876
"	(e)	"	14 to 18	132.878
"	(f)	"	19 to 20	132.869
"	(g)	"	21, 22	132.878
"	(h)	"	23 to 25	132.877
		Mean value		132.878

The "probable error" of this result, obtained according to the method of least squares, is only about ± 0.001, if each average is counted as a single determination. Of course, however, this "probable error" gives no clew as to the presence or absence of a possible *constant* error. The accuracy of the work and the purity of the silver may be better tested by comparing the weight of silver taken with that of the argentic chloride obtained; thus 22.93252 grams of silver gave 30.46482 grams of chloride, or 75.275 to 100.00; while Stas found the ratio 75.276 to 100.00. This agreement is as close as could be desired; it shows that no weighable trace of iodine remained in the preparations.

The means (d) and (h) represent by far the most careful work, and the six experiments included in them give the average 132.877 (± 0.0007). This seems to be the most trustworthy value of the atomic weight of caesium as obtained by the analysis of the chloride.

IV. The Analysis of Potassic Chloride.

As a further test of the purity of the materials and the accuracy of the method it seemed worth while to make a few similar determinations of the atomic weight of potassium. Since this value is already well known from the accurate work of Stas, the present work upon this element was far less searching than the preceding investigation.

For this purpose pure potassic chloride was prepared as follows. "Chemically pure" potassic chloride was dissolved in pure water, and to this was added a carefully prepared solution of chloroplatinic acid solution. The potassium chlorplatinate was well washed and reduced by ignition. The potassic chloride was dissolved with hot water in a platinum vessel and the solution was allowed to stand for some time in order that any chlorplatinate which had not been reduced might separate out. The solution was then evaporated, and the potassic chloride repeatedly precipitated with hydrochloric acid gas. No better method could be found for eliminating the last traces of chlorplatinate; for since this latter salt yields a dissimilar anion, it is even more soluble in strong hydrochloric acid than in water, while potassic chloride is almost insoluble in concentrated hydrochloric acid. The successive precipitation was repeated long after the last apparent traces of platinum had disappeared, and the last snow white crystals were washed with a little water and thoroughly dried over potash in a desiccator.

THE COMPARISON OF POTASSIC AND ARGENTIC CHLORIDES.

Ag = 107.930; Cl = 35.455.

No. of Analysis.	Weight of KCl in Vacuum.	Weight of AgCl in Vacuum.	Ratio AgCl : KCl = 100.000 : x.	Atomic Weight of Potassium.
			x =	
1	2.50019	4.80600	52.022	39.137
2	2.50391	4.81325	52.021	39.136
	Average		52.022	39.137

THE COMPARISON OF POTASSIC CHLORIDE AND SILVER.

No. of Analysis.	Weight of KCl in Vacuum.	Weight of Ag in Vacuum.	Ratio Ag : KCl = 100.000 : x.	Atomic Weight of Potassium.
			x =	
1	2.50019	3.61747	69.114	39.140
2	2.50391	3.62283	69.115	39.141
	Average		69.115	39.141
	Mean value of Series I and II . . .			39.139

The method of analysis was precisely similar to that used in the earlier analyses of caesic chloride, and the same sample of silver was used. The only difference worthy of note is in the fact that while drying the potassic chloride for analysis, fusion did not take place at the highest temperature attainable in the hard glass tube; but the salt was finely powdered, and the heating was maintained for a considerable length of time at about 700°, so as to afford as much opportunity as possible for the escape of the last traces of water and hydrochloric acid. Moreover since the mother liquor contained almost as much chlorine as the potassic chloride, weight for weight, the inclusion of a trace would not greatly affect the result.

The results for the potassic chloride are given in the foregoing table. The mean value 39.139 agrees very closely with the value obtained by Stas, whose later experiments with the chloride gave values ranging *from 39.142 to 39.15.*

Thus the evidence of this work affirms the trustworthiness of the experiments with caesic chloride.

V. THE QUANTITATIVE DECOMPOSITION OF NITRATES.

If a process could be devised which involved the use of a different set of apparatus, and another series of salts, any constant error which might possibly have crept into the previous method would probably not appear, and the new results would furnish a severe test of the correctness of the values found from the chlorine determinations. Hence a search was instituted for an entirely new method, involving different manipulation.

It is well known that when silicic oxide is heated with the nitrate of such a metal as caesium, the salt breaks up according to the following reaction: —

$$2CsNO_3 + SiO_2 = Cs_2O\,(SiO_2) + N_2O_5$$

the nitrogen pentoxide decomposing and passing off, leaving the caesic silicate. Therefore, from the loss in weight of a mixture containing a known weight of caesic nitrate and an excess of silica one might compute the atomic weight of caesium. This method, commonly used to determine nitric acid, has never before served for this exact purpose.

A series of preliminary experiments were of course necessary to determine the limit of accuracy attainable.

The silicic dioxide was prepared from a very pure specimen of natural sand. This material was washed several times with boiling water and then digested with hot concentrated sulphuric acid for several days. Subsequently it was again washed with water and covered with boiling aqua regia. It was allowed to stand on the steam bath in this solution for at least a week. Upon washing and drying the product, it was found to be beautifully white, and when heated to a high temperature in the blast lamp, it came almost at once to a constant weight. After powdering very finely in an agate mortar and heating again it was ready for use. Subsequently very pure artificial silica made from alkaline silicate was tried, but its use was abandoned because of the very great difficulty found in expelling every trace of water from it. All the experiments described below were made with the pure powdered quartz sand.

A rough experiment was first made in order that one might see where errors were likely to appear and what special precautions would have to be taken.

A large platinum crucible containing a short platinum spatula was half filled with the silicic dioxide, ignited for a short time, cooled in a desiccator and weighed, using another platinum crucible containing an equal amount of sand as a tare. Some pure powdered potassic nitrate was then added to the silicic dioxide, and the two were thoroughly mixed with the platinum spatula. The whole was again weighed in order to find the weight of nitrate added. Subsequently a large amount of the dioxide was spread over the top until the crucible was three-quarters full, and the whole was weighed again. This layer on top serves to catch any of the nitrate which might otherwise be carried away with the escaping vapors of the oxides of nitrogen. In order to find whether or not any moisture had been taken up during the mixing, the crucible was heated in the electric oven for two hours at about 150° C., cooled in a desiccator and weighed. The loss actually found after this additional drying never amounted to more than 0.00004 of a gram. The last weight was of course taken as the true one.

The mixture was now very carefully heated in the flame of the Bunsen burner, keeping the temperature at a comparatively low point for several hours and gradually increasing the heat until the temperature of the blast lamp was reached.

The melting point of potassic nitrate is about 339° C., while the caesic nitrate melts near 414° C.*; the full heat of a Bunsen burner should bring the silicate to a constant weight in either case.

It was soon noticed that the nitrate had a tendency to creep over the sides of the crucible, when a little above the fusing point. That part of the nitrate next the platinum, not being entirely surrounded by the sand, evidently had not the same opportunity of combining with the silica as the remainder of the salt; the exposure of this nitrate obviously might cause a loss by sublimation at a high temperature.

This danger was eliminated by using a small platinum crucible inside the larger one, in which to mix the nitrate and sand; this small crucible was completely surrounded and covered with a layer of silicic dioxide.

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* This temperature was found in a capillary hard-glass tube by means of a nitrogen-filled mercury thermometer, using a bath of potassic nitrate. The thermometer was suitably standardized and corrected for the cool column.

If with this arrangement the heating is conducted slowly and cautiously, no evidence appeared of loss by sublimation. No trace of deposition was found upon the under surface of the clean cool platinum lid of the crucible.

Potassic and caesic silicate both seem to be essentially permanent and non-volatile at 1000°, as far as the present experiments were concerned, and constant weight was easily reached.

The next problem to be solved was the finding of the best source of heat. In the first place, in order to avoid the danger of contamination from the impurities in illuminating gas, an electric furnace giving the desired temperature was used. We soon found, however, that the platinum heat-producing resistance was volatilized to a serious extent. The fire-clay of the oven was soon partially coated by a black film of sublimed platinum, and an empty crucible heated in the oven for an hour gained 0.0002 gram. Qualitative analysis showed beyond doubt the nature of this sublimate, when deposited upon pure porcelain. This tendency of platinum under certain circumstances to volatilize has been studied by Hall;* it brought an unwelcome complication into the present work. Evidently platinum cannot safely be used as a resistance for obtaining high temperatures in accurate work; we had almost introduced a large error into the investigation while trying to eliminate a small one. Of course the temperature of the wire in the electric drying oven previously mentioned is too low to cause any danger from this source.

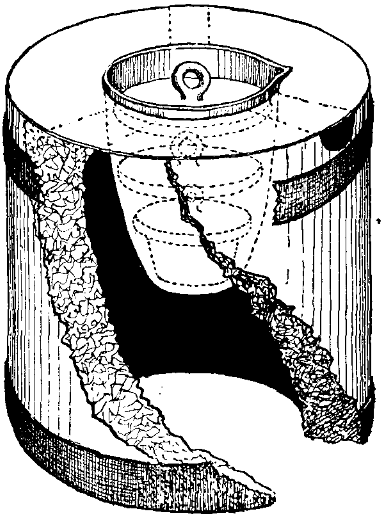

FIGURE 2.

Recourse was now had to a contrivance which had been used in this Laboratory in other work. Its general character will be seen from the diagram taken from the original paper. *

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* Hall, J. Am. Chem. Soc., **22**, 8, 494 (1900).

A large especially made crucible of Berlin porcelain fits tightly into a jacket or furnace of fire clay; the platinum crucible to be ignited is placed inside this, being protected from the porcelain by a spiral of platinum wire between the two crucibles. As a further protection from the impurities of the gas flame, the platinum crucible is fitted with a small porcelain cover, while a larger cover is placed above it. A circular opening is cut in two ignited asbestos sheets, lying on top of the furnace, into which the larger outside crucible fits tightly; these sheets serve as a further means of deflecting the impurities from the flame, since the only escape for the products of combustion is through the draught holes in the side. The asbestos sheets are not shown in the diagram. By means of this furnace the crucible can be heated at a uniform temperature for any length of time.

Experiments were made to ascertain if in this furnace any platinum volatilized from the crucible at high temperatures. When half full of silicic dioxide, no appreciable loss or gain was observed, either in the Bunsen flame or in the full flame of the blast lamp. When empty, however, although the crucible lost nothing in the Bunsen flame, it lost 0.00012 grams when heated for an hour in the blast. Evidently traces of platinum were volatilized from the *inside* of the vessel into the open air space. The outside annular space was probably kept saturated with platinum vapor from the platinum wire. Since the crucible was always more than half full of silica when placed in the furnace during the work described below, it was assumed that no platinum was lost by volatilization.

When the precautions indicated in this chapter are carefully followed no further complications arise, and the method easily yields constant results which seem to be unimpeachable.

The chief objections to the method lie in the fact that the nitrate must be wholly free from water (an end easily attained by gentle fusion in the present case †), and the further fact that any error which may occur is intensified in the calculation, as is always the case when the two substances weighed both contain either the element whose atomic weight is sought or the standard of reference.

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* Richards, Proc. Amer. Acad., **33**, 399 (1898); Am. Chem. J., **20**, 701 (1896).

† Stas (Aronstein), Untersuchungen, 235 (1867).

VI. THE ANALYSIS OF POTASSIC NITRATE.

The details of experimentation having been mastered, three careful analyses of potassic nitrate were made.

The salt lends itself easily to purification by recrystallization, on account of the great temperature coefficient of its solubility; it was recrystallized eight times in Jena glass vessels and four more times in platinum. The product thus obtained was carefully dried and fused in a platinum dish at the lowest temperature possible, in order to avoid the formation of even a trace of nitrite. The pure white solid was finely powdered and again dried. Although, as Hempel has shown,* a hard mineral causes considerable abrasion in an agate mortar, a soft substance like potassic nitrate may be powdered safely in one.

In every case at least five times as much of the silica by weight was used as of the nitrate.

In order to reduce the observed weights to the vacuum standard, the specific gravity of the potassic nitrate was taken as 2.09.† It was assumed that the contraction which takes place when potassic oxide combines with silica is negligible as far as the reduction to vacuum is concerned, and hence that the specific gravity of potassic oxide may be taken as 2.65, according to Karsten.‡

Following are the results of the three consecutive determinations of the atomic weight of potassium, using the nitrate prepared in the manner described above.

RESULTS OF ANALYSIS OF POTASSIC NITRATE.
$O = 16.000$; $N = 14.040$.

No. of Analysis.	Weight of N_2O_5 in Vacuum.	Weight of KNO_3 in Vacuum.	Ratio $N_2O_5 : K_2O = 100.00 : x$.	Atomic Weight of Potassium.
	grams.	grams.	$x =$	
30	0.96692	1.81034	87.227	39.138
31	1.68005	3.14564	87.235	39.142
32	1.36512	2.55598	87.235	39.142
	Average		87.232	39.141

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* Hempel, Chem. Centralbl., **2**, 719 (1901).

† Landolt and Börnstein, Tabellen (1894), 134. This is the most probable average of many determinations.

‡ Karsten, Schweigger's Ann., **65**, 419 (1832).

The average of these results (39.141) agrees well with that obtained from the chloride (39.139), and the constancy of the results indicates that the method is adequate for its purpose.

VII. The Analysis of Caesic Nitrate.

The decomposition of caesic nitrate may be effected in precisely the same fashion as that of potassic nitrate. The only points to be considered especially are the preparation of the pure salt and the data necessary for reduction to the vacuum standard.

Very pure caesic nitrate prepared from the four times recrystallized dichloriodide, and subsequently twice recrystallized, had already been prepared for us by Professor Wells. Like potassic nitrate, it is very easy to recrystallize.*

After gentle fusion this preparation was finely powdered, and served for the first two analyses. In order to test its purity, a portion was twice recrystallized in platinum vessels, very carefully fused in platinum, powdered as before, and subjected to analysis. Because the results were identical with those obtained from the first sample, further purification seemed unnecessary.

No measurements of the specific gravity of caesic nitrate could be found, therefore this constant was determined, since its value is needed in order to find the weight of air displaced. 1.7638 grams of fused caesic nitrate displaced upon one occasion 0.4210 gram and at another time 0.4208 gram of rectified benzol at 28° C. The specific gravity of the benzol was 0.880, 20°/4° ; therefore the specific gravity of the caesic nitrate must be 3.687.

In determining the air displaced by caesic oxide when combined with silica, the only fact available is the density of the hydrate. If we assume that the same contraction takes place when caesic oxide combines with water as when it combines with silica, the specific gravity of caesic oxide under these conditions is easily computed to be 4.9 if caesic hydroxide has a specific gravity 4.0,† as follows. Twice the molar volume of caesic hydroxide is 57 c.c. greater than that of water, and if the molar weight of caesic oxide is divided by this difference, the result given above is obtained. This assumption would have given in the case of potassium about the same result as that used under that head ; and while it is of course inexact, it will answer sufficiently well the present purpose. In applying the corresponding correction, it was subtracted from that computed for the nitrate, and the difference added to the final weighing, in order to determine the true loss of weight which corresponds to the nitric peutoxide.*

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* Wells, Am. J. Sci., **3**, 46, 186.

† Bekétoff, Bull. Acad. St. Petersb., **2**, 171 (1890) ; Ch. Centralbl., **2**, 451 (1891).

RESULTS OF THE ANALYSIS OF CAESIC NITRATE.

O = 16.000; N = 14.040.

No. of Analysis.	Sample of $CsNO_3$.	Weight of N_2O_5 in Vacuum.	Weight of $CsNO_3$ in Vacuum.	Ratio $N_2O_5 : Cs_2O = 100.000 : x$.	Atomic Weight of Caesium.
		grams.	grams.		
33	I	1.04273	3.76112	260.699	132.882
34	I	0.92416	3.33334	260.689	132.876
35	II	1.33590	4.81867	260.706	132.886
36	II	1.39960	5.04807	260.679	132.871
		Average		260.693	132.879

In this case, as in all the other cases treated in this paper, the analyses were consecutive, and the table contains all that were made excepting such as were intentionally of a preliminary nature and not worthy of detailed mention. The results will be discussed in connection with the other data at the close of the paper.

VIII. THE ANALYSIS OF CAESIC BROMIDE.

While the preceding results seemed to be conclusive as to the atomic weight of caesium, it was thought desirable to study the bromide also, because this salt is capable of especially exact analysis.

The bromide is easily prepared. In the first place, by adding to the nitrate an excess of hydrobromic acid and bromine, the tribromide is obtained on cooling in the form of deep orange crystals. Pure materials, prepared by the well known methods of Stas, were used in this process. After having been freed from chlorine and iodine the bromine was redistilled five times; and the hydrobromic acid made from it was thoroughly washed in the gaseous state as well as twice distilled in aqueous solution, rejecting the first and last portions. Hence there is no possibility that the product could have contained either chlorine or iodine. The trace of mother liquor included in the crystals of tribromide was eliminated by means of several successive recrystallizations and washings with hydrobromic acid. After drying the crystals as thoroughly as possible in a centrifugal machine, they were placed in a porcelain dish and kept at a temperature of 80° C. in an electric drying oven until all obvious signs of the extra bromine had disappeared. The pure white bromide was then heated for eight hours at 150°, and subsequently dissolved in a little water and precipitated by means of pure concentrated hydrobromic acid. The crystals were provisionally dried over potash and were then ready to fuse. Like the chloride, they were not deliquescent in the ordinary air of the laboratory.

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* Let a = weight of nitrate in air, and m = its correction to the vacuum standard; b = the weight of the oxide in air, and n = its correction to the vacuum standard; c = weight of silica. Then the weight of nitric peroxide in vacuum = true loss of weight = $(a + c) + m - [(b + c) + n] = (a - b) + (m - n)$.

The apparatus for fusing, bottling, and weighing the bromide was similar to that used in the case of the chloride, except that an addition was provided for the sake of introducing a slight amount of perfectly dry hydrogen bromide into the nitrogen during the fusion. * This was done because the loss of a trace of bromine was feared, although indeed none was ever noticed. After partial cooling the acid was wholly replaced by pure dry nitrogen and finally by air, which had passed through a freshly filled train of purifiers and freshly sublimed phosphoric oxide. The fused product, like the chloride, affected neither methyl orange nor phenol phthalein in the preliminary tests made to prove the normality of the salt. Caesic bromide fuses at a point somewhat above 600° C., although below the softening point of hard glass, but the exact point was not determined.

The analysis of the salt was less troublesome than the analysis of the chloride, because of the very meagre solubility of argentic bromide, which served as the means of precipitating the bromine. The silver used was from the same sample as that used in the case of the chloride, its weight was determined as before, and the very faint trace of bromide in the last pure wash-waters was estimated with the nephelometer as in the case of analyses 11, 12, and 13.

The specific gravity of caesic bromide was found by Setterberg † to be 4.37; in verification of this value the following determinations of this constant were made. At different times 0.9964 and 0.8456 gram of

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* Richards and Baxter, Proc. Am. Acad., **33**, 124 (1897).

† Oefvers. Stockh. Acad. Forh., **39**, No. 6, 23 (1882). We are indebted to Mr. F. R. Fraprie for this reference, which has been generally overlooked.

the salt displaced 0.2004 and 0.1699 gram of benzol respectively, therefore the two results (since the benzol had the specific gravity 0.880 referred to water at 4°) were 4.382 and 4.378 respectively, the mean value being 4.380. The temperature was 20°. This value was used in correcting the weight of caesic bromide to the vacuum standard.

Comparison of Caesic and Argentic Bromides.

Ag = 107.93; Br = 79.955.

No. of Analysis.	Weight of CsBr in Vacuum.	Weight of AgBr in Vacuum.	Atomic Weight of Cs.
	grams.	grams.	
37	3.49820	3.08815	132.878
38	6.20409	5.47673	132.883
39	7.17300	6.33213	132.880
			Mean = 132.880

Comparison of Caesic Bromide and Silver.

Ag = 107.93; Br = 79.955.

No. of Analysis.	Weight of CsBr in Vacuum.	Weight of Ag in Vacuum.	Atomic Weight of Caesium.
	grams.	grams.	
40	3.49820	1.77402	132.873
41	6.20409	3.14606	132.885
42	7.17300	3.63740	132.884
			Mean = 132.881

The agreement of these analyses with one another and with the previous results seems to indicate their trustworthiness. Further emphasis may be laid upon this consistency by pointing out that the 8.55748 grams of silver produced 14.8970 grams of argentic bromide, which must then have contained 57.444 per cent of silver. Stas found 57.445 per cent, or essentially the same value.

IX. THE ATOMIC WEIGHTS OF CAESIUM, POTASSIUM, AND NITROGEN.

In the preceding description are recorded analytical results involving the atomic weights of caesium, potassium, nitrogen, chlorine, bromine, silver, and oxygen. Of these, by common consent, the value for oxygen is fixed; the others may be calculated in many ways from the data in connection with the data obtained by other experimenters.

No attempt was made to explore any such complete experimental field as that suggested by Clarke* in his recent paper on the calculation of atomic weights. This omission was due not so much to lack of time as to the feeling that some of the results would have been fruitless. Experimental skill in attaining uniformity of conditions can almost always reduce the so called "probable error" of manipulation to a vanishingly small quantity; but constant errors of a chemical nature are much harder to avoid and much more serious when perpetrated. Hence the "probable error" is little or no clue to the trustworthiness of the results, and can rarely be used as a reliable measure of the relative preponderance to be ascribed to the respective members of a series of conflicting figures.† Instead of attempting to observe the weight of every element which combines with a given weight of every other without discrimination, it seems much wiser to select those operations in which the constant errors, by long and thorough study, have been discovered and rendered susceptible of elimination. A single series of experiments, and thoroughly investigated and properly executed, is of more value than ten series containing unknown and incorrigible errors.

The preceding figures involve seven different ratios for determining the atomic weight of caesium, assuming in each case one or more of the other atomic weights to be known. Five of these are obvious in the results given; the other two refer the salts of caesium to those of potassium. These latter ratios are of interest because they eliminate, at least in part, any possible constant errors in the processes immediately employed, by virtue of the parallelism between the experiments with the two metals. The atomic weight of caesium should therefore be influenced only by those constant errors which already affect the assumed atomic weight of potassium. 100.000 parts of silver were found to be equivalent to 155.964 and 69.115 parts of caesic and potassic chlorides respectively, hence these two weights must be equivalent to one another, or 100.000

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* Am. Chem. J., **27**, 321 (1902).

† Richards, A Table of Atomic Weights, Am. Chem. J., **20**, 543 (1898).

of potassic to 225.659 of caesic chloride. Again, 100.000 parts of argentic chloride were found to result from the precipitation of 117.398 and 52.022 parts of the same chlorides, a result which gives the almost equal ratio 100.000 : 225.670. The average of these two, 225.665, is given below.

In the same way, 100.000 parts of nitric anhydride were found to be equivalent to 87.232 and 260.693 parts of potassic and caesic oxides respectively.

A similar calculation might be performed with the bromide, using Stas's data obtained from potassic bromide; but this result would not have the same significance because the experimental work would not then have been done all by the same hands under the same conditions.

Below is given a table containing the results for the constant in question computed from the several ratios. After each standard substance is given its assumed molecular weight, and after each value of the atomic weight of caesium is given the "mean error" or average deviation from the mean. This is chosen rather than the "probable error" because the latter is in most cases too small to possess important significance. In the cases of the last two ratios the average of the mean errors of the potassium and caesium series is recorded.

From	AgCl	(143.385)	: CsCl,	Cs = 132.876	± 0.003
"	Ag	(107.93)	: CsCl,	Cs = 132.877	± 0.003
"	AgBr	(187.885)	: CsBr,	Cs = 132.880	± 0.002
"	Ag	(107.93)	: CsBr,	Cs = 132.881	± 0.004
"	N_2O_5	(108.08)	: Cs_2O,	Cs = 132.879	± 0.005
"	KCl	(74.595)[*]	: CsCl,	Cs = 132.879	± 0.002
"	K_2O	(94.280)	: Cs_2O	Cs = 132.879	± 0.003
	Average			132.879[†]	

It is of interest to note that not one of these seven independent figures deviates from the average of all by an amount greater than its mean experimental error. The inference to be drawn from this is that constant errors have been essentially eliminated from the various chemical processes involved.

For the atomic weight of potassium we have three direct ratios, which give the following results: —

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[*] This value of potassic chloride is known directly with reference to oxygen from the analysis of potassic chlorate, which (according to Clarke) contains 39.154 per cent of oxygen.

[†] If O = 15.879, Cs = 131.874.

$AgCl$ (143.385) : KCl gives $K = 39.137 \pm 0.001$
Ag (107.93) : KCl gives $K = 39.141 \pm 0.001$
N_2O_5 (108.08) : K_2O gives $K = 39.141 \pm 0.002$
Average 39.140

This result confirms the results of Stas, but disagrees with Clarke's value (39.11) by nearly one tenth of a per cent. The reason for the difference is to be traced chiefly to the influence of some relatively inexact analyses of potassic iodide by Marignac, which have seriously reduced Clarke's average value; moreover Clarke's estimate of chlorine is somewhat lower than ours. The value given above, 39.14, seems to us to represent more nearly the true atomic weight of potassium.

On the other hand, assuming from the experiments with the halides, that the atomic weights of caesium and potassium are respectively 132.879 and 39.139, it becomes possible to calculate the atomic weight of nitrogen from the analysis of the nitrate. Since the atomic weight sought is not included in both of the substances weighed, an error is not magnified by the calculation.

Cs_2O (281.758) : N_2O_5, $N = 14.040$
K_2O (94.278) : N_2O_5, $N = 14.037$
Average . . . $N = 14.039$

This value affords another confirmation of the remarkable work of Stas.

Many other combinations of the preceding data with those of other experimenters might have been made, but the preceding are enough to show the most important features of the new investigation.

It is a pleasure to acknowledge our indebtedness to the Cyrus M. Warren Fund for Research in Harvard University for some of our apparatus, and to Dr. Wolcott Gibbs for invaluable platinum utensils, as well as to Professor Wells for his great generosity in providing the caesium material.

X. Summary.

1. As an outcome of forty-two analyses involving seven different ratios and three compounds of caesium (the chloride, bromide, and nitrate), the atomic weight of caesium is found to be 132.879, if oxygen is 16.000.

2. Incidentally it is found that the atomic weights of potassium and nitrogen most consistent with the new data are nearly 39.14 and 14.04.

3. In confirmation of Wells, caesium dichloriodide is found to afford

a convenient and exceptionally thorough means of purifying caesium material.

4. No evidence was found indicating the presence of any similar element of higher atomic weight, except a trace of thallium.

5. The specific gravities of caesic chloride, bromide, and nitrate were found to be respectively 3.972, 4.380, and 3.687; and the melting point of the nitrate was found to be 414° C.

6. The precautions necessary to effect the precisely quantitative decomposition of alkaline nitrates are described.

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ON THE MULTIPLE POINTS OF TWISTED CURVES.

BY JOHN N. VAN DER VRIES.

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I.

INTRODUCTION.

TWISTED curves have been studied in detail by Cayley,[*] Salmon,[†] Halphen,[‡] and Nöther,[§] and to some extent by Genty,[||] Picquet, Rohn, Weyr, Kohn,[¶] Hoppe, Stäckel, and others. In the consideration of these curves attention has been paid not only to the surfaces on which they lie and the surfaces which contain them as their complete or partial intersections, but also to the singularities of the curves themselves. Among these singularities we wish to emphasize in particular the multiple points and the apparent double points. It has in general been overlooked that multiple points may be of different kinds according to the way in which the tangents at these points lie. It has been stated that

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[*] Considérations générales sur les courbes en espace. Comptes Rendus, LIV. pp. 55, 396, 672. On Halphen's characteristic n in the theory of curves in space. Jour. für Math. CXI. (1893), pp. 347–352.

[†] Geometry of Three Dimensions (1882), pp. 278–382.

[‡] Mémoire sur les courbes gauches algébriques. C. R., LXX. p. 380.

[§] Zur Grundlegung der Theorie der algebraischen Raumcurven. Kronecker Journal, XCIII. pp. 271–318.

[||] Étude sur les courbes gauches unicursales. Bull. de la Soc. Math., t. 9.

[¶] Ueber algebraische Raumcurven. Wien. Ber. LXXXII. pp. 755–770.

double points and points of higher multiplicity than two have no effect on the number of apparent double points. Halphen* gives as the lower limit of the number of apparent double points of a curve of order m the greatest integer contained in $\left(\frac{m-2}{2}\right)^2$; the curve then being the intersection of a quadric cone and a surface of order $\frac{m}{2}$ or $\frac{m+1}{2}$, according as m is even or odd. According to this, the lower limit of the number of apparent double points of a quintic is four; and Salmon† gives four quintics having four or more apparent double points. Genty,‡ in his consideration of the maximum number of actual double points of a twisted curve, makes use of the lower limit of the number of apparent double points as given by Halphen and applies it to all curves. Picquet, in a note to Genty's paper, states that Halphen's lower limit holds "pour une courbe gauche quelconque." Dr. Williams,§ however, gives a quintic curve that has a triple point and only three apparent double points; thus one less than the number given by Halphen's formula. This diminution is due to the fact that the curve has a triple point at which the tangents do not lie in one plane.

It is the object of this paper to consider curves that have multiple points of different kinds and to investigate the effect of these different kinds of points on the other singularities of the curves; to consider the curves first as the intersection, partial or complete, of any two surfaces whatever that may contain them, and then as the intersection, partial or complete, of a cone and a monoid; to consider the number of actual and apparent double points to which a multiple point may be equivalent; and finally to consider in detail all quintic, sextic, and septimic curves that have multiple points. In the consideration of the curve as the intersection of a cone and a monoid, we are led to the consideration of the different kinds of lines possible on a monoid and the effect on the curve of intersection when these lines are common to the cone and the monoid. In the consideration of the composition of multiple points we shall only consider points of multiplicity not greater than seven.

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* Loc. cit. p. 381.

† Loc. cit. p. 318.

‡ Loc. cit. p. 151.

§ Geometry on Ruled Quartic Surfaces. Proceedings of the American Academy of Arts and Sciences, Vol. XXXVI. No. 3, July, 1900, p. 60.

II.

CONSIDERATION OF CURVES THAT HAVE MULTIPLE POINTS, THE TANGENTS AT WHICH DO OR DO NOT LIE IN ONE PLANE.

1. Every algebraic curve is the complete or partial intersection of two algebraic surfaces. Two surfaces of orders μ and ν, say S_μ and S_ν, respectively, intersect in a curve of order $\mu\nu$, which may, however, break up into components of lower orders. This curve or these curves may have points that are not ordinary points, but points of higher multiplicity. The number of these points on any component is generally finite; if infinite, the component is a multiple curve. Points that are ordinary points on both surfaces are in general ordinary points on the curve of intersection. The tangent line to the curve at any ordinary point of it is the intersection of the tangent planes to the two surfaces at that point. There may be points of multiplicity ρ on the curve for which the tangents all lie in one plane. Such a point does not in any way imply a singularity on any surface that contains the curve. It simply means that the two surfaces lie so close together in the neighborhood of the point that they intersect in ρ directions from the point, i. e. that there are ρ lines that meet the curve there in $\rho+1$ points each. Such a multiple point, say the point A, that has its tangent lines lying in one plane may, however, be a multiple point on one of the two surfaces that contain the curve, but it must be an ordinary point on the other surface. Namely, if a curve having a multiple point A lie on a surface S_μ that has the point A as an ordinary point, the tangents to the curve at A are tangent lines to S_μ at that point, i. e. they lie in the tangent plane to S_μ at A. If A be also an ordinary point on the surface S_ν the tangents lie in the tangent plane common to S_μ and S_ν, but if it is a multiple point on S_ν the tangents are the intersection of the tangent cone to S_ν by the tangent plane to S_μ at that point. The curve obtained in the latter case can, however, always be obtained as the intersection of two surfaces, each of which has the point A as an ordinary point, e. g. by the intersection of the surface S_μ with the surface $S_\mu + S_{\mu-\nu} S_\nu$, if $\nu \stackrel{=}{<} \mu$, or with the surface $S_\mu S_{\nu-\mu} + S_\nu$, if $\mu < \nu$; where $S_{\mu-\nu}$ and $S_{\nu-\mu}$ are surfaces of orders $\mu-\nu$ and $\nu-\mu$, respectively, and where $S_{\nu-\mu}$ does not contain the point A. If, however, the point A is a point of multiplicity k on the surface S_μ and of multiplicity k' on S_ν, it will be a point of multiplicity kk' on the complete intersection. The kk' tangents to the curve at this point will be given by the intersection of the tangent cones of order k

and k', respectively, to the two surfaces at that point, i. e. they will be the common generators of the two cones, and no three will in general lie in one plane. Thus, a multiple point at which the tangents lie in one plane will in general be an ordinary point on one of the two surfaces that contain the curve, whereas a multiple point at which the tangents do not lie in one plane must be a multiple point on both surfaces that contain the curve.

2. If the tangents at a k-tuple point of a curve all lie in one plane, the curve will be met k times at that point by any line that lies in the plane of the tangents and passes through the point; a line, however, not in this tangent plane can meet the curve only once there. A cone drawn from an arbitrary point as a vertex with the curve as base will have the line joining the vertex to the k-tuple point as a k-tuple edge. The number of actual double points of the curve is therefore affected by a point of this kind, but the number of apparent points is not. For, a line drawn from an arbitrary point in space has two points the same for both surfaces in the case of an apparent double point, whereas in the case of a point of the above kind the line has only one point the same for both surfaces. We assume that the arbitrary point does not lie in the tangent plane nor on a tangent line to the curve at the multiple point. Nor does it lie on the cone having the multiple point as vertex and the curve as base, for the line to the multiple point would then meet the curve again. If, however, the multiple point is such that the tangents do not lie in one plane, the line from an arbitrary point in space will meet the surfaces in two or more points that are the same for both surfaces, and the number of apparent double points is affected. We shall investigate the number of apparent double points of a curve that has such a multiple point; we shall use the method of Salmon.* We consider two surfaces U and V of orders μ and ν respectively. They intersect in a curve of order $\mu\nu$ which may break up into a number of component curves. The points on the "lines through two points" that pass through an arbitrary point are given by the intersection of the curve UV and a certain surface, which we shall call S. The equation of this cone S with its vertex at the point (1) is obtained by eliminating $\lambda : \kappa$ between

$$\kappa^{\mu-1}\Delta_1 U + \frac{\kappa^{\mu-2}\lambda}{2!}\Delta_1^2 U + \frac{\kappa^{\mu-3}\lambda^2}{3!}\Delta_1^3 U + \ldots\ldots = 0,$$

$$\kappa^{\nu-1}\Delta_1 V + \frac{\kappa^{\nu-2}\lambda}{2!}\Delta_1^2 V + \frac{\kappa^{\nu-3}\lambda^2}{3!}\Delta_1^3 V + \ldots\ldots = 0;$$

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* Salmon's Geometry of Three Dimensions (1882), p. 309.

where $\Delta_1 \equiv x_1 \frac{\partial}{\partial x} + y_1 \frac{\partial}{\partial y} + z_1 \frac{\partial}{\partial z} + s_1 \frac{\partial}{\partial s}$, etc. We then have

$$S \equiv \left| \begin{array}{llllll}
\Delta_1 U, & \frac{1}{2}\Delta_1^2 U, & \ldots\ldots\ldots, & U_1, & & \\
& \Delta_1 U, & & & \cdot & \\
& & \ddots & & & \ddots \\
& & & \Delta_1 U, & \ldots\ldots\ldots, & U_1 \\
\Delta_1 V, & \frac{1}{2}\Delta_1^2 V, & \ldots\ldots, & V_1, & & \\
& \Delta_1 V, & & & V_1, & \\
& & \ddots & & & \ddots \\
& & & & \Delta_1 V, \ldots\ldots, & V_1
\end{array} \right|$$

If the point (0, 0, 0, 1) is a k-tuple point on U and a k'-tuple point on V, the equation $U = 0$ will be of the form

$$\phi_\mu + \ldots\ldots + \phi_{k+1} s^{\mu-k-1} + \phi_k s^{\mu-k} = 0,$$

and $V = 0$ will be of the form

$$\psi_\nu + \ldots\ldots + \psi_{k'+1} s^{\nu-k'-1} + \psi_{k'} s^{\nu-k'} = 0;$$

where $\phi_\mu, \ldots\ldots, \phi_k, \psi_\nu, \ldots\ldots, \psi_{k'}$ are functions of degrees $\mu, \ldots\ldots, k, \nu, \ldots\ldots, k'$, respectively, in x, y, and z. The terms of $\Delta_1{}^r U$ that are of lowest degree in x, y, and z are thus of degree $k - r$, if $r \leqq k$, and of degree 0, if $k + 1 \leqq r$. It is now necessary to find the terms of S that are of lowest degree in x, y, and z in order to know the multiplicity of (0, 0, 0, 1) on S. We know that the eliminant of two homogeneous equations is homogeneous in the coefficients of those equations. The terms of S that are of lowest degree in x, y, and z are thus those that are of highest degree in s. We shall therefore look for the terms of highest degree in s in the following scheme, where we have taken account only of the terms of lowest degree in x, y, and z, and therefore of highest degree in s. In this scheme $f_1, f_2, \ldots\ldots$ are polynomials of degrees 1, 2, ..., respectively, in x, y, and z; similarly for the g's. We have filled in the lacking members of our scheme with terms having zero-coefficients. The scheme then really consists of four parts, the first having terms of degree $\mu - k$ in s; the second, terms of degree $\nu - k'$ in s; the third, terms in descending powers of s from $s^{\mu-k}$ down; and the fourth, terms in descending powers of s from $s^{\nu-k'}$ down. Our scheme is then the following: —

$$
\begin{array}{l}
f_{k-1}\,s^{\mu-k},\ \ldots,\ f_1\,s^{\mu-k},\ a_k\,s^{\mu-k},\ a_{k+1}\,s^{\mu-k-1},\ \ldots,\ a_\mu,\ 0\,s^{-1},\ \ldots\ldots\ldots,\ 0\,s^{-\nu+2} \\
0\,s^{\mu-k},\ \ldots \qquad \ddots \qquad \vdots \\
\vdots \qquad \ddots \qquad \ldots,\ 0\,s^{-1} \\
0\,s^{\mu-k},\ \ldots\ldots,\ 0\,s^{\mu-k},\ f_{k-1}\,s^{\mu-k},\ \ldots,\ f_1\,s^{\mu-k},\ a_k\,s^{\mu-k},\ a_{k+1}\,s^{\mu-k-1},\ \ldots,\ a_\mu \\
g_{k'-1}\,s^{\nu-k'},\ \ldots\ g_1\,s^{\nu-k'},\ b_{k'}\,s^{\nu-k'},\ \ldots,\ b_\nu,\ 0\,s^{-1},\ \ldots\ldots\ldots\ldots,\ 0\,s^{-\mu+2} \\
0\,s^{\nu-k'},\ \ldots \qquad \ddots \qquad \vdots \\
\vdots \qquad \ddots \qquad \ldots,\ 0\,s^{-1} \\
0\,s^{\nu-k'},\ \ldots\ldots\ldots\ldots,\ 0\,s^{\nu-k'},\ g_{k'-1}\,s^{\nu-k'},\ \ldots,\ g_1\,s^{\nu-k'},\ b_{k'}\,s^{\nu-k'},\ \ldots,\ b_\nu
\end{array}
$$

Looking for terms of highest degree in s and lowest degree in x, y, and z, we first take $\nu - 1$ constituents diagonally, beginning with the first row and the kth column, then $k - 1$ constituents diagonally, beginning with the νth row and the first column, and finally $\mu - k$ constituents from the last $\mu - k$ rows and columns. These last $\mu - k$ constituents must be constants from the main diagonal, otherwise a zero-coefficient will be brought in, causing the product to vanish. This choice will give us the terms or S of highest degree in s. For, if we begin with the constituent of the first row and $(k + 1)$st column, we can bring in one more constituent in our second group, and one less constituent from the main diagonal in our last group. Doing this we diminish the degree of s by unity in each of the first $\nu - 1$ rows, and bring in one new term in $s^{\nu-k'}$; this does not, however, compensate for the loss of $s^{\nu-1}$. A similar loss will occur every time we move one place to the right. If we begin with the constituent of the first row and $(k - 1)$st column, we do not change the power of s in the first $\nu - 1$ rows, but lose a multiple of $s^{\nu-k'}$ in the second group, and only bring in an additional constant constituent in the third group. A similar loss will occur whenever we move a column to the left. Nor can we obtain a term of higher degree in s by taking any other combination of constituents of our scheme. For, if one term of a determinant is known, another term can be obtained by interchanging any two rows or columns of the determinant and taking the constituents corresponding to those used in the determinant before it was so changed, e. g. considering the two rows and columns: —

$$\begin{vmatrix} & \vdots & & \vdots & \\ \cdots & a, & \cdots & b, & \cdots \\ & \vdots & & \vdots & \\ \cdots & c, & \cdots & d, & \cdots \\ & \vdots & & \vdots & \end{vmatrix}$$

if a and d are factors of the known term, we can get another term by substituting b and c for a and d. This process can be repeated until any desired term is obtained. Compare the term we have chosen with any other term that can be obtained in this way. If a and d are both in the first group, and d is p columns to the right of a, the degree of b in s is p less than that of a, and c is of a degree the same as that of d. Therefore when we substitute b and c for a and d respectively we do not increase the degree of this term in s. Similarly, if both constituents come from the second or third group. If b and c are in the first and second groups respectively, a is of the same degree in s as the degree of b, whereas d is of the same degree in s as the degree of c, if $c - d \gtreqless k'$, but of a degree less by $c - d - k'$ if $k' < c - d$. Thus the substitution of a and d for b and c respectively does not increase the degree in s of our term. If a and d are in the first and third groups, respectively, the substitution of b for a decreases the degree in s by $b - a$, whereas the substitution of c for d increases the degree by $b - a$ only if $b - a < \nu - k'$, and by less than $b - a$ if $\nu - k' \gtreqless b - a$. These interchanges of pairs of corresponding constituents, however often performed, do not increase the degree of s in our term. A zero-coefficient, whenever it occurs, causes the term to vanish. We can therefore in no way get a term of higher degree in s than the term first selected. In this term, s appears to the degree

$$(\nu - 1)(\mu - k) + (k - 1)(\nu - k'), \quad \text{if } k' < k;$$

and to the degree

$$(\mu - 1)(\nu - k') + (k' - 1)(\mu - k), \quad \text{if } k < k'.$$

These are both, however, equivalent to

$$\nu\mu - kk' - \mu - \nu + k + k',$$

which is thus the highest degree to which s can occur in S. As the eliminant is homogeneous of degree $(\mu - 1)(\nu - 1)$ in the variables, the lowest degree to which x, y, and z together occur is

$$(\mu - 1)(\nu - 1) - (\nu\mu - kk' - \mu - \nu + k + k') = (k - 1)(k' - 1).$$

The point $(0, 0, 0, 1)$ is thus a $[(k - 1)(k' - 1)]$-tuple point on the surface S.

The points of intersection of U, V, and S are the "points" of the "lines through two points;" they are in general $\mu\nu(\mu - 1)(\nu - 1)$ in number. The number h of "lines through two points" or of apparent double points being one-half of this number, we have in general

$$2h = \mu\nu(\mu - 1)(\nu - 1).^*$$

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* Salmon's Geometry of Three Dimensions (1882), p. 309.

These points are thus determined by the intersection of the curve $U\,V$ with the surface S. In the case we are considering, the curve $U\,V$ has a $k\,k'$-tuple point at (0, 0, 0, 1), i. e. there are $k\,k'$ branches of the curve passing through this point. This point is also a $[(k-1)\ (k'-1)]$-tuple point on S. As an ordinary point of the curve does not in general lie on S, no branch of the curve through the multiple point lies on S. It can also be easily shown that no branch of the curve in general touches S at the point (0, 0, 0, 1). Every branch of the curve thus meets the surface S $(k-1)\ (k'-1)$ times at the point (0, 0, 0, 1), that is U, V, and S intersect $k\,k'\ (k-1)\ (k'-1)$ times at the point (0, 0, 0, 1). These points are thus included among the $\mu\,\nu\ (\mu-1)\ (\nu-1)$ points common to U, V, and S. We therefore have

$$\mu\,\nu\ (\mu-1)\ (\nu-1) = 2\,h + k\,k'\ (k-1)\ (k'-1)$$

i. e.
$$h = \tfrac{1}{2}\,[\mu\,\nu\,(\mu-1)\ (\nu-1) - k\,k'\,(k-1)\ (k'-1)].$$

Therefore: —

A point of multiplicity $k\,k'$ on a curve that is the complete intersection of two surfaces containing the point as points of multiplicities k and k', respectively, reduces the number of apparent double points of the curve by $\tfrac{1}{2}\,k\,k'\ (k-1)\ (k'-1)$.

3. If two surfaces of orders μ and ν that intersect in a curve of order $\mu\,\nu$ having a $k\,k'$-tuple point of the above kind have stationary contact at a point, the line drawn to this point from an arbitrary point is a cuspidal edge on the cone that has the curve as base and this arbitrary point as vertex. Similarly, the line drawn to a point of ordinary contact of the two surfaces is a double edge on the cone. If the surfaces have t points of ordinary contact and β points of stationary contact, we have as the order of the cone $m = \mu\,\nu$, as its number of cuspidal edges, $\beta = \beta$, and as its number of double edges,

$$\delta = h + t = \tfrac{1}{2}\ [\mu\,\nu\ (\mu-1)\ (\nu-1) - k\,k'\ (k-1)\ (k'-1)] + t.$$

Then, by Art. 327, Salmon's Geometry of Three Dimensions (1882), we have

$$r = \mu\,\nu\ (\mu+\nu-2) - 2\,t - 3\,\beta + k\,k'\ (k-1)\ (k'-1),$$

$$n = 3\,\mu\,\nu\ (\mu+\nu-3) + 3\,k\,k'\ (k-1)\ (k'-1) - 6\,t - 8\,\beta,$$

$$a = 2\,\mu\,\nu\ (3\,\mu+3\,\nu-10) + 6\,k\,k'\ (k-1)\ (k'-1) - 12\,t - 15\,\beta,$$

$$\begin{aligned}2y = \mu\nu\,\{\mu\nu\,(\mu+\nu-2)^2 + 2\,[k k'\,(k-1)\,(k'-1) - (2t+3\beta+5)] \\ (\mu+\nu-2) + 8\} + \{k k'\,(k-1)\,(k'-1) \\ - (2t+3\beta+5)\}^2 - (3\beta+25),\end{aligned}$$

$$\begin{aligned}2x = \mu\nu\,\{\mu\nu\,(\mu+\nu-2)^2 + 2\,[k k'\,(k-1)\,(k'-1) - (2t+3\beta+2)] \\ (\mu+\nu-2)\} + \{k k'\,(k-1)\,(k'-1) - (2t+3\beta+2)\}^2 - (\beta+4),\end{aligned}$$

$$\begin{aligned}2g = \mu\nu\,\{9\,\mu\nu\,(\mu+\nu-3)^2 + 2\,(\mu+\nu-3)\,[9\,k k'\,(k-1)\,(k'-1) \\ - 3\,(6t-8\beta)]\} + \{3\,k k'\,(k-1)\,(k'-1) - (6t+8\beta)\}^2 \\ - 22\,\{\mu\nu\,(\mu+\nu-3) + k k'\,(k-1)\,(k'-1) - 2t - 3\beta\} \\ + 5\,(\mu\nu - 2\beta)\,;\end{aligned}$$

where r, n, α, y, x, and g are the rank, class, number of stationary planes, number of planes through two lines, number of points on two lines, and number of lines in two planes, respectively, of the system.

4. If the intersection of the two surfaces U and V breaks up into two components of orders m and m', we have $\mu\nu = m + m'$. If the point $(0, 0, 0, 1)$ is a ρ-tuple point on the component of order m and a ρ'-tuple point on the component of order m', we have $\rho + \rho' = k k'$. We shall represent the number of lines through two points on the component of order m by h, those through two points on the component of order m' by h', and those through a point on each component by H''. Considering the intersection of the first component with the surface S, we have

$$m\,(\mu-1)\,(\nu-1) = 2\,h + H'' + \rho\,(k-1)\,(k'-1).$$

Similarly, for the other component we have

$$m'\,(\mu-1)\,(\nu-1) = 2\,h' + H'' + \rho'\,(k-1)\,(k'-1).$$

From these, by addition and subtraction, we obtain

$$\mu\nu\,(\mu-1)\,(\nu-1) = 2\,(h + h' - H'') + k k'\,(k-1)\,(k'-1)\,;$$

and $\quad (m-m')\,(\mu-1)\,(\nu-1) = 2\,(h-h') + (\rho-\rho')\,(k-1)\,(k'-1).$

Therefore, summing for all points that are k-tuple points on U and k'-tuple points on V, we have, if the curve is the complete intersection,

$$h = \tfrac{1}{2}\,[\mu\nu\,(\mu-1)\,(\nu-1) - \sum k k'\,(k-1)\,(k'-1)]\,; \qquad \text{(I)}$$

or, if the curve breaks up into two components of orders m and m' that contain the points as ρ-tuple and ρ'-tuple points, respectively,

$$h - h' = \tfrac{1}{2}\left[(m - m')(\mu - 1)(\nu - 1) - \sum (\rho - \rho')(k - 1)(k' - 1)\right]; \quad \text{(II)}$$

also,

$$h + h' + H'' = \tfrac{1}{2}\left[\mu\,\nu\,(\mu - 1)(\nu - 1) - \sum k k' (k - 1)(k' - 1)\right]. \quad \text{(III)}$$

It is evident from these formulae that multiple points of the curve that are ordinary points on either surface have no effect on the number of apparent double points.

Thus, if a curve is the complete intersection of two surfaces, we can at once find its number of apparent double points by means of (I). A sextic with a quadruple point that is the complete intersection of a quadric cone and a cubic surface that has the vertex of the cone as a double point has four apparent double points. From formula (II) it is evident that, when the number of apparent double points of one component is known, the number of the other component can be obtained at once. Thus, if the quadric and cubic surfaces mentioned above have a line in common, the residual intersection will be a quintic curve having a triple point. We then have

$$\mu = 3,\ \nu = 2,\ k = 2,\ k' = 2,\ \rho = 3,\ \rho' = 1,\ h' = 0;$$

then, substituting these in (II), we obtain $h = 3$. The quintic with a triple point thus has three apparent double points.

5. We have next to find the number of points in which two surfaces S_μ and S_ν can touch when they have a point in common that is a k-tuple point on S_μ and a k'-tuple point on S_ν in the case where the complete intersection does not break up. This intersection is then of order $\mu\,\nu$ and has on it a point of multiplicity $k k'$. A cone of order $\mu\,\nu$ drawn from an arbitrary point to a curve of order $\mu\,\nu$ cannot have more than $\frac{(\mu\nu - 1)(\mu\nu - 2)}{2}$ double edges. The edge to the $k k'$-tuple point counts as $\frac{k k' (k k' - 1)}{2}$ double edges. If we subtract from the remaining number the number due to apparent double points, we have the maximum number that can be due to contacts. We have then

$$\frac{(\mu\nu - 1)(\mu\nu - 2)}{2} - \frac{k k' (k k' - 1)}{2} - \frac{\mu\,\nu\,(\mu - 1)(\nu - 1)}{2} + \frac{k k' (k - 1)(k' - 1)}{2}$$
$$= \frac{\mu\,\nu\,(\mu + \nu - 4)}{2} - \frac{k k' (k + k' - 2)}{2} + 1.$$

Every point of multiplicity $k k'$ of this kind reduces the possible number of actual double points of the curve by $\dfrac{k k' (k + k' - 2)}{2}$. Therefore: —

A $k k'$*-tuple point on a curve that is the complete intersection of two surfaces that contain the point as points of multiplicities* k *and* k', *respectively, is equivalent to* $\dfrac{k k' (k + k' - 2)}{2}$ *actual and* $\dfrac{k k' (k - 1)(k' - 1)}{2}$ *apparent double points.*

The quadruple point on the sextic curve mentioned on page 483 is equivalent to four actual and two apparent double points. This will be discussed more in detail later.

6. It is also possible to find an upper limit to the number of actual intersections of the two components of orders m and m' into which the curve may break up; where $m + m' = \mu \nu$. Two curves of orders m and m' have $m m'$ intersections, actual and apparent. A point that is a ρ-tuple point on one curve and a ρ'-tuple point on the other curve counts as $\rho \rho'$ intersections of the two curves. We have then

$$m m' = t + H'' + \rho \rho', \qquad \text{(IV)}$$

where t is the number of actual intersections of the two curves. Substituting the value of H from (III) into (IV), we get

$$t = m m' - \rho \rho' - \frac{\mu \nu (\mu - 1)(\nu - 1)}{2} + \frac{k k' (k - 1)(k' - 1)}{2} + h + h'.$$

The maximum or minimum number of actual intersections of the two curves thus corresponds to the maximum or minimum number of apparent double points of the two components. A twisted curve of order m with a ρ-tuple point can be projected into a plane curve of order m with a ρ-tuple point. This plane curve of order m cannot have more than $\frac{1}{2}(m - 1)(m - 2) - \frac{1}{2}\rho(\rho - 1)$ double points. The component of order m cannot therefore have more than $\frac{1}{2}(m - 1)(m - 2) - \frac{1}{2}\rho(\rho - 1)$ actual and apparent double points. If it have δ actual double points, we must have

$$h \overline{\gtrless} \tfrac{1}{2}(m - 1)(m - 2) - \tfrac{1}{2}\rho(\rho - 1) - \delta.$$

Similarly, for the component of order m' with a ρ'-tuple point and δ' actual double points, we have

$$h' \overline{\gtrless} \tfrac{1}{2}(m' - 1)(m' - 2) - \tfrac{1}{2}\rho'(\rho' - 1) - \delta'.$$

Substituting these limits for h and h' in (III) we obtain

$$H \geqq m\,m' - \rho\,\rho' - \frac{\mu\,\nu\,(\mu + \nu - 4)}{2} + \frac{k\,k'\,(k + k' - 2)}{2} + \delta + \delta' - 2.$$

$$\therefore t \leqq \frac{\mu\,\nu\,(\mu + \nu - 4)}{2} = \frac{k\,k'\,(k + k' - 2)}{2} - \delta - \delta' + 2.$$

If the intersection of a quartic cone and a quadric surface, not passing through the vertex of the cone, break up into two quartic curves, and neither quartic has a double point, we have $t \leqq 10$, i. e. the two quartics cannot intersect each other in more than ten points. Similarly, the quintic curve with a triple point mentioned above cannot meet the line that is the residual intersection of the quadric and the cubic surfaces in more than one point in addition to the vertex.

7. An irreducible curve C_m of order m lying on a surface S_ν of order ν can be cut out of this surface by some surface S_μ of order μ. If the curve C_m is not the complete intersection of the two surfaces, it may be necessary in the classification of the curves to consider the residuals and classify according to them also. In such a case it is possible, when the one surface S_ν is given, to determine the second surface in such a way that the residual may take a particular form. It is always possible to find a value of μ that will suffice to make S_μ cut C_m out of S_ν. A surface of order μ is determined by $\frac{1}{6}\,(\mu + 1)\,(\mu + 2)\,(\mu + 3) - 1$ points. These points must be taken in such a way that the surface S_μ contains the curve. This can be done by making S_μ contain $m\,\mu + 1$ points of C_m. There are in general enough points at our disposal to do this if μ satisfies the inequality : —

$$m\,\mu + 1 \leqq \tfrac{1}{6}\,(\mu + 1)\,(\mu + 2)\,(\mu + 3) - 1.$$

If, however, this inequality gives a value of μ that is greater than ν, care must be taken that S_μ does not contain S_ν as a factor. Thus, if the points left at our disposal after we have made S_μ contain C_m are greater in number than the number necessary to determine a surface of order $\mu - \nu$, these can be taken in such a way that S_μ does not break up into S_ν and a surface $S_{\mu-\nu}$ of order $\mu - \nu$. We must then determine μ from the inequality : —

$$\tfrac{1}{6}\,(\mu - \nu + 1)\,(\mu - \nu + 2)\,(\mu - \nu + 3)$$
$$\leqq \tfrac{1}{6}\,(\mu + 1)\,(\mu + 2)\,(\mu + 3) - 1 - (m\,\mu + 1).$$

In the case we are concerned with in particular, the surface S_μ has a k-tuple point at a ρ-tuple point of the curve. S_μ is then determined by $\frac{1}{6}(\mu+1)(\mu+2)(\mu+3)-\frac{1}{6}k(k+1)(k+2)-1$ additional points. The multiple point counts in general as $\rho\,k$ intersections of C_m and M_μ. Therefore, in order to make S_μ contain C_m, it is only necessary for us to make it contain $m\,\mu-\rho\,k+1$ points of the curve in addition to the multiple point. Therefore μ must in general satisfy the inequality: —

$$m\,\mu-\rho\,k+1 \overset{=}{<} \tfrac{1}{6}(\mu+1)(\mu+2)(\mu+3)-\tfrac{1}{6}k(k+1)(k+2)-1.$$

If, however, this inequality gives a value of μ that is greater than the value of ν, care must be taken that the surface S_μ having a k-tuple point does not break up into the surface S_ν having a k'-tuple point and a surface $S_{\mu-\nu}$ of order $\mu-\nu$ having a $(k-k')$-tuple point; that is of the points necessary to determine S_μ one more than enough to determine the surface $S_{\mu-\nu}$ must be taken as not lying on S_ν. This can be done if μ satisfies the inequality: —

$$\tfrac{1}{6}(\mu-\nu+1)(\mu-\nu+2)(\mu-\nu+3)-\tfrac{1}{6}(k-k')(k-k'+1)(k-k'+2)$$
$$\overset{=}{<} \tfrac{1}{6}(\mu+1)(\mu+2)(\mu+3)-\tfrac{1}{6}k(k+1)(k+2)-1-(m\,\mu-\rho\,k+1).$$

Summing for all points that are ρ-tuple points on C_m while k-tuple points on S_μ and k'-tuple points on S_ν, we have μ given in general by the inequality: —

$$m\,\mu-\sum\rho\,k+1 \overset{=}{<} \tfrac{1}{6}(\mu+1)(\mu+2)(\mu+3)-\tfrac{1}{6}k(k+1)(k+2); \quad \text{(V)}$$

or, if this gives a value of μ such that $\nu \overset{=}{<} \mu$, by the inequality: —

$$\begin{aligned} m\mu-\sum\rho k+1 \overset{=}{<} \tfrac{1}{6}(\mu+1)(\mu+2)(\mu+3)-\tfrac{1}{6}k(k+1)(k+2) \\ -\tfrac{1}{6}(\mu-\nu+1)(\mu-\nu+2)(\mu-\nu+3) \\ +\tfrac{1}{6}(k-k')(k-k'+1)(k-k'+2), \end{aligned}$$

that is by

$$\sqrt{\frac{m^2}{\nu^2}-m\left(\frac{4}{\nu}-1\right)-\frac{\nu^2}{12}+\frac{1}{3}+\frac{4}{\nu}+\Sigma\frac{k'}{\nu}\{(k+1)(k-k'+1)+\tfrac{1}{3}(k'+1)(k'-1)\}-2\,\Sigma\,\rho\frac{k}{\nu}}$$
$$\overset{=}{<}\mu-\left(\frac{\nu}{2}+\frac{m}{\nu}-2\right). \quad \text{(VI)}$$

It is thus possible to find a surface S_μ that will cut the curve from the given surface S_ν, where μ is the smallest integer that satisfies (V) or (VI); using (V) if it gives a value $\mu<\nu$, otherwise using (VI). It may be possible to cut the curve out of S_ν by a surface of lower order than the smallest integer that satisfies (V) or (VI), but we need never take a surface of higher order.

If, however, the surface S_ν is not considered as given, we can determine its order as the smallest integer that satisfies the inequality: —

$$m\nu - \sum \rho k' + 1 \leqq \tfrac{1}{6}(\nu+1)(\nu+2)(\nu+3) - \tfrac{1}{6}\sum k'(k'+1)(k'+2) - 1. \qquad \text{(VII)}$$

The order of S_ν being obtained from (VII), the order of S_μ can be obtained from (VI) above. From the above three inequalities we can always find the orders of two surfaces that will contain a given curve as their complete or partial intersection.

III.

CONSIDERATION OF A CURVE AS THE INTERSECTION OF A CONE AND A MONOID.

A.

1. If a twisted curve is not the complete intersection of two surfaces, difficulties may arise in the determination of the characteristics of the residual and therefore of the curve itself. If the residual is a proper curve of an order less than four it is definitely determined, but if of an order as great as four there will be difficulty in determining to what species it belongs. A convenient method of representing a curve in general is as the intersection of a cone and a monoid, a monoid being a surface that has a point of multiplicity one less than the order of the surface. This method is due to Cayley.* The advantage of this method lies in the fact that the residual consists entirely of straight lines through a point.

The cone K drawn from an arbitrary point in space to a curve C_m of order m will in general be of order m, that is, every edge † of the cone will meet the curve in one, and only one, point. A multiple edge counts for the number of simple edges to which it is equivalent; the number of points of C_m on a k-tuple edge of K_m thus being k. There are, however, curves that are met by lines from certain points in two, three, or more points, e.g. any curve that is wound around a cone a number of times, or a curve, say the curve C_{pq} of order pq, that is the complete intersection of two cones of orders p and q not having the same vertex. Every edge of the first cone meets the curve q times and every edge of the second cone meets it p times. As Professor Story has proved, all simple generators of a cone of any order whatsoever meet any curve lying on it in the same number of points. Thus if a generator of a cone K_a of order a meets a curve lying on it in β points, every generator will meet it in β points, and the curve, if it does not pass through the vertex, will be of order $a\beta$. The cone K_m in this case is the cone K_a taken β times. It is, however, only for special positions of the vertex that this breaking up of the cone K_m can occur. In the case of the curve C_{pq} of order pq mentioned above, if we take the point (0, 0, 0, 1) as the vertex of the one cone and the point (0, 0, 1, 0) as the vertex of the other cone, and eliminate y between the equations of the two cones, we shall get a homogeneous expression of degree pq in x, z, and s. This is in general the equation of a cone of order pq having the point (0, 1, 0, 0) as its vertex, and the curve C_{pq} as its base. This vertex is an arbitrary point, and so a line from a point off the curve meets the curve in general in only one point. For the eliminant above will not in general break up into two, three, or more equal factors, as would be necessary in order to have every edge of the cone from the point (0, 1, 0, 0) to the curve meet the curve in two, three, or more points. There may be certain points that, taken as vertices of this cone, will cause it to break up, but for an arbitrary point it will not generally do so. Thus in the case of the quartic of the first kind, if we consider it as the intersection of two quadric cones, there are at most two other quadric cones on which the curve may lie, i.e. there are at most two other points off the curve from which an infinite number of lines can be drawn to meet the curve in two points. Excluding these quadric cones, any other cone whose base is the quartic in question will be a cone of the fourth order, every edge of which meets the curve once. Thus we can always take the vertex of the cone K in such a way that the cone will be of the same order as the curve, an edge meeting the curve in only one point.

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* Comptes Rendus, t. LIV. (1862) pp. 55, 396, 672.

† Hereafter, when we speak of an edge, we shall mean a simple edge unless it is otherwise designated.

There is, however, from an arbitrary point in space a finite number of lines that meet the curve C_m in two distinct points. These correspond to the apparent double points of the curve and are double edges on the cone K_m. As a curve has only a finite number of apparent double points, it will not in general have an apparent multiple point of multiplicity greater than two. A simple edge of the cone is thus due to an ordinary point on the curve, a double edge to an actual or apparent double point on the curve, and a multiple edge of multiplicity greater than two to an actual multiple point of the same multiplicity. As before, it is not necessary to consider the case where the line from the vertex to a multiple point crosses the curve again. Nor shall we consider the case where the vertex lies on a tangent to the curve, these tangents generating a definite developable surface on which the arbitrary vertex may also be assumed not to lie.

The fact that we can always find a point such that an edge of the cone having this point as vertex and the curve as base generally meets the curve in only one point enables us to obtain any curve C_m as the partial intersection of a cone K_m and a monoid M_μ of order μ having the same vertex as K_m. For, every line through the vertex of M_μ, and therefore every edge of K_m, meets M_μ in one point distinct from the vertex; viz., the point where C_m crosses the edge of K_m. An ordinary edge of K_m is not necessarily a line on M_μ; a certain number of them, however, generally lie on the monoid. A double edge of the cone due to an apparent double point of the curve always lies on the monoid. For it meets C_m, and therefore the monoid M_μ on which C_m lies, in two points distinct from the vertex. It thus meets M_μ in $\mu + 1$ points and therefore lies on it. A line to an actual double point or to a multiple point that has its tangents lying in one plane meets the curve in general only once at that point, which is not enough to make it lie on M_μ. The line from the vertex to a multiple point that does not have its tangents all lying in one plane is a line on M_μ. This will be shown more fully later.

The general equation of M_μ is of the form: —

$$u_{\mu-1}\, s + u_\mu = 0;$$

where $u_{\mu-1}$ and u_μ are homogeneous functions of x, y, and z of degrees $\mu - 1$ and μ respectively. These functions thus represent cones, known as the inferior and superior cones, respectively, of the monoid. The number of independent constants in the above equation is $\mu(\mu + 2)$. In order to make M_μ contain C_m, we must make it contain $m\mu + 1$ points of the curve. We can do this if

$$m\mu + 1 \leqq \mu(\mu + 2),$$

i. e. if
$$\frac{m-2}{2} + \tfrac{1}{2}\sqrt{m^2 - 4m + 8} \leqq \mu,$$

i. e. if
$$m - 1 \leqq \mu;$$

that is, we can always obtain the curve as the intersection of a cone of order m and a monoid of order $m - 1$. We may be able to obtain the curve as the intersection of the cone with a monoid of lower order, but we need never take a monoid of order higher than $m - 1$. Thus far we have not taken into account the multiple points of the curve. It will be shown later that even when this is done the monoid need not be of an order higher than $m - 1$. The lines of intersection of the inferior and superior cones, $\mu(\mu - 1)$ in number, are commonly known as the lines of the monoid. The cones $u_{\mu-1}$ and K_m have $m(\mu - 1)$ edges in common, all of which meet C_m and therefore M_μ in a point distinct from O, where O is the point $(0, 0, 0, 1)$. They therefore lie on M_μ, and with the curve C_m form the complete intersection of K_m and M_μ. Since $\mu \overset{=}{<} m - 1$, more lines are common to K_m and M_μ than there are lines on M_μ. Some lines of M_μ must therefore count two or more times as lines common to K_m and M_μ, that is, they must be double lines or lines of higher multiplicity on K_m corresponding to apparent multiple points on C_m. As the vertex of K_m has been taken in such a way that the curve C_m has no apparent multiple points of multiplicity greater than two, we see that at least $(m - \mu)(\mu - 1)$ lines of M_μ must be double edges on K_m. There will be just this number if all lines of M_μ lie on K_m, an additional double edge on K_m being necessary for every line of M_μ that does not lie on K_m. The curve C_m of order m that is the partial intersection of K_m and M_μ thus has in general at least $(m - \mu)(\mu - 1)$ apparent double points. Assuming $\mu = m - 1$, we see that a curve of order m having no multiple points never has less than $m - 2$ apparent double points. We wish now to investigate the effect of a multiple point of C_m on this number of apparent double points. If the point is a multiple point on M_μ, the line joining it to the vertex will be a line on M_μ, for it meets M_μ in enough points distinct from the vertex to make it lie on M_μ. We are thus led to the consideration of lines and, in particular, of multiple lines on M_μ, and to the investigation of the effect of these on the curve of intersection when they are common lines of K_m and M_μ.

2. Ordinary or multiple lines on any surface may be of different kinds. A line is torsal or scrolar on a sheet of the surface containing it according as the tangent plane to that sheet of the surface is or is not the same at every point of the line.* If the line xy is a k-tuple line on a surface, the equation of the tangent planes along that line will be given by the terms of the kth degree in x and y in the equation of the surface. According as the equations of these planes, separately or collectively, do or do not contain z and s will these planes be dependent or independent

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* See Cayley, Collected Papers, VII 334.

of the position of the point of tangency on the line. A multiple line may thus be torsal on all sheets, torsal on some and scrolar on the remaining, or scrolar on all sheets of the surface that contain it. A line that is scrolar on any or all sheets containing it may, moreover, be scrolar of different kinds. By scrolarity of the first kind we mean that the tangent plane revolves through an angle of 180° as we pass along the line. A generator, say the line $x\,y$, of the quadric x (linear function of z and s) $+\ y$ (linear function of z and s) is a line of this kind. A k-tuple line on a surface may be scrolar of the first kind on each of a number of sheets separately, or it may be scrolar of this kind on a number of sheets taken together. In the latter case the sheets are inseparably connected, the tangent planes to them together revolving through 180°. If a line is scrolar of the kth kind, the tangent plane revolves through $k \times 180°$. There are then k points along this line that have the same tangent plane, that is, every tangent plane is a k-tuple tangent plane. A number of sheets in this case may also be inseparably connected, as in the case of lines scrolar of the first kind.

As the lines of M_μ are the lines common to the inferior and superior cones of M_μ, it is evident that for a line to be a line on M_μ it must be a line on both cones, and for it to be a k-tuple line on M_μ it must be a line of multiplicity k on one cone and of multiplicity not less than k on the other cone. A number of cases must therefore be considered according to the relative multiplicities of the line on the two cones of M_μ.

I. We shall first consider the case where the line $x\,y$ is a k-tuple line on the superior cone and a $(k+1)$-tuple line on the inferior cone. The equation of M_μ can then be put into the form: —

$$(v_{k+1}\, z^{p-1} + v_{k+2}\, z^{p-2} + \dots)\, s + (w_k\, z^{p+1} + w_{k+1}\, z^{p} + \dots) = 0,$$

where $p = \mu - k - 1$; v_α and w_α being homogeneous functions of degree α in x, y, and z. The tangent planes along the line $x\,y$ are given by $w_k = 0$; the line is therefore torsal on all k sheets passing through it. There are no points on the line of higher multiplicity than k distinct from the vertex. We assume $w_k \neq 0$, otherwise the line $x\,y$ is a $(k+1)$-tuple line on both cones, and therefore also on M_μ; this is a case to be considered later. The line $x\,y$ therefore counts as $k\,(k+1)$ of the $\mu\,(\mu-1)$ lines on M_μ. This case includes the case where

$$v_{k+1} = v_{k+2} = \dots\dots = v_{k+g-1} = 0,$$

i.e., where the line is a $(k+g)$-tuple line on the inferior cone, while only a k-tuple line on the superior cone. We assume $2 \leqq g \leqq \mu - k - 1$.

The line is still a k-tuple line on M_μ, torsal on every sheet through it, but counts as $k(k+g)$ lines of M_μ. The polynomials v_{k+g} and w_k may have any number κ (where $\kappa \leqq k$) of their linear factors equal, i. e. κ sheets of M_μ may touch κ sheets of both $u_{\mu-1}$ and u_μ along $x\,y$. The line in this case counts as $k(k+g)+\kappa$ lines of M_μ. There is no difficulty in seeing the effect of any combination of tangent planes on the number of lines on M_μ. The quantity w_k may break up in a great variety of ways, e. g. $x^\alpha y^\beta \ldots\ldots$; $x\,y$ being torsal on all k sheets of M_μ, the number of lines adjacent to $x\,y$ alone being affected by the way in which v_{k+g} and w_k break up.

II. The next case is where the line $x\,y$ is a k-tuple line on both cones. The equation of the monoid can then be put into the form: —

$$(v_k z^p + v_{k+1} z^{p-1} + \ldots\ldots)\, s + (w_k z^{p+1} + w_{k+1} z^p + \ldots\ldots) = 0. \quad (1)$$

We assume $w_k \neq 0$; otherwise the line will not be a k-tuple line on both cones, but will belong to a later case. We also assume v_k ⦶ w_k * (and $\therefore v_k \neq 0$); otherwise we shall have

$$(v_k z^p + v_{k+1} z^{p-1} + \ldots\ldots)\, s + (a\, v_k z^{p+1} + w_{k+1} z^p + \ldots\ldots) = 0.$$

Substituting $s - a\,z$ for s, this reduces to

$$(v_k z^p + v_{k+1} z^{p-1} + \ldots\ldots)\, s + [(w_{k+1} - a\, v_{k+1})\, z^p + \ldots\ldots] = 0,$$

which is a later case. The line $x\,y$ is a k-tuple line on the monoid whose equation is (1), and has no point on it of multiplicity greater than k. The tangent planes along this line are given by the terms $v_k\, s + w_k\, z$. As we have assumed v_k ⦶ w_k, s and z cannot factor out, and the line $x\,y$ must be scrolar of the first kind on one or more sheets of M_μ. If v_k and w_k have no common factor, that is if the line $x\,y$ is torsal on no sheet of M_μ, it is scrolar on all k sheets. These k sheets, however, are inseparably connected, that is the tangent planes to all of them together revolve through 180°. If v_k and w_k have α factors in common (where $\alpha \leqq k-1$), the line is torsal on α sheets of M_μ and scrolar of the first kind on the remaining $k-\alpha$ sheets together. The inferior and superior cones then touch the monoid along this line on α sheets. The line $x\,y$ counts in general for k^2 lines on M_μ, but for $k^2+\alpha$ lines if v_k and w_k have α

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* The sign ⊙ should be read: contains as a factor; the sign ⊙ should be read: does not contain as a factor.

factors in common. It may count for more lines in the same way as the line $x\,y$ in the preceding case.

III. The third case is where the line $x\,y$ is a k-tuple line on the inferior cone and a $(k+1)$-tuple line on the superior cone. The equation of the monoid will then be of the form

$$(v_k\,z^p + v_{k+1}\,z^{p-1} + \ldots\ldots)\,s + (w_{k+1}\,z^p + w_{k+2}\,z^{p-1} + \ldots\ldots) = 0. \quad (2)$$

We assume $v_k \not= 0$, otherwise the line will belong to the preceding case. We also assume $w_{k+1} \mathrel{\phi} v_k$ (and $\therefore\ w_{k+1} \not= 0$), otherwise we shall have

$$(v_k\;z^p + v_{k+1}\,z^{p-1} + \ldots\ldots)\;s + (v_k\,a\,z^p + w_{k+2}\,z^{p-1} + \ldots\ldots) = 0,$$

where a is a linear function of x and y. Substituting $s - a$ for s, this reduces to our next case. The line $x\,y$ is a k-tuple line on the monoid whose equation is (2) above, and has a point (0, 0, 1, 0) of multiplicity $k+1$ on it. The tangent planes along $x\,y$ are given by v_k; the line is thus torsal on all the sheets that pass through it. The tangent cone at the $(k+1)$-tuple point is given by the terms $v_k\,s + w_{k+1}$. The line $x\,y$ is also a k-tuple line on this tangent cone. This cone can never break up into $k+1$ planes, as this would require $w_{k+1} \odot v_k$. Nor can it break up into factors of which more than one are proper cones, as it could not then contain the line $x\,y$ as a k-tuple line. If it breaks up at all, it must break up into factors of which all but one are planes, and this remaining factor a proper cone of a certain order, say σ, having the line $x\,y$ as a $(\sigma - 1)$-tuple line. The plane or planes into which the tangent cone at the $(k+1)$-tuple point may break up belong to the k tangent planes through the k-tuple line. The line $x\,y$ counts in general for $k\,(k+1)$ lines of M_μ, but may count for more as in the preceding two cases.

IV. The next case is where the line is a k-tuple line on the inferior cone, and a $(k+2)$-tuple line on the superior cone. The equation of the monoid can then be put into the form

$$(v_k\,z^p + v_{k+1}\,z^{p-1} + \ldots\ldots)\;s + (w_{k+2}\,z^{p-1} + w_{k+3}\,z^{p-2} + \ldots\ldots) = 0.$$

We assume $v_k \not= 0$, otherwise the line will belong to the preceding case. The tangent planes along the line $x\,y$ are given by the term v_k; the line is thus torsal on all sheets that contain it. The point (0, 0, 1, 0) is a $(k+1)$-tuple point on M_μ, the tangent cone at it being given by $v_k\,s$. The tangent cone thus breaks up into the k tangent planes along the k-tuple line, and the plane s. The line $x\,y$ counts here for $k\,(k+2)$

lines of M_μ. The case where $w_{k+2} = \ldots\ldots = w_{k+g-1} = 0$, that is where the line is a $(k+g)$-tuple line on the superior cone while only a k-tuple line on the inferior cone belongs to this case. We assume $3 \overline{\overline{<}} g \overline{\overline{<}} \mu - k$. The line now counts for $k\,(k+g)$ lines of M_μ, otherwise it is not different from the general line belonging to this case.

We thus have in reality four kinds of lines, as shown by the four cases above. We shall designate these lines as of kinds I, II, III, and IV, respectively. Lines of kinds I, III, and IV are torsal on all sheets of M_μ that contain them, whereas lines of kind II are scrolar on at least one sheet of the monoid. Lines of kind III differ from lines of kind IV only in regard to the breaking up of the tangent cone at the $(k+1)$-tuple point.

3. Consider now the character of the curve of intersection of K_m and M_μ in the neighborhood of lines of the above kinds when they are common lines of K_m and M_μ. The point where the line is crossed by a branch of the curve can be determined by considering the intersection of M_μ by the tangent plane to a sheet of K_m along that line. We can, however, determine more than this. If a plane, say $x = 0$, is taken to be the tangent plane to a sheet of the cone along the line xy, we can develop x in terms of y.* We neglect all terms in the development after the first, as we are concerned only with points in the immediate neighborhood of the line xy. Substituting the value of x from this development in the equation of the monoid that has the line xy as a line of one of the above four kinds, we obtain not only the point of crossing of the line by the curve but also the direction of the tangent to this curve at that point. The equation of the cone K_m having the line xy as a k-tuple line can be put into the form

$$t_k\, z^{m-k} + t_{k+1}\, z^{m-k-1} + \ldots\ldots + t_{m-1}\, z + t_m = 0,$$

where t_a is a homogeneous function of x and y of degree a. This equation can be transformed into an equation in x and y alone (putting $z = 1$). If the plane x is a tangent plane to an ordinary sheet of K_m through the line $x\,y$, the development will be of the form

$$x = -\,a\,y^2 + b\,y^3 + \text{etc.}$$

If the plane x is a tangent plane along a keratoidal cuspidal edge of K_m, we shall have

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* This can be done by the Newton-Cramer method, explained in Salmon's Higher Plane Curves (1873), p. 44.

$$x = -cy^{\frac{3}{2}} - dy^2 + \text{etc.}$$

If the line xy is a tacnodal edge, having the plane x as tangent plane, we shall obtain two developments, viz.: —

$$x = ey^2 + \text{etc., and } x = fy^2 + \text{etc.}$$

If the line is a ramphoidal cuspidal edge, the development will be of the form

$$x = gy^2 + hy^{\frac{5}{2}} + \text{etc.}$$

In all these cases, a, b, c, d, e, f, g, h depend on the coefficients of the equation of K_m. It is evident that, using only the first term in each development, these cases reduce to two, viz.: —

1°. $x = -ay^2$ where the sheet of K_m simply touches the tangent plane along the line.

2°. $x = -cy^{\frac{3}{2}}$, where the sheet in question touches the line, and then turns back leaving the tangent plane on the opposite side from that on which it approached it.

We shall consider the different kinds of lines separately, treating one case of a line of kind I in detail, and giving the results in all other cases. If the line xy is an ordinary line of kind I on M_μ, the equation of M_μ in non-homogeneous coördinates will be of the form

$$(\alpha_1 x^2 + \beta_1 xy + \gamma_1 y^2 + \text{etc.})\,s + (\alpha_2 x + \beta_2 y + \gamma_2 x^2 + \delta_2 xy + \epsilon_2 y^2 + \text{etc.}) = 0.$$

Substituting $x = -ay^2$ in this equation, we get an equation in y and s, which is the equation of a curve that in the neighborhood of xy is the same as the curve of intersection of K_m and M_μ. Of this equation y is a factor, the line xy thus counting once as a line common to K_m and M_μ. From the resultant equation, taking account only of the lowest terms, we obtain

$$s = \frac{\delta_2\, a y^2 + \alpha_2\, a y - \epsilon_2\, y - \beta_2}{\gamma_1\, y}$$

Putting $y = 0$ in this expression, we obtain $s = \infty$; the line xy thus meets the curve above at the vertex, i. e. the curve of intersection of K_m and M_μ passes through the vertex. (Similarly if $x = -cy^{\frac{3}{2}}$, there is a point of the curve at the vertex.) If, however, $\beta_2 = 0$, i. e. if the cone and the monoid touch along xy, we get a point of the curve distinct from the vertex. For y factors out once more from the above, that is the line xy counts twice as a line common to K_m and M_μ, and we have

$$s = \frac{\delta_2\, a y + (\alpha_2\, a - \epsilon_2)}{\gamma_1}.$$

This reduces to $s = \frac{\alpha_2\, a - \epsilon_2}{\gamma_1}$ for $y = 0$. The curve of intersection thus crosses $x y$ at the point $\left(x = y = 0,\, s = \frac{\alpha_2\, a - \epsilon_2}{\gamma_1}\right)$. Transforming our coördinates so that $\alpha_2\, a = \epsilon_2$, we obtain $(x = y = s = 0)$ as the point of crossing and $\gamma_1\, s - \delta_2\, a y = 0$, $x = 0$ as the equations of the tangent line to the curve at that point. Thus it is evident that in the case of lines of kind I, whatever be the multiplicity of $x y$ on M_μ, if no sheet of K_m touches a sheet of M_μ along this line, the curve of intersection will always pass through the vertex. As we are considering a point off the curve as vertex such cases need not be considered. In the case where two sheets of K_m unite to form a cuspidal sheet that touches a sheet on M_μ along $x y$, the curve of intersection will either pass through the vertex or it will have $x y$ as tangent line at a point of it. As we have also assumed the vertex not to lie on a tangent to the curve, these cases may both be avoided. There will be a point of the curve on $x y$ distinct from the vertex and not having $x y$ as tangent at it, only in the case where a sheet of K_m touches a sheet of M_μ along the line $x y$ and the sheet of K_m is one for which development 1° on page 35 holds. It does not matter whether the sheet of M_μ is single, cuspidal, or tacnodal. There will be as many branches of the curve crossing $x y$ as there are such sheets of K_m touching sheets of M_μ. As the vertex has been taken in such a way that the curve has no apparent multiple point of multiplicity greater than two, the case where there are more than two distinct points of the curve on $x y$ may be avoided. For special relations between the coefficients, κ of the points on $x y$ may coincide and form a κ-tuple point on the curve. This will, however, in general necessitate the monoid to be of an order higher than $m - 1$. For the equation of M_{m-1} that has a line as a κ-tuple line of kind I contains in general $m^2 - \kappa^2 - 2\kappa - 2$ arbitrary constants. In order to make this monoid contain a curve C_m that has a point of the κ-tuple line as a κ-tuple point we must make it contain $m\,(m - 1) - \kappa^2 + 1$ additional points of C_m. We can therefore always make M_μ cut C_m out of K_m, if

$$m\,(m-1) - \kappa^2 + 1 \gtreqless m^2 - \kappa^2 - 2\kappa - 2,$$

i. e. if

$$2\kappa + 3 \gtreqless m.$$

Thus if $\kappa = 3$, it is only possible when $9 \gtreqless m$; if $\kappa = 4$, when $11 \gtreqless m$; etc. As will be shown later, the curve C_m that has a multiple point can always be obtained as the partial intersection of K_m with a monoid of order $m - 1$ that has a line of kind III or IV. We shall therefore not use a line of kind I to obtain a curve with a multiple point.

If M_μ contains a line of kind II, every sheet of K_m that passes through this line intersects M_μ in a branch of a curve that crosses $x\,y$ at the point where the tangent plane to the sheet of K_m coincides with the tangent plane to the scrolar sheet of M_μ. If $x\,y$ is a κ-tuple line on K_m, there will be κ branches of the curve crossing the line due to the scrolar sheet of M_μ. No torsal sheet of M_μ can cause an additional branch of the curve; for this would necessitate the line to be of a multiplicity greater than κ on K_m, which in turn would cause more than κ branches of the curve to be due to the scrolar sheet alone; and so on *ad infinitum*. We therefore need a line on M_μ that has torsal sheets passing through it in addition to the scrolar sheet. If two sheets of K_m unite to form a cuspidal sheet along $x\,y$, there is either a branch of the curve touching $x\,y$ at the point where the tangent plane to the cuspidal sheet coincides with the tangent plane to M_μ, or a branch of the curve passing through the vertex. Both of these cases may be avoided. A number of conditions must be imposed on the coefficients of M to make the κ points of crossing of the line by the curve coincide. We shall therefore not use a line of kind II to obtain a multiple point on the curve. We need not therefore in general consider the case where the line $x\,y$ is more than a double line on K_m. Lines of kind II are only necessary on M_μ to produce the apparent double points of the curve, and only ordinary lines of this kind are then needed. The line to an apparent double point counts thus in general for two lines common to K_m and M_μ, while only for one line on M_μ. If we consider M_μ as of order $m - 1$, it is evident that K_m must have at least $m - 3$ double edges in addition to the double edge caused by this apparent double point. The curve must therefore have at least $m - 3$ apparent double points in addition, i. e. at least $m - 2$ in all, as on page 490.

If M_μ has a line $x\,y$ of kind III on it, there will be a branch of the curve through the multiple point for every sheet of K_m that passes through $x\,y$. If $x\,y$ is a κ-tuple line on K_m and a k-tuple line of kind III on M_μ, there will be κ branches of the curve through the $(k + 1)$-tuple point of M_μ. The point will thus be a κ-tuple point on the curve having its κ tangents lying in general on a cone of order $k + 1$ that has the line $x\,y$ as a k-tuple edge. For the tangents to the curve at this multiple point are the lines of intersection of the tangent cone at this multiple point by the κ tangent planes to K_m along $x\,y$, each plane intersecting the tangent cone in the line $x\,y$ k times and in one other line, which is a tangent to the curve. If the tangent cone breaks up into g planes and a cone of order $k - g + 1$ (where $2 \overset{=}{<} k - g + 1$) that has the line $x\,y$ as a $(k - g)$-tuple edge, the κ tangents to the curve will lie on this cone of order $k - g + 1$. In this case it is only necessary to have the line $x\,y$ as a $(k - g)$-tuple line of kind III on M_μ. We need not consider the case where one or more sheets of K_m touch M_μ and therefore the tangent cone at the multiple point along the line $x\,y$, for this line is then tangent to the curve. Nor can any sheet whose tangent plane is one of the components of the tangent cone at the multiple point cause a branch of the curve by having contact with a sheet of the cone.

If M_μ contains a line $x\,y$ of kind IV, every sheet of k_m that passes through this line cuts out of M_μ a branch of the curve that passes through the multiple point on M_μ. The tangent lines to the curve in this case all lie in one plane, viz. the plane that is tangent to M_μ at the multiple point but does not contain the line $x\,y$. Thus, if the line $x\,y$ is a κ-tuple edge on K_m, the curve of intersection will have a κ-tuple point, the tangents at which all lie in one plane. As the only sheet of M_μ that affects the curve is the sheet that does not pass through the multiple line, it is not necessary to have the line from the multiple point to the common vertex of K_m and M_μ as a line on M_μ. The single sheet of M_μ simply cuts the κ branches of the curve out of the κ sheets of the cone, its tangent planes intersecting the κ tangent planes to K_m in the κ tangent lines to the curve at that point.

4. We can therefore always obtain a curve of order m having a κ-tuple point, the tangents at which lie on a cone of order $k + 1$ that has the point as vertex and has an arbitrary line through this point as a k-tuple line, as the partial intersection of a particular cone and a particular monoid. The cone is of order m, has its vertex on the k-tuple edge of the cone of order $k + 1$, and has this edge as a k-tuple edge. The monoid is of order μ, and has the multiple point of the curve as a $(k + 1)$-tuple point and the line from this point to the common vertex of the two surfaces as a k-tuple line. The line from the common vertex to the multiple point counts thus as $k\,\kappa$ lines common to K_m and M_μ, and as $k\,(k + 1)$ lines of M_μ. The cone and the monoid have $m\,(\mu + 1) - k\,\kappa$ additional lines in common, whereas there are only $\mu\,(\mu - 1) - k\,(k + 1)$ additional lines on M_μ. The cone K_m must therefore have at least

$$m\,(\mu - 1) - k\,\kappa - \mu\,(\mu - 1) + k\,(k + 1)$$

i. e. $$(m - \mu)\,(\mu - 1) - k\,(\kappa - k - 1) \equiv a$$

double edges, i. e. the curve C_m must have at least a apparent double points. It is thus evident that the multiple point reduces the *necessary* number of apparent double points by $k(\kappa - k - 1)$. This also holds true when the tangents lie in one plane. For we then have $k + 1 = 1$, i. e. $k = 0$. The line from the multiple point to the common vertex is neither a line on the monoid nor a line in the tangent plane. We then have $k(\kappa - k - 1) = 0$; that is, such a multiple point does not affect the *necessary* number of apparent double points. It is thus evident that multiple points whose tangents do not lie in one plane affect the number of apparent double points, whereas multiple points whose tangents lie in one plane do not affect this number, as well in the case of curves that are the partial intersection of a cone and a monoid as in the case of curves that are the complete intersection of two surfaces.

This is also shown if we employ the method used in determining the number of apparent double points of the complete intersection of two surfaces of orders μ and ν.* This number was found to be one half of the number of intersections of the curve with a surface s of order $(\mu - 1)(\nu - 1)$. The complete intersection in the case under consideration consists of the curve C_m of order m, a number of lines that are κ-tuple lines on K_m and k-tuple lines on M_μ, a number h of lines that are double lines on K_m and ordinary lines on M_μ, and finally a number [viz. $m(\mu - 1) - \sum k\kappa - 2h$] † of lines that are ordinary lines on both surfaces. There are certain points of apparent intersection of C_m with the lines common to K_m and M_μ that are included among the points of intersection of C_m and S. Among these are $(m - k)k\kappa$ points for every line to a κ-tuple point, $2(m - 2)$ points for every line to an apparent double point of the curve when viewed from the vertex of K_m and $m - 1$ points for every line that is an ordinary line on both surfaces. The κ-tuple point of C_m, moreover, is a point of multiplicity $k(\kappa - 1)$ on S and therefore counts as $k\kappa(\kappa - 1)$ of the $m(m - 1)(\mu - 1)$ points of intersection of C_m and S. Calling h' the number of apparent double points of C_m when viewed from an arbitrary point in space, we have

$$m(m-1)(\mu-1) - \sum_{\kappa}[(m-\kappa)\kappa k + (\kappa-1)\kappa k] - 2(m-2)h \\ - (m-1)[m(\mu-1) - \sum_{\kappa}\kappa k - 2h] = 2h';$$

i. e.

$$h = h'.$$

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* See page 481.

† In the Σ we take account of all lines to points that are κ-tuple points on C_m and $(k+1)$-tuple points on M_μ.

Thus, as a multiple point at which the tangents do not lie in one plane is a point on S, a number of apparent double points disappear when it is formed on the curve. A multiple point, however, at which the tangents all lie in one plane, can be obtained, as we have seen, by means of a monoid that has it as an ordinary point. It is therefore a 0-tuple point on S and has no apparent double points in its composition. This will be shown more in detail later.

Every κ-tuple point, however, whether its tangents do or do not lie in one plane, has the same effect on the *possible* number of apparent double points; each reduces it by $\frac{1}{2}\kappa(\kappa-1)$. Thus a curve of order m having a number of κ-tuple points can never have more than

$$\tfrac{1}{2}(m-1)(m-2)-\tfrac{1}{2}\sum\kappa(\kappa-1)$$

apparent double points; where the summation includes all points for which $2 \overline{\gtrless} \kappa \overline{\gtrless} m-2$.

5. There is an upper limit to the *necessary* order of the cone on which the tangents to a curve at a multiple point lie. The equation of a cone of order $k+1$ that has a certain line as a k-tuple edge has $2k+2$ arbitrary constants, that is, such a cone can in general be made to pass through $2k+2$ arbitrary lines through the multiple point. We can therefore always pass a cone of order $k+1$, having an arbitrary line through the κ-tuple point of the curve as a k-tuple edge, through the κ tangent lines at the κ-tuple point of the curve, if $\kappa \overline{\gtrless} 2k+2$, i. e. if $\frac{\kappa-2}{2} \overline{\gtrless} k$, or $\frac{\kappa}{2} \overline{\gtrless} k+1$. We need, therefore, never take k greater than $\frac{\kappa-2}{2}$ if κ is even, nor greater than $\frac{\kappa-1}{2}$ if κ is odd. The tangents may, however, in some cases lie on cones of lower orders than those given by the expressions above. We can in fact classify multiple points according to the orders of cones of the lowest order that can be passed through the tangents at them, this order taking any value from 1 to $\frac{\kappa}{2}$ or $\frac{\kappa+1}{2}$, according as κ is even or odd. We must also bear in mind that the order of the cone places a limit on the number of the tangent lines that can lie in one plane. If λ tangents at a κ-tuple point of a twisted curve of order m lie in one plane, the curve is met by that plane in at least $2\lambda+\kappa-\lambda$ or $\kappa+\lambda$ points. We must therefore have $\kappa+\lambda \overline{\gtrless} m$, i. e. $\kappa \overline{\gtrless} m-\lambda$, or $\lambda \overline{\gtrless} m-\kappa$. A curve of order six can therefore not have a quadruple point, three of the tangents at which lie

in one plane. If a cone of order g can be passed through the κ tangents at a κ-tuple point, the cone is met by the curve in at least $(g + 1)\,\kappa$ points. We must therefore have

$$(g + 1)\,\kappa \leqq m\,g; \quad \text{i.e. } \kappa \leqq \frac{g}{g+1}\,m,$$

if we do not wish the curve to lie entirely on the cone. Thus, if $g = 1$, $\kappa \leqq \frac{m}{2}$; otherwise the curve is a plane curve. A twisted curve of order m can therefore never have a point of multiplicity greater than $\frac{m}{2}$ if m is even, or greater than $\frac{m-1}{2}$ if m is odd, if the tangents all lie in one plane. If $g = 2$, $\kappa \leqq \frac{2\,m}{3}$; that is, if a twisted curve of order m having a point of multiplicity greater than $\frac{2\,m}{3}$ has the tangents at this point lying on a quadric cone the curve will lie entirely on that cone. Similarly for cones of higher orders.

6. We shall now determine the order μ of the monoid of lowest order that will in general cut C_m out of K_m in the manner described above; that is, that value of μ that will always suffice to obtain C_m. The equation of a monoid of order μ that contains a k-tuple line of kind *III* has just $(\mu + 1)^2 - (k + 1)^2 - 1$ arbitrary constants. We can make this monoid contain the curve C_m that has a κ-tuple point at the $(k + 1)$-tuple point of M_μ if we make it contain $m\,\mu - \kappa\,(k + 1) + 1$ additional points of C_m. This is always possible if

$$m\,\mu - \kappa\,(k + 1) + 1 \leqq (\mu + 1)^2 - (k + 1)^2 - 1;$$

$$\text{i.e. if } \frac{m-2}{2} + \tfrac{1}{2}\sqrt{m^2 - 4\,m + 4\,k^2 + 8\,k - 4\,k\,\kappa - 4\,\kappa + 12} \leqq \mu.$$

Summing for all κ-tuple points on C_m, we have

$$\frac{m-2}{2} + \tfrac{1}{2}\sqrt{m^2 - 4\,m + 8 - 4\sum(k + 1)\,(\kappa - k - 1)} \leqq \mu; \qquad \text{(I)}$$

where in the summation each κ-tuple point has its own value of k. We need never take the order of M_μ greater than the smallest integer value of μ that will satisfy this inequality. The curve C_m may nevertheless in certain cases be cut out of K_m by a monoid of lower order. If the κ-tuple points are all of the most general kind, that is, if the corresponding values of k are determined by $\frac{\kappa - 1}{2}$ or $\frac{\kappa - 2}{2}$, according as κ is odd or even, we must have in general

1°. If the κ's are all even,

$$\frac{m-2}{2} + \tfrac{1}{2}\sqrt{(m-2)^2 - \sum \kappa^2 + 4} \overset{=}{<} \mu; \tag{II}$$

2°. If the κ's are all odd,

$$\frac{m-2}{2} + \tfrac{1}{2}\sqrt{(m-2)^2 - \sum \kappa^2 + 5} \overset{=}{<} \mu. \tag{III}$$

It is to be noticed that $\sum (k+1)(\kappa - k - 1)$ in (I) above is always greatest when the κ-tuple points are all of the most general kind. The *sufficient* value of μ is therefore always least in this case, that is when μ is given by (II) or (III) above. The quantity $(k+1)(\kappa - k - 1)$ in (I) is never less than zero; the quantity under the radical sign is thus never greater than $m^2 - 4m + 8$; therefore $\mu = m - 1$ will always satisfy the conditions. We need therefore never take μ greater than $m - 1$.

7. The inequalities above give values of μ greater than which we *need never* take the order of the monoid. We can, moreover, find values of μ smaller than which we *can never* take the order. As M_μ has $\mu(\mu - 1)$ lines on it, we must always have

$$h + \sum k(k+1) + \sigma \overset{=}{<} \mu(\mu - 1); \tag{IV}$$

where h corresponds to the ordinary lines of M_μ that are double lines on K_m, $\sum k(k+1)$ to the k-tuple lines of M_μ that are κ-tuple lines on K_m, and σ to the ordinary lines of M_μ that are ordinary lines on K_m. As K_m intersects M_μ in $m(\mu - 1)$ lines, we must have

$$2h - \sum k\kappa + \sigma = m(\mu - 1). \tag{V}$$

By eliminating h from (IV) and (V), we obtain

$$\sum k(2k - \kappa - 2) + \sigma \overset{=}{<} (2\mu - m)(\mu - 1),$$

$$\text{i.e.} \quad \frac{m+2}{4} + \tfrac{1}{4}\sqrt{(m-2)^2 + 8\sum k(2k - \kappa - 2) + 8\sigma} \overset{=}{<} \mu; \tag{VI}$$

that is, μ must always be taken as great as the quantity on the left side of this inequality. The smallest value of this quantity occurs when $\sigma = 0$, and otherwise depends on the values of $k\,(2\,k - \kappa + 2)$, that is on the orders of the cones on which the tangents at the multiple points lie.

I). If κ is even, k can take any value from zero up to $\frac{\kappa - 2}{2}$ inclusive. The greatest value of $k\,(2\,k - \kappa - 2)$ will then be $-\left(\frac{\kappa - 2}{4}\right)\left(\frac{\kappa - 2}{2}\right)$ if $\kappa - 2$ is an evenly even number, or

$$-\left(\frac{\kappa - 4}{4}\right)\left(\frac{\kappa}{2}\right), \left\{\text{or} - \left(\frac{\kappa}{4}\right)\left(\frac{\kappa - 4}{2}\right)\right\},$$

if $\kappa - 2$ is an oddly even number. We must therefore always have

$$\frac{m + 2}{4} + \tfrac{1}{4}\sqrt{(m - 2)^2 - \sum(\kappa - 2)^2} \leqq \mu, \text{ if every } \kappa = 2\,n,$$

or $$\frac{m + 2}{4} + \tfrac{1}{4}\sqrt{(m - 2)^2 - \sum\kappa\,(\kappa - 4)} \leqq \mu, \text{ if every } \kappa = 4\,n'.$$

(Here n is an odd number, whereas n' is any integer.) We can thus never take μ less than the quantities on the left-hand sides of these inequalities, if the κ's are all even numbers. It is also evident from (VI) that, if the points are all such that the tangents at them lie in planes or on cones of order $\frac{\kappa}{2}$, we must have

$$\frac{m + 2}{4} + \tfrac{1}{4}\sqrt{(m - 2)^2} \leqq \mu,$$

$$\text{i.e.} \quad \frac{m}{2} \leqq \mu.$$

The lower limit for μ when the κ's are all even numbers thus varies between $\frac{m}{2}$ and $\frac{m + 2}{4} + \frac{1}{4}\sqrt{(m - 2)^2 - \sum(\kappa - 2)^2}$, according to the way in which the tangents to the curve at the different multiple points lie.

II). If κ is odd, k can take any value from 0 up to $\frac{\kappa-1}{2}$ inclusive. The largest possible value of $k\,(2\,k-\kappa+2)$ for any multiple point will then be $-\left(\frac{\kappa-3}{4}\right)\left(\frac{\kappa-1}{2}\right)$, or $-\left(\frac{\kappa-1}{4}\right)\left(\frac{\kappa-3}{2}\right)$, according as κ is one more than an oddly even or an evenly even number. The two products are, however, equal. The greatest possible value of $\sum k\,(2\,k-\kappa+2)$ will thus occur when all the points are such that $k=\frac{\kappa-3}{4}$ or $\frac{\kappa-1}{4}$, according as κ is one more than an oddly even or an evenly even number. If κ's are all odd numbers, we must thus have

$$\frac{m+2}{4}+\tfrac{1}{4}\sqrt{(m-2)^2-\sum(\kappa-1)\,(\kappa-3)}\leqq\mu.$$

It is evident from (VI) above that

$$\frac{m+2}{4}+\tfrac{1}{4}\sqrt{(m-2)^2+4\sum(\kappa-1)}\leqq\mu,\quad\text{if } k=\frac{\kappa-1}{2}\text{ for each }\kappa;$$

or

$$\frac{m+2}{4}+\tfrac{1}{4}\sqrt{(m-2)^2}\leqq\mu,\quad\text{if } k=0\text{ for each }\kappa.$$

The lower limit for μ when κ is odd thus varies between

$$\frac{m+2}{4}+\tfrac{1}{4}\sqrt{(m-2)^2-\sum(\kappa-1)\,(\kappa-2)}$$

and

$$\frac{m+2}{4}+\tfrac{1}{4}\sqrt{(m-2)^2+4\sum(\kappa-1)},$$

according to the way in which the tangents to the curve at the different multiple points lie.

It is thus evident that μ can in no case be taken less than

$$\frac{m+2}{4}+\tfrac{1}{4}\sqrt{(m-2)^2-\sum(\kappa-2)^2}.$$

Using the formulae found above we can tabulate as follows: —

A curve C_m of order . . . . . . $m \equiv$	5	6	6	7	7	7	7	7	8	8	8	8	8	8	8	8	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
having a point (or points) of multiplicity . . . . $\kappa \equiv$	3	4	3	5	5	4	3	3, 3	6	6	5	5	4	3	4, 3	3, 3	7	7	7	6	6	5	5	4	3	5, 3	5, 3	4, 4	4, 3	3, 3	3, 3, 3
can be cut out of a cone of order m having C_m as base by a monoid of order . . . . $\mu \equiv$	3	3	4	4	4	5	5	5	4	5	5	5	6	6	6	6	6	5	6	6	6	7	7	7	7	5	6	6	7	7	7
that has the multiple point (or points) of C_m as point (or points) of multiplicity . . $k - 1 \equiv$	2	2	2	3	2	2	2	2, 2	3	2	3	2	2	2	2, 2	2, 2	4	3	2	3	2	3	2	2	2	3, 2	2, 2	2, 2	2, 2	2, 2	2, 2, 2

B.

1. Every curve C_m of order m can also be obtained as the partial intersection of a cone of order $m - 1$ and a monoid of order μ, where μ is to be determined. The cone K_{m-1} is a cone that has an ordinary point of C_m as its vertex and C_m as its base. There being an infinite number of ordinary points on C_m, the special positions of the vertex from which every line drawn to C_m meets C_m in two or more points can be avoided. The vertex can therefore be taken in such a way that the cone of order $m - 1$ does not break up into cones of orders that are submultiples of $m - 1$. Every edge of K_{m-1} thus has on it one point of C_m in addition to that at the vertex; C_m can therefore be cut out of K_{m-1} by a monoid M_μ. We shall consider the case of a curve that has points of multiplicity κ, at which the tangents lie on cones of order $k + 1$ that have the lines from the multiple points to the vertex of K_{m-1} as k-tuple edges; the case of curves with no multiple points being but a special case of this. It can be shown, as in the previous case, that the lines from the common vertex of K_{m-1} and M_μ to these κ-tuple points are κ-tuple lines on K_{m-1} and k-tuple lines of kind III on M_μ. The

equation of a monoid of order μ that has a k-tuple line of kind III contains in general $(\mu + 1)^2 - (k + 1)^2 - 1$ arbitrary consonants. In order to make this monoid M_μ contain a curve C_m that has the vertex of M_μ as an ordinary point and a $(k + 1)$-tuple point of M_μ as a κ-tuple point, we must make it contain $m\mu - (\mu - 1) - \kappa(k + 1) + 1$ additional points of C_m. This is possible if

$$m\mu - (\mu - 1) - \kappa(k + 1) + 1 \leqq (\mu + 1)^2 - (k + 1)^2 - 1,$$

i.e. if $\dfrac{m - 3}{2} + \frac{1}{2}\sqrt{m^2 - 6m + 17 - 4(k + 1)(\kappa - k - 1)} \leqq \mu$;

or, summing for all κ-tuple points of C_m that are $(k + 1)$-tuple points on M_μ, we must have

$$\frac{m - 3}{2} + \frac{1}{2}\sqrt{m^2 - 6m + 17 - 4\sum(k + 1)(\kappa - k - 1)} \leqq \mu. \qquad \text{(I)}$$

The smallest value of μ that satisfies (I) will in general suffice as the order of a monoid that can be made to cut C_m out of K_{m-1}. As $(k + 1)(\kappa - k - 1)$ is never less than 0, it is evident that $\mu = m - 2$ will always suffice. As in the previous section, k can take any value from 0 to $\dfrac{\kappa - 2}{2}$ or $\dfrac{\kappa - 1}{2}$ inclusive. We can show in the same way as there that μ can never be less than the smallest integer that satisfies

$$\frac{m - 1}{4} + \frac{1}{4}\sqrt{m^2 - 6m + 1 + 8\sum k(2k - \kappa + 2) + 8\sigma} \leqq \mu. \qquad \text{(II)}$$

Using (I) and (II), we can tabulate as follows: —

A curve of order . . . . . $m \equiv$	5	6	6	7	7	7	7	8	8	8	8	8	8	8
having a point (or points) of multiplicity . . . . . . $\kappa \equiv$	3	4	3	5	4	3	3, 3	6	5	5	4	3	4, 3	3, 3
can be obtained as the intersection of a cone of order . . . . . . $m - 1 \equiv$	4	5	5	6	6	6	6	7	7	7	7	7	7	7
and a monoid of order . . $\mu \equiv$	2	2	3	2	4	4	4	3	4	4	5	5	4	5
that has the κ-tuple point (or points) of the curve as points of multiplicity . . . . $k + 1 \equiv$	2	2	2	2	2	2	2, 2	2	3	2	2	2	2, 2	2, 2

Thus a quintic curve with a triple point can be obtained as the partial intersection of a quartic cone that has a triple edge and a quadric monoid that has the triple point of the curve as a double point; that is, as the partial intersection on a quartic and a quadric cone not having the same vertex but having a line in common that is a triple edge on the quartic and an ordinary edge on the quadric cone. This edge and the quintic curve constitute the complete intersection of the two surfaces.

2. The complete intersection of K_{m-1} and M_μ is of order $(m-1)\mu$. The cone K_{m-1} meets the inferior cone of M_μ in $(m-1)(\mu-1)$ lines, of which all but one (namely the tangent to C_m at the vertex O of K_{m-1}) meets C_m in points different from O and not consecutive to it. These lines therefore lie on M_μ, and together with C_m constitute the complete intersection of K_{m-1} and M_μ. As there are only $\mu(\mu-1)$ lines on M_μ, it is evident that K_{m-1} has in general at least

$$(m-1)(\mu-1)-1-\mu(\mu-1),$$

that is, $m\mu-\mu^2-m$, double edges. C_m has therefore in general at least $m\mu-\mu^2-m$ apparent double points when viewed from a point of the curve, that is, at least $m\mu-\mu^2-2$ when viewed from an arbitrary point in space.* If, however, the curve has κ-tuple points that are $(k+1)$-tuple points on M_μ, the lines from these points to the vertex count as $\sum k\kappa$ lines common to K_{m-1} and M_μ and as $\sum k(k+1)$ lines on M_μ. The cone K_{m-1} must therefore have at least

$$(m-1)\mu-m-\sum k\kappa-\mu(\mu-1)+\sum k(k+1)$$

double edges, that is, C_m must have at least $m\mu-\mu^2-2-\sum k(\kappa-k-1)$ apparent double points. Each multiple point thus reduces the necessary number of apparent double points by $k(\kappa-k-1)$, as in the previous section. If we take $\mu=m-2$, it is evident that we must always have

$$2m-6-\sum k(\kappa-k-1) \overline{\gtrless} h.$$

C.

1. A curve C_m having a point A as a κ-tuple point lies in general on a cone K of order $m-\kappa$ that has the point A as its vertex and the curve C_m as its base. This excludes those curves that are met by every line through the multiple point in two or more additional points if at all,

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* See Salmon's Geometry of Three Dimensions (1882), No. 330, Ex. 2.

e. g. curves of order m that are wound a times around a cone of order $\frac{m-\kappa}{a}$ and pass κ times through the vertex of this cone. Such curves cannot be obtained as the partial or complete intersection of a cone and a monoid that have this multiple point as their common vertex. They can be obtained as the intersection, partial or complete, of a cone of order $\frac{m-\kappa}{a}$ and a surface of order μ that has the vertex of the cone as a $(\mu - a)$-tuple point; every line through the $(\mu - a)$-tuple point, and therefore every edge of the cone, meets this surface in a additional points, which are the a points of the curve on this edge. We can avoid doing this by considering these curves as lying on cones of order m or $m - 1$, as we did in the previous two sections. We shall treat in this section only those curves that are met by an edge of $K_{m-\kappa}$ in one point distinct from the vertex. Such curves can be cut out of $K_{m-\kappa}$ by some monoid of order μ.

I. Suppose $m - \kappa \leqq \mu$. Then in order to make M_μ contain C_m without breaking up into $K_{m-\kappa}$ and a monoid of order $\mu - m + \kappa$, we must have

$$m\mu - \kappa(\mu - 1) + 1 \leqq (\mu + 1)^2 - (\mu - m + \kappa + 1)^2 - 1;$$

the vertex counting as $\kappa(\mu - 1)$ points of intersection of C_m and M_μ. We must therefore have in general

$$\frac{(m - \kappa - 1)^2 + (\kappa + 1)}{m - \kappa} \leqq \mu.$$

If the curve C_m has in addition to this κ-tuple point certain κ'-tuple points that are $(k' + 1)$-tuple points on M_μ, it is evident from reasoning similar to that on page 506, that we must have

$$\begin{aligned} m\mu - \kappa(\mu - 1) - \sum(k' + 1)k' + 1 \leqq \Big[(\mu + 1)^2 - \sum(k' + 1)^2\Big] \\ - \Big[(\mu - m + \kappa + 1)^2 - \sum(k' - \kappa' + 1)^2\Big] - 1, \end{aligned}$$

i. e.

$$\frac{(m - \kappa - 1)^2 + (\kappa + 1) - \sum \kappa'(\kappa' - k' + 1)}{m - \kappa} \leqq \mu; \qquad \text{(I)}$$

where the summation extends over all multiple points of C_m (except the one at the vertex of $K_{m-\kappa}$) that are $(k' + 1)$-tuple points on M_μ. The smallest value of μ that satisfies (I) is the order of the monoid that will in this case cut C_m out of $K_{m-\kappa}$.

II. Suppose $m - \kappa > \mu$. We must then have

$$m\mu - \kappa(\mu - 1) + 1 \leqq (m + 1)^2 - 1,$$

i.e. $$\frac{m - \kappa - 2}{2} + \tfrac{1}{2}\sqrt{(m - \kappa - 2)^2 + 4(\kappa + 1)} \leqq \mu,$$

if we wish to make M_μ contain C_m. If, however, the curve C_m has in addition to this κ-tuple point certain κ'-tuple points that are $(k' + 1)$-tuple points on M_μ, we must have

$$\frac{m - \kappa - 2}{2} + \tfrac{1}{2}\sqrt{(m - \kappa - 2)^2 + 4(\kappa + 1) - 4\sum(\kappa' + 1)(\kappa' - k' - 1)} \leqq \mu, \tag{II}$$

where the summation extends over all the multiple points except the one at the vertex.

We know that $m \leqq \mu(m - \kappa)$, i.e. $\dfrac{m}{m - \kappa} \leqq \mu$. Thus $m - \kappa \leqq \mu$, whenever $(m - \kappa)^2 \leqq m$. We therefore use (I) whenever $(m - \kappa)^2 \leqq m$, that is, when $m - \sqrt{m} \leqq \kappa$, and (II) whenever $(m - \kappa)^2 > m$, that is, when $\kappa < m - \sqrt{m}$. We can then tabulate as follows: —

A curve of order . . . . $m \equiv$	5	6	6	7	7	7	7	8	8	8	8	8	8	8
having a point of multiplicity . $\kappa \equiv$	3	4	3	5	4	3	3	6	5	4	3	4	3	3
and a point of multiplicity . . $\kappa' \equiv$							3					3	3	4
can be obtained as the intersection of a cone of order having the κ-tuple point as vertex $m - \kappa \equiv$	2	2	3	2	3	4	4	2	3	4	5	4	5	5
and a monoid of order . . . . $\mu \equiv$	3	4	3	4	3	4	3	4	3	4	4	3	4	3
having the κ-tuple point as vertex and the κ'-tuple point as a point of multiplicity $k' + 1 \equiv$							2					2	2	2

D.

1. We shall now consider in particular the case where the multiple point that is taken as the vertex of the cone is an $(m - 2)$-tuple point. Every such twisted curve C_m of order m that has an actual $(m - 2)$-tuple point is unicursal. For a plane passing through any line that

joins this $(m-2)$-tuple point to any point of C_m never contains more than one other point of C_m; nevertheless during its revolution around the line, it contains successively all the other points of the curve. If we take an ordinary point of C_m as vertex and construct a cone with C_m as base, this cone will be of the $(m-1)$st order. This cone contains the line joining the $(m-2)$-tuple point to the vertex as an $(m-2)$-tuple edge, and can therefore have no double edges. The curve can therefore have no apparent double points when viewed from an ordinary point of the curve. We therefore have $h = m - 2$;* thus a twisted curve of order m that has an $(m-2)$-tuple point has $m-2$ apparent double points.

2. Let C_m be a twisted curve of order m having an $(m-2)$-tuple point at A (where $4 \gtreqless m$). This curve will lie on a cone of order two, say K_2, whose vertex is A; for any plane through the point A can contain only two other points of C_m, the lines joining these points to A being generators of a quadric cone. Not more than two tangents to C_m at A can lie in one plane, for the plane would then meet the curve in more than m points, which is only possible if the plane contains the curve. Every unicursal curve of order m with an actual $(m-2)$-tuple point thus lies on a quadric cone. We can show that C_m can be cut out of this cone K_2 by a monoid. Clearly C_m is the complete or partial intersection of K_2 with some surface of order μ, say S_μ, having a k-tuple point at A. Now $m - 2 \gtreqless 2k$, since $2k$ is the multiplicity of A on the complete intersection of S_μ and K_2. It is evident that k is the order of the cone that contains the $m-2$ tangents to C_m at A that lie on K_2, provided $m - 2 \gtreqless \frac{1}{2} k (k+3) - \frac{1}{2} (k-2)(k+1) - 1$, where $2 \gtreqless k$, i. e. provided $m - 2 \gtreqless 2k$, where $2 \gtreqless k$. We can thus always take $k = \dfrac{m-2}{2}$ or $\dfrac{m-1}{2}$, according as m is even or odd. The surface S_μ with a k-tuple point at A is determined by $\frac{1}{6}(\mu+1)(\mu+2)(\mu+3) - \frac{1}{6} k (k+1)(k+2) - 1$ points. In order not to have S_μ break up into K_2 and a component of order $\mu - 2$ having a $(k-2)$-tuple point at A, we must take $\frac{1}{6}(\mu-1)\mu(\mu+1) - \frac{1}{6}(k-2)(k-1)k$ of these points off K_2. There are $\mu(\mu+2) - k^2$ points remaining that can be taken arbitrarily, but in such a way that S_μ shall contain C_m. S_μ meets C_m in $k(m-2)$ points at A, and will therefore contain C_m entirely if it contains $m\mu - (m-2)k + 1$ additional points of C_m. S_μ can therefore be determined so as to contain C_m if

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* Salmon's Geometry of Three Dimensions (1882), No 330, Example 2.

$$m\mu - (m-2)\,k + 1 \overset{=}{<} \mu\,(\mu + 2) - k^2.$$

If there are still points left at our disposal, we may use them in different ways, and may cause the residual curve to break up in different ways. The complete intersection of K_2 and S_μ is of order $2\,\mu$; and as C_m is of order m the residual is of order $2\,\mu - m$. As the complete intersection has a point of multiplicity $2\,k$ at A, the residual has a point of multiplicity $2\,k - m + 2$ there. Every edge of K_2 meets S_μ in $k + 1$ points of which k are at A. If we can take $\mu - k$ additional points on some of these edges, these edges will contain $\mu + 1$ points in all and will lie completely on S_μ. Therefore if there are enough points left at our disposal to enable us to take $\mu - k$ additional points on each of $2\,\mu - m - 1$ edges of K_2, the residual will consist of these $2\,\mu - m - 1$ edges of K_2 and (since the entire residual is of order $2\,\mu - m$) another straight line, which can only be another edge of K_2. The residual can then be made to consist entirely of straight lines if

$$(\mu - k)\;(2\,\mu - m - 1) \overset{=}{<} \mu\,(\mu + 2) - k^2 - m\mu + (m-2)\,k - 1,$$

i. e. if

$$(\mu - k - 1)\;(\mu - k - 2) \overset{=}{<} 1\,;$$

which can only occur if $k = \mu - 1$, or $k = \mu - 2$. We may then always take $k = \mu - 1$, that is take S_μ to be a monoid. Thus $\mu = k + 1 = \frac{m}{2}$ or $\frac{m+1}{2}$ according as m is even or odd. Therefore: —

A unicursal curve of order m that has a point of multiplicity m — 2 can always be considered as the partial intersection of a quadric cone and a monoid of order μ*, where* $\mu = \frac{m}{2}$ *or* $\frac{m+1}{2}$ *according as m is even or odd.*

Thus the quintic curve with a triple point can be obtained, as we have seen before, as the partial intersection of a quadric cone and a cubic monoid that have the same vertex. The cubic monoid is determined by 15 points, but 4 of these must be taken off the quadric in order not to have the monoid contain the quadric as a component. We thus have 11 points at our disposal. In order to make the cubic monoid contain a quintic curve that has a triple point at the vertex of the monoid, we must make it contain $3 \cdot 5 - 2 \cdot 3 + 1$ or 10 additional points of the curve. We can do this and still have one point left at our disposal. This point can be taken on any generator of the cone, causing this generator to be the residual intersection of the cone and the monoid.

Similarly, a sextic curve with a quadruple point can be obtained as the complete intersection of a quadric cone and a cubic monoid. The cubic monoid is determined by 15 points, of which 4 must be taken off the quadric to insure the monoid not containing it as a factor. In order to make this monoid contain the sextic that has a quadruple point at the vertex of the monoid, we must make it contain $6 \cdot 3 - 4 \cdot 2 + 1$ or 11 additional points. This is the exact number at our disposal.

IV.

On the Composition of Multiple Points.

1. If a curve of order m has an $(m - 2)$-tuple point, it can have no other multiple point. The curve is unicursal. We assume $5 \gtreqless m$. The tangents to the curve at this multiple point cannot lie in a plane, but they lie on a quadric cone. As we saw on page 510, this curve has $m - 2$ apparent double points. It can have no more apparent singularities.

A curve with an $(m - 3)$-tuple point can have at most one other actual multiple point, viz. a double point. If it had more it would necessarily be a plane curve. Thus a twisted quintic cannot have three actual double points. A curve of order m having an $(m - 3)$-tuple point and a double point lies in general on a cone of order $m - 2$ having the double point as vertex and the curve as base. This curve cannot have more than $\frac{(m-3)(m-4)}{2}$ double edges. The edge joining the vertex to the $(m - 3)$-tuple point counts for just this number of double edges. The cone can therefore have no more double edges; that is, when viewed from the double point of the curve, the number of apparent double points of this curve is zero. According to Salmon,* the number of apparent double points of the curve is $2m - 6$ more when viewed from an arbitrary point in space than when viewed from a double point of the curve. A curve of order m with an $(m - 3)$-tuple point and an actual double point has therefore just $2m - 6$ apparent double points. It can have no other actual multiple point or apparent singularity. It is thus evident that when one of the branches through the double point moves into the $(m - 3)$-tuple point, the curve loses $m - 4$ apparent double points, that is an $(m - 2)$-tuple point may be formed from an $(m - 3)$-tuple point, a double point and $m - 4$ apparent double points. A triple point on a quintic curve is thus equivalent to two actual double points and one apparent double point. This triple point does not have its tangents lying in one plane.

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* Salmon's Geometry of Three Dimensions (1882), No. 330, Example 2.

Consider now a curve of order m with an $(m-4)$-tuple point and a triple point. The curve can have no other actual multiple point. We assume $6 \gtreqless m$. In this case we take the vertex of the cone at the triple point, and must therefore know the reduction in the number of apparent double points of a curve when it is viewed from a triple point of the curve. We shall, however, consider the general case and consider the reduction when the curve is viewed from a k-tuple point on it. Plucker's formulae * give

$$\kappa = 3\nu(\nu-2) - 6\tau - 8i, \tag{1}$$

$$3\mu = 3\nu(\nu-1) - 6\tau - 9i, \tag{2}$$

$$i = 3\mu(\mu-2) - 6\delta - 8\kappa, \tag{3}$$

$$\kappa = \tfrac{1}{3}\left[\mu(\mu-1) - 2\delta - \nu\right]. \tag{4}$$

In the case we are considering,

$$\mu = m - k, \quad \nu = r - 2k, \quad \kappa = \beta.$$

Substituting these values in (1) and (2), and combining, we have

$$i = \beta - 3(m-k) + 3(r-2k).$$

Substituting the same values in (3), we have

$$i = 3(m-k)(m-k-2) - 6\left(\delta_1 + \sum_2 \frac{\rho(\rho-1)}{2}\right) - 8\beta;$$

where $\sum_2$ extends to all multiple edges due to all multiple points other than the vertex, and where δ_1 is the number of double edges due to apparent double points. Eliminating i between these last two equations, we have

$$2\left(\delta_1 + \sum_2 \frac{\rho(\rho-1)}{2}\right) = (m-k)^2 - m + 3k - r - 3\beta. \tag{5}$$

Now take the vertex of the cone at an arbitrary point in space. If δ is the number of double edges on this cone, we have

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* Salmon's Geometry of Three Dimensions (1882), No. 325, footnote.

$$\delta = h + \sum_2 \frac{\rho(\rho - 1)}{2} + \frac{k(k - 1)}{2}.$$

Substituting in formulae (4), we have

$$\beta = \tfrac{1}{3}\left[m(m - 1) - 2\left(h + \frac{k(k - 1)}{2} + \sum_2 \frac{\rho(\rho - 1)}{2}\right) - r\right]. \quad (6)$$

Comparing (5) and (6), we have

$$\delta_1 = h - m k + k(k + 1);$$

that is, the number of apparent double points is less by $m k - k(k + 1)$ when viewed from a k-tuple point on the curve than when viewed from an arbitrary point in space.

Consider now a curve of order m that has an $(m - k - 1)$-tuple point and a k-tuple point. Then the curve can have no more actual double or multiple points. It will in general lie on a cone of order $m - k$ that has the k-tuple point as vertex and the curve as base. This cone cannot have more than $\frac{(m - k - 1)(m - k - 2)}{2}$ double edges. The $(m - k - 1)$-tuple edge to the $(m - k - 1)$-tuple point counts for just this number. The cone can therefore have no double edges, that is the curve can have no apparent double points when viewed from the k-tuple point. It has therefore just $m k - k(k + 1)$ apparent double points when viewed from an arbitrary point in space. It can have no more apparent singularities.

Let P_n denote an n-tuple point on the curve. A curve of order m can thus have a P_{m-k-1}, a P_k, and $[m k - k(k + 1)]$ apparent double points, or a P_{m-k}, a P_{k-1}, and $[m(k - 1) - k(k - 1)]$ apparent double points. We can therefore write symbolically

$$P_{m-k} + P_{k-1} = P_{m-k-1} + P_k + (m - 2k) \text{ apparent double points.} \quad \text{(I)}$$

[We obtained $m k - k(k + 1)$ apparent double points by assuming the vertex to be taken at P_k. If we interchange and take the vertex at P_{m-k-1}, we get the same result, viz.:

P_k, P_{m-k-1}, and $m(m - k - 1) - (m - k - 1)(m - k)$

apparent double points, i. e.

P_k, P_{m-k-1}, and $m k - k(k + 1)$ apparent double points.]

If we use a and b, where $a + b = m - 1$, in (I), we obtain

$$P_a + P_b + (m - 2\,b) \text{ apparent double points} = P_{a+1} + P_{b-1}.$$

If m is odd, we have

$$P_{\frac{m-1}{2}} + P_{\frac{m-1}{2}} + \left[\frac{m\,(m-1)}{2} - \frac{(m-1)\,(m+1)}{4}\right]$$

apparent double points

$$= P_{\frac{m+1}{2}} + P_{\frac{m-3}{2}} + \left[\frac{m\,(m-3)}{2} - \frac{(m-3)\,(m-1)}{4}\right]$$

apparent double points, i. e.

$$2\,P_{\frac{m-1}{2}} + 1 \text{ apparent double point} = P_{\frac{m+1}{2}} + P_{\frac{m-3}{2}}.$$

Thus, if $m = 2\,a + 1$

$$2\,P_a + 1 \text{ apparent double point} = P_{a+1} + P_{a-1}.$$

If m is even, we have

$$P_{\frac{m}{2}} + P_{\frac{m-2}{2}} + \left[\frac{m\,(m-2)}{2} - \frac{m\,(m-2)}{4}\right] \text{ apparent double points}$$

$$= P_{\frac{m+2}{2}} + P_{\frac{m-4}{2}} + \left[\frac{m\,(m-4)}{2} - \frac{(m-4)\,(m-2)}{4}\right]$$

apparent double points, i. e.

$$P_{\frac{m}{2}} + P_{\frac{m-2}{2}} + 2 \text{ apparent double points} = P_{\frac{m+2}{2}} + P_{\frac{m-4}{2}}.$$

Thus if $m = 2\,a$,

$$P_a + P_{a-1} + 2 \text{ apparent double points} = P_{a+1} + P_{a-2}.$$

Therefore, in general,

If $m = 2\,a + 1$,

$$2\,P_a + k^2 \text{ apparent double points} = P_{a+k} + P_{a-k};$$

also,

$$P_{a+k} + P_{a-k} + g\,(g + 2\,k) \text{ apparent double points} = P_{a+k+g} + P_{a-k-g}:$$

If $m = 2\,a$,

$$P_a + P_{a-1} + k\,(k + 1) \text{ apparent double points} = P_{a+k} + P_{a-k-1};$$

also,

$$P_{a+k} + P_{a-k-1} + g\,(g + 2\,k + 1) \text{ apparent double points} = P_{a+k+g} + P_{a-k-g-1}.$$

A sextic curve can thus have a quadruple point and four apparent double points, or a triple point, a double point, and six apparent double points. A quadruple point on a sextic curve is thus equivalent to a triple point, a double point, and two apparent double points.

Similarly a septimic curve can have a quintuple point and five apparent double points, or a quadruple point, a double point, and eight apparent double points, or two triple points and nine apparent double points. A quintuple point on a septimic is thus equivalent to a quadruple point, a double point, and three apparent double points, or to two triple points and four apparent double points. Likewise a quadruple point and a double point on a septimic curve are equivalent to two triple points and one apparent double point.

2. We shall now examine different kinds of multiple points and determine the number of actual and apparent double points to which they are equivalent. We shall consider in detail all kinds of multiple points from the triple point to the septuple point inclusive, and shall then draw some conclusions for multiple points in general.

I. *Triple Points.*

A triple point on a curve can, as we have seen, be of two kinds according as *the tangents to the curve at this point do or do not lie in one plane.* We shall consider the way in which a triple point can be formed. A curve C_m with a double point can be obtained as the partial intersection of a cone K and a monoid M, where the double point of C_m is a double point on K and an ordinary point of M. The line from this point to the common vertex of K and M is a double edge on K but does not lie on M. Let another sheet of K that passes near this line intersect M in a new branch of C_m. This new sheet of K intersects the two sheets through the double line in two lines that are also double edges on K. These double edges must be due to actual or apparent double points of C_m. If K is deformed in such a way that these two edges coincide with the first double edge and form a triple edge, C_m will have a triple point. The tangents at this triple point will all lie in one plane, as they are cut out of the three tangent planes to K along the triple line by the one tangent plane to M at this point. The last two double edges of K must have been due to actual double points of C_m, these double points being the points of intersection of the ordinary sheet of M by the double edges of K. This triple point is thus equivalent to *three actual* double points. If, however, when we deform K we also deform M in such a way that it has a double point at the double point of C_m, it is evident that the three tangents in the limit will not lie in one plane. For they are the intersections of the tangent cone at the double point of M by the three tangent planes to K along the triple line. The tangent line to the third or new branch intersects the plane of the tangents to the other two branches in the limit at an angle; the new branch must therefore be considered as meeting the first two branches together once at that point. The two double edges of K caused by the third sheet cannot in this case both be considered as due to actual double points of C_m, as the three tangents would then in the limit lie in one plane. Nor can both double edges be considered as due to apparent double points, for the new branch actually meets the other two branches together in the limit. One must therefore be considered as due to an actual and one to an apparent double point. This kind of triple point is thus equivalent to *two actual* and *one apparent* double point. This agrees with what we obtained on page 512.

II. *Quadruple Points.*

(1). If the four tangents all lie in one plane, as in 1,* the quadruple point is equivalent to 6 *actual* double points. This is shown in the same way as the case of the triple point of the first kind.

(2). If three tangents lie in one plane, as in 2,* the quadruple point is equivalent to 4 *actual* and 2 *apparent* double points. For if a curve having such a quadruple point is obtained as the partial intersection of a cone K and a monoid M, the point must be at least a triple point and the line from it to the vertex at least a double line of kind III on M; otherwise three tangents cannot lie in one plane. Let a curve C_m with a triple point at which the tangents lie in one plane be obtained as the partial intersection of a cone K and a monoid M that has the point as a triple point. This means that the three lines in which (in addition to the line from the multiple point to the vertex 6 times) the three tangent planes to K intersect the tangent cone at the triple point of M lie in one plane. Let a new branch of C_m passing near the triple point be caused by another sheet of K intersecting M. This sheet meets the three sheets through the triple edge of K in three lines, which are thus double lines on K. These lines must be due to actual or apparent double points of C_m. As in the case of the triple point it is evident that one and no more must in the limit be considered as due to an actual double point. This quadruple point is thus equivalent to the triple point, one actual double point, and two apparent double points, that is, to four actual and two apparent double points.

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* These numbers and succeeding ones refer to figures on the accompanying plate.

(3). If no three tangents lie in one plane, as in 3, the quadruple point is equivalent to 3 *actual* and 3 *apparent* double points. For, a curve having a triple point at which the tangents lie on a quadric cone but not in a plane can be obtained as the partial intersection of a cone K and a monoid M that has the point as a double point and a line from it to the vertex as an ordinary line of kind III. A new branch of the curve can be added, as in the previous cases, to form a quadruple point. The tangent to this branch will be cut out of the tangent cone at the double point of M by the tangent plane to the new sheet of K. As this tangent does not in general lie in the plane of any two other tangents, the new branch must be considered as meeting the three branches together once at the triple point. This quadruple point is therefore equivalent to the triple point and one actual and two apparent double points, that is, to three actual and three apparent double points. This agrees with what we obtained on pages 501 and 516.

We can go on in this way obtaining the different kinds of points of any multiplicity by adding a new branch to points of one less multiplicity.

III. *Quintuple Points.*

(1). If all the tangents lie in one plane, as in 4, the quintuple point is equivalent to 10 *actual* double points. This is obtained by adding a new branch to a II (1)* in such a way that its tangent lies in the plane of the four tangent lines at the quadruple point.

(2). If four tangents lie in one plane, as in 5, the quintuple point is equivalent to 7 *actual* and 3 *apparent* double points. This can be obtained from a II (1).

(3). If three tangents lie in one plane, and the other two lie in a plane with one of these, as in 6, the quintuple point is equivalent to 6 *actual* and 4 *apparent* double points. This can be obtained from a II (2) by adding a new branch in such a way that its tangent in the limit lies in a plane with the tangents to two other branches. It is therefore equivalent to a II (2), two actual double points and two apparent double points. In Figure 6, let us consider the line 123 as the cross section of a plane that contains the lines whose bases are 1, 2, and 3, and the line 145 as the cross section of a plane that contains the lines whose bases are 1, 4, and 5; where the five lines have not yet united to form a quintuple point. The lines then have six actual intersections, viz. 12, 13, 23, 14, 15, and 45, and four apparent intersections, viz. 24, 25, 34, and 35. Considering these lines as the tangents to a curve we see that this quintuple point can in the limit be considered as composed of six actual and four apparent double points.

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* We mean by II (1) a quadruple point of kind (1), etc.

(4). If three tangents lie in one plane and the other two do not lie in a plane with one of these, as in 7, the quintuple point is equivalent to 5 *actual* and 5 *apparent* double points. This quintuple point can be obtained from a II (2) by adding a new branch with its tangent in the limit not in the plane of two other tangents, or from a II (3), by adding a new branch with its tangent in the limit in the plane of two other tangents.

(5). If no three tangents lie in one plane, as in 8, the quintuple point is equivalent to 4 *actual* and 6 *apparent* double points. This is the kind of quintuple point mentioned on page 516. It can be obtained from a II (3) by adding a new branch in such a way that its tangent does not in the limit lie in the plane of two other tangents at the quadruple point. It does not matter whether this new branch is added in such a way that the five tangents in the limit lie on a quadric cone that contains an arbitrary line as an edge or on a cubic cone that contains this line as a double edge. The multiplicity of the point on the monoid only affects the composition of the multiple point on the curve by placing an upper limit on the number of tangent lines that can lie in one plane. The only relations between the tangent lines that directly affect the number of actual and apparent double points to which the multiple point is equivalent are the number of lines that lie in one plane and the number that lie in two or more planes containing two or more other tangent lines.

IV. *Sextuple Points.*

(1). If six tangents lie in one plane, as in 9, the sextuple point is equivalent to 15 *actual* double points. This can be obtained from a III (1).

(2). If the tangents lie by threes in four planes, as in 10, the sextuple point is equivalent to 12 *actual* and 3 *apparent* double points. For let 1, 2, 3, 4, 5, and 6 represent not only the bases of the tangent lines, as in 10, but also the tangent lines themselves, where the branches have not yet united to form the sextuple point. We suppose 1, 2, and 3 to lie in one plane, and 1, 4, and 5 in another plane, but no three of these lines to meet in one point. The line 6 is added in such a way that it lies in the plane not only of 2 and 5 but also of 3 and 4. The lines before we have reached the limit then lie as in the tetrahedron in Figure 11. The actual intersections are 12, 13, 14, 15, 23, 25, 26, 34, 36, 45, 46, and 56, and the apparent intersections are 16, 24, and 35; thus twelve actual and three apparent double points in the composition of the sextuple point.

(3). If five tangents lie in one plane, as in 12, the sextuple point is equivalent to 11 *actual* and 4 *apparent* double points. This can be obtained from a III (1).

(4). If four tangents lie in one plane, and the other two lie in a plane with one of the four, as in 13, the point is equivalent to 9 *actual* and 6 *apparent* double points. This can be obtained from a III (2).

(5). If the tangents lie by threes in three planes, as in 14, the point is equivalent to 9 *actual* and 6 *apparent* double points. This can be obtained in the same way as case (2) above. The lines before we reach the limit lie as in Figure 15. The actual intersections are 12, 13, 14, 15, 23, 25, 26, 45, and 56, and the apparent intersections are 16, 24, 34, 35, 36, and 46.

(6). If four tangents lie in one plane and the other two do not lie in one plane with one of these, as in 16, the sextuple point is equivalent to 8 *actual* and 7 *apparent* double points. This can be obtained from a III (2).

(7). If three tangents lie in one plane and two lie in a plane with one of these, and the six in no plane with two others, as in 17, the sextuple point is equivalent to 7 *actual* and 8 *apparent* double points. This can be obtained from a III (3).

(8). If three tangents lie in one plane and the other three in a plane that does not contain one of the first three, as in 18, the point is equivalent to 7 *actual* and 8 *apparent* double points. This can be obtained from a III (4).

(9). If three tangents lie in one plane and none of the remaining lie in a plane with two others, as in 19, the point is equivalent to 6 *actual* and 9 *apparent* double points. This can be obtained from a III (4).

(10). If no three lie in one plane, as in 20, the point is equivalent to 5 *actual* and 10 *apparent* double points. This can be obtained from a III (5).

V. *Septuple Points.*

(1). If the tangents all lie in one plane, as in 21, the point is equivalent to 21 *actual* double points. This can be obtained from a IV (1).

(2). If the tangents lie by threes in six planes, as in 22, the point is equivalent to 18 *actual* and 3 *apparent* double points. This will be explained with the following case.

(3). If the tangents lie by threes in five planes, as in 23, the point is equivalent to 16 *actual* and 5 *apparent* double points. The branches of a sextuple point of kind (2) before they unite to form the sextuple point have their tangents lying as edges of a tetrahedron, as in 11. Adding 7 in the plane of 3 and 5 causes the intersection not only of 7 with 3 and 5, and of 3 with 5, but also of 1 with 6. The lines 1, 2, 3, 4, 5, and 6 now meet in one point. This septuple point is thus equivalent to a IV (2), four actual and two apparent double points, that is, to 16 actual and 5 apparent double points. If moreover 7 is added in such a way that it also lies in the plane of 1 and 6, it meets 1 and 6 and two more actual double points must be added. A septuple point of kind (2) is thus equivalent to 18 actual and 3 apparent double points.

(4). If six tangents lie in one plane, as in 24, the point is equivalent to 16 *actual* and 5 *apparent* double points. This can be obtained from a IV (1).

(5). If four tangents lie in one plane, and the others as in 25, the point is equivalent to 15 *actual* and 6 *apparent* double points. This can be obtained from a IV (2).

(6). If the tangents lie as in 26, the point is equivalent to 15 *actual* and 6 *apparent* double points. The tangent lines before the branches of the curve unite to form the septuple point lie as in 27, which have twelve points of intersection.

(7). If five tangents lie in one plane, and the other two lie in a plane with one of these, as in 28, the point is equivalent to 13 *actual* and 8 *apparent* double points. This can be obtained from a IV (3).

(8). If six of the tangents lie by threes in four planes, and the seventh is not in a plane with two others, as in 29, the point is equivalent to 13 *actual* and 8 *apparent* double points. This can be obtained from a IV (2).

(9). If five tangents lie in one plane, and the other two do not lie in a plane with one of these, as in 30, the point is equivalent to 12 *actual* and 9 *apparent* double points. This can be obtained from a IV (3).

(10). If four tangents lie in one plane and the other three lie in a plane with one of these, as in 31, the point is equivalent to 12 *actual* and 9 *apparent* double points. This can be obtained from a IV (4).

(11). If four tangents lie in one plane, and the other three as in 32, the point is equivalent to 12 *actual* and 9 *apparent* double points. This can be obtained from a IV (5).

(12). If the tangents lie by threes in four planes, as in 33, the point is equivalent to 12 *actual* and 9 *apparent* double points. This can be obtained from a IV (5) by adding 7 so as to lie in a plane with 6 and 4; thus causing 6 and 4 to meet.

(13). If four tangents lie in one plane, and two lie in a plane with one of these, and the seventh lies in no plane with two others, as in 34, the point is equivalent to 10 *actual* and 11 *apparent* double points. This can be obtained from a IV (7).

(14). If four tangents lie in one plane, and the other three lie in a plane not containing one of the first four, as in 35, the point is equivalent to 10 *actual* and 11 *apparent* double points. This can be obtained from a IV (8).

(15). If the tangents lie as in 36, the point is equivalent to 10 *actual* and 11 *apparent* double points. This can be obtained from a IV (5).

(16). If four tangents lie in one plane, and none of the other three lie in a plane with any two, as in 37, the point is equivalent to 9 *actual* and 12 *apparent* double points. This can be obtained from a IV (9).

(17). If the tangents lie by threes in three planes, as in 38, the point is equivalent to 9 *actual* and 12 *apparent* double points. This can be obtained from a IV (7).

(18). If the tangents lie as in 39, the point is equivalent to 8 *actual* and 13 *apparent* double points. This can also be obtained from a IV (7).

(19). If three tangents lie in one plane, and three in another plane, and the seventh in no plane with two others, as in 40, the point is equivalent to 8 *actual* and 13 *apparent* double points. This can be obtained from a IV (8).

(20). If three tangents lie in one plane, and none of the remaining four lie in a plane with any two, as in Figure 41, the point is equivalent to 7 *actual* and 14 *apparent* double points. This can be obtained from a IV (10).

(21). If no three tangents lie in one plane, as in 42, the point is equivalent to 6 *actual* and 15 *apparent* double points. This can also be obtained from a IV (10).

VI. *Multiple Points in General.*

Considering multiple points in general, it is evident that a k-tuple point is equivalent to at least $k-1$ actual double points. For, every branch of the curve has in the limit at least one actual intersection with the totality of $k-1$ other branches. It is also evident that if the tangents at the multiple point lie in a number of planes of which no two have a tangent line in common the totality of the branches whose tangents lie in one plane must be considered as having at least one connection or intersection with the totality of branches whose tangents lie in the other planes. The whole system of tangent lines or of branches themselves may thus in the limit be considered as forming a connected system. The difference between the number of actual double points and $\frac{k\,(k-1)}{2}$ is the number of apparent double points contained in the composition of the k-tuple point.

A $(k+1)$-tuple point can be obtained by causing a new branch of the curve to pass through a k-tuple point of the curve. If this new branch is added in such a way that its tangent lies in a plane with the tangents to λ of the branches through the k-tuple point, it must be considered as meeting the k branches together λ times at the k-tuple point; and we have: —

$$P_{k+1} = P_k + \lambda \text{ actual d. pts.} + (k-\lambda) \text{ apparent d. pts.}$$

If the new branch is added in such a way that its tangent in the limit lies in σ planes that contain two other tangents to the curve at the k-tuple point, we must in general add three actual double points to the value of the multiple point for each of these σ planes, viz. two for the intersection of this new branch with the two whose tangents lie in the same plane with its tangent and one for the intersection of these two branches. This is shown if we consider a V (2) as obtained from a IV (5). The new branch is added in such a way that its tangent lies in the limit in three planes each of which contains two other tangents to the curve; nine actual double points thus being added to the value of a IV (5) to obtain the value of a V (2). If, however, in the k-tuple point γ connections were necessary involving branches whose tangents lie in these σ planes, we must add only $3\,\sigma - \gamma$ actual double points. Thus, if we consider a V (6) as obtained from a IV (8), we add only eight actual double points, for the three branches through the sextuple point whose tangents lie in one plane are considered as having one connection in the limit with the branches whose tangents lie in the other plane. If the new branch is added in such a way that its tangent lies in the limit in ρ planes that contain respectively $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots \lambda_\rho$ other tangent lines, where $3 \overline{\gtrless} \lambda_\nu$, we must in general add $\delta \equiv \lambda_1 + \lambda_2 + \lambda_3 + \ldots\ldots + \lambda_\rho$ actual double points. If, however, in the limit there were α connections necessary between these different sets of branches of the k-tuple point, we must add only $\delta - \alpha$ actual double points. Thus, if we consider a

V (10) as obtained from a IV (8), we add only five actual double points, for the branches of the sextuple point whose tangents lie in one plane are considered as having one connection in the limit with the branches whose tangents lie in the other plane.

V.

CLASSIFICATION OF QUINTIC, SEXTIC, AND SEPTIMIC CURVES THAT HAVE MULTIPLE POINTS.

A.

Quintic Curves.

A quintic curve, according to the enumeration of Salmon,[*] can have

a) 2 and 4[†]; a′) 1 and 5; a″) 0 and 6;
b) 1 and 4; b′) 0 and 5;
c) 0 and 4.

In addition to these there is the quintic that has a triple point at which the tangents do not lie in one plane.[‡] This quintic can have no actual double point in addition to the triple point. It can be obtained as the partial intersection of the quadric cone $a\,b - d^2$ and the cubic monoid $a\,b\,d + b^2 c + (b^2 + d^2)\,e$ that have the point $a\,b\,d$ as the common vertex. The residual intersection is the line $b\,d$. The point $a\,b\,d$ being a quadruple point on the complete intersection and an ordinary point on the line $b\,d$ is a triple point on the quintic curve. As the tangent lines at the triple point are the three lines that in addition to the line $b\,d$ form the complete intersection of the quadric cones $a\,b - d^2$ and $b^2 + d^2$, it is evident that they do not lie in one plane.[§] As the quadric cone $a\,b - d^2$ can have no double edge, the quintic curve can have no apparent double point when viewed from the triple point. It has, therefore, $m\,k - k\,(k + 1) \equiv 5 \cdot 3 - 3 \cdot 4$ or 3 apparent double points when viewed from an arbitrary point. The triple point on the quintic is equivalent, as we have seen, to two actual and one apparent double points. The quintic can therefore be obtained directly from a) above, as there are just three apparent double points when two actual and one apparent double points are taken to form the triple point. The curve can be obtained from a$'$) or a$''$) only by having one or two apparent double points change to actual double points in the limit; this produces no new kind of curve. The curve cannot be obtained from b), b$'$), or c) as there will not be enough apparent double points remaining. There is thus only one species of quintic curve having a triple point, and it has three apparent double points.

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[*] Salmon's Geometry of Three Dimensions (1892), p. 318.

[†] The two numbers in each case correspond to the number of actual double points and apparent double points, respectively.

[‡] Salmon's Geometry of Three Dimensions (1882), p. 320.

[§] No two of the three lines can coincide, as the plane through this line and the third line would meet the curve in six points.

B.

Sextic Curves.

A sextic curve, according to the classification of Salmon,* can have

a) 4, 6; a$'$) 3, 7; a$''$) 2, 8; a$'''$) 1, 9; a$^{\text{iv}}$) 0, 10;
b) 3, 6; b$'$) 2, 7; b$''$) 1, 8; b$'''$) 0, 9;
c) 2, 6; c$'$) 1, 7; c$''$) 0, 8;
d) 1, 6; d$'$) 0, 7;
e) 0, 6.

In addition to these are the sextics that have multiple points.

1. A sextic curve may have a quadruple point. The tangents at this point cannot lie in one plane, as this plane would meet the curve in eight points. A quadruple point at which three tangents lie in one plane is equivalent, as we have seen, to four actual and two apparent double points. A sextic with such a quadruple point thus seems to be derivable from a curve of class a) above. The plane of the three tangents would, however, meet the curve in seven points, which is not possible in the case of a twisted sextic. It is thus evident that although a curve may have actual and apparent double points sufficient to form a certain multiple point nevertheless there are cases where the points cannot unite to form this multiple point. This is analogous to the case of plane curves.† The only quadruple point possible on a sextic curve is one at which no three tangents lie in one plane. This curve has been considered on page 511, as the complete intersection of a quadric cone and a cubic monoid. It has four apparent double points. The quadruple point on this curve is equivalent to three actual and three apparent double points. The curve can thus be obtained directly from a$'$) above. It cannot be formed from a) as it cannot have a double point in addition to

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* Salmon, Cambridge & Dublin Mathematical Journal, Vol. V.

† Salmon's Higher Plane Curves (1873), p. 27.

the quadruple point. It can be obtained from one of the remaining a)'s only by having some of the apparent double points change to actual double points in the limit. This can give no new species of sextic curve. It is not possible to obtain the curve from any b), c), d), or e), as there will not be enough apparent double points remaining. There is thus only one species of sextic curve with a quadruple point and it has four apparent double points.

2. A sextic curve may have a triple point at which the tangents do not lie in one plane. This sextic lies on a cone of order three that has the triple point as vertex and the curve as base. This cone can have at most one double edge. The curve can therefore have at most one actual double point or one apparent double point when viewed from the triple point. As the number of apparent double points of a sextic when viewed from an arbitrary point in space is six more than when viewed from a triple point of the curve it is evident that a sextic having such a triple point may have either 0 and 6, 1 and 6, or 0 and 7. These sextics can be cut out of the cubic cone by a cubic monoid. A cubic monoid is determined by fifteen arbitrary points other than the vertex. In order to make this monoid contain the curve we must make it contain nineteen points of it. As the vertex, however, counts for six points common to the curve and the monoid, it is only necessary to make the monoid contain thirteen other points of the curve. There are thus two points left at our disposal. The cubic cone meets the inferior cone of the monoid in six lines, of which three are the tangent lines to the curve at the triple point and three are lines common to the cone and the monoid. These three lines and the sextic make up the complete intersection of the two cubic surfaces. As the tangent lines at the triple point lie on the quadric inferior cone of the monoid they do not lie in one plane. As there are six lines on the monoid and only three lines common to cone and monoid it is evident that it is not necessary for the cone to have a double edge due to an apparent double point. If the cone and the monoid have three distinct lines in common the curve has no apparent double points when viewed from the triple point. It may or may not have an actual double point in addition according as the cone does or does not have a double edge. If the cone and the monoid have only two lines in common, the curve must either have an apparent double point when viewed from the triple point, or one of the lines common to the cone and the monoid must be a line to an actual double point of the curve. All three kinds above can therefore be obtained as the partial intersection of a cubic cone by a cubic monoid. These three kinds can

be obtained directly from b′), a′), and a″) above. The other cases either give us these in the limit, or give a curve with more actual double points or fewer apparent double points than the sextic curve can or must have.

The sextic with a double point in addition to the triple point can be obtained as the partial intersection of a quartic cone, having the double point as vertex and the curve as base, and a cubic monoid having the triple point of the curve as a double point and the line from it to the vertex as an ordinary line of kind III. The quartic cone meets the inferior cone of the monoid in eight lines, of which two are tangent lines at the double point and six are lines common to cone and monoid. The line to the triple point counts as three of the six lines of the monoid. The cone need therefore have no double edges, and in fact cannot have, as it already has a triple edge. Three lines in addition to the line to the triple point must therefore be common to cone and monoid. The three branches of the curve at the triple point are cut out of the monoid at the double point by the three sheets of the cone that pass through the triple line. The tangent lines at the triple point are the three lines in which, in addition to the line from the triple point to the vertex three times, the tangent planes to the three sheets of the cone intersect the tangent cone at the double point of the monoid. As the curve can have no apparent double points when viewed from the double point, it has $2m - 6$ or 6 apparent double points, when viewed from an arbitrary point in space.

3. A sextic curve can have a triple point at which the tangents lie in one plane. Such a sextic can be obtained as the intersection of a cubic cone and a quadric (monoid) that passes through the vertex of the cone but has no line in common with it. The vertex, being a triple point on the cone and an ordinary point on the monoid, is a triple point on the curve. The tangents at this point lie in one plane, for they are the intersections of the cubic cone by the tangent plane to the quadric at this point. The cone can have no double edge due to an apparent double point of the curve, for this would cause the sextic to break up into a quartic curve and the line doubled. It may, however, have a double edge due to an actual double point of the curve. The curve, whether it has an actual double point or not, has therefore no apparent double points when viewed from the triple point. We thus obtain two species of sextics with such a triple point, viz., 1) 0 and 6, and 2) 1 and 6.

A sextic curve with such a triple point can also be obtained as the partial intersection of a quintic cone and a cubic monoid. The quintic cone has an ordinary point of the curve as vertex and the line from this point to the triple point of the curve as a triple edge. The monoid is determined by 15 points. In order to make it contain the curve we must make it contain 19 points of it. As the vertex counts for two of these points, it is only necessary to make the monoid contain 17 additional points of the curve. If we take the triple point of the curve as one of the 15 points at our disposal, it is evident that there are just enough points to make the monoid contain the curve. The quintic cone meets the inferior cone of the monoid in 10 lines, of which one is the tangent to the curve at the vertex and the other nine are lines common to the two surfaces. As there are only six lines on the monoid, it is evident that at least three lines common to cone and monoid are double lines on the cone due to apparent double points of the curve. The cone can, however, not have more than three double edges in addition to the triple edge; every line of the monoid must therefore lie on the cone. This sextic has therefore three apparent double points when viewed from an ordinary point of the curve, or seven apparent double points when viewed from an arbitrary point.

We can obtain the three species directly from b), a), and a′), respectively. As the triple point is equivalent to three actual double points, it is evident that no other species can be obtained. The sextic that has a double point in addition to this triple point can also be obtained as the intersection of a quartic cone and a cubic monoid that have the double point of the curve as their common vertex.

C.

Septimic Curves.

A septimic curve, according to Genty,* always has at least 9 apparent double points. The possible cases are then: —

a) 6,9; a′) 5,10; a″) 4,11; a‴) 3,12; a$^{\text{iv}}$) 2,13; a$^{\text{v}}$) 1,14; a$^{\text{vi}}$) 0,15
b) 5,9; b′) 4,10; b″) 3,11; b‴) 2,12; b$^{\text{iv}}$) 1,13; b$^{\text{v}}$) 0,14.
c) 4,9; c′) 3,10; c″) 2,11; c‴) 1,12; c$^{\text{iv}}$) 0,13;
d) 3,9; d′) 2,10; d″) 1,11; d‴) 0,12; d$^{\text{iv}}$);
e) 2,9; e′) 1,10; e″) 0,11;
f) 1,9; f′) 0,10;
g) 0,9.

We shall describe in detail the septimic curves that have multiple points.

1. A septimic curve can have a quintuple point provided no three of the tangents lie in one plane. Such a quintuple point is equivalent to

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* Bull. de la Soc. Math., t. 9, 10 (1881), p. 153.

four actual and six apparent double points. This septimic can be obtained as the partial intersection of a quadric cone and a quartic monoid, the residual intersection being a straight line. This curve has, as we have seen, five apparent double points. It is thus obtainable from a$''$) above and is the only species of twisted septimic with a quintuple point.

2. A septimic curve can have a quadruple point at which three of the tangents lie in one plane. Such a quadruple point is equivalent to four actual and two apparent double points. This curve will lie on a cubic cone that has the quadruple point of the curve as vertex and the curve as base. If we wish to cut this curve out of the cubic cone by means of a monoid, we must take a monoid that has the multiple point of the curve as a point of multiplicity not less than three; otherwise the three tangents cannot lie in one plane. The monoid must therefore be of an order as great as four. A quartic monoid is determined by 24 points. Four of these points must be taken off the cubic cone to insure the monoid not breaking up into this cone and a plane. In order to make this quartic monoid contain the septimic curve that has the vertex of the monoid as a quadruple point, we must make it contain $4 \cdot 7 - 3 \cdot 4 - 1$ additional points of the curve. There are thus still three points left at our disposal. The cubic cone meets the inferior cone of the monoid in nine lines, of which four are tangent lines to the curve at the quadruple point and five are lines common to the two surfaces. It is evident, as in previous cases, that either four or five of the lines of the monoid are edges of the cubic cone, this cone having at most one double edge. Septimic curves with quadruple points of the above kind are thus of three kinds, viz., 1) 0, 8; 2) 1, 8; and 3) 0, 9. These can be obtained directly from b$'$), a$'$), and a$''$) above; there are no other possible cases.

3. A septimic curve with a quadruple point at which no three tangents lie in one plane can be obtained as the partial intersection of a cubic cone and a cubic monoid. The cubic cone has the quadruple point of the curve as vertex and the curve as base. The cubic monoid is determined by 15 arbitrary points. In order to make it contain a septimic curve that has the vertex of the monoid as a quadruple point, we must make it contain $3 \cdot 7 - 2 \cdot 4 + 1$ or 14 additional points of the curve. There is thus one point left at our disposal. The cubic cone meets the inferior cone of the monoid in six lines, of which four are tangents to the curve at the quadruple point and two are lines common to the cone and the monoid. These two lines and the septimic make up the complete intersection of the two surfaces. As in the previous case, it is evident that

this septimic can have: 1) 0, 8; 2) 1, 9; and 3) 0, 9. The quadruple point in this case is equivalent to three actual and three apparent double points. The above three species can thus be obtained directly from curves of kinds b''), a''), and a'''), respectively; no other species are possible.

4. A septimic curve with two triple points, the tangents at each of which lie in one plane, can be obtained as the partial intersection of a quartic cone having a triple edge and a quadric (monoid) that has a line in common with the cone. The vertex is a point of multiplicity four on the complete intersection and therefore, since the line passes through the point, a point of multiplicity three on the septimic curve. As the point is an ordinary point on the quadric, the three tangents to the curve at the triple point lie in one plane. The other triple point is the point distinct from the vertex where the sheet of the quadric crosses the triple line. The quartic cone having a triple edge can have no double edge in addition, due either to an actual or an apparent double point. The curve has therefore just $7 \cdot 3 - 3 \cdot 4$ or 9 apparent double points. This species can be obtained directly from a); it is the only possible species.

5. A septimic curve with two triple points, one of each kind, can be obtained as the partial intersection of a sextic cone and a quartic monoid. The sextic cone has an ordinary point of the curve as vertex and the curve as base; and the quartic monoid has the triple point at which the tangents do not lie in one plane as a double point, and the line from it to the vertex as an ordinary line of kind III. This quartic monoid is determined by twenty arbitrary points in addition to its vertex and the double point. In order to make this monoid contain the septimic that has the vertex as an ordinary point and the double point as a triple point, we must make it contain $4 \cdot 7 - 3 - 2 \cdot 3 + 1$, or 20 additional points on the curve. This is just the number at our disposal. The sextic cone meets the inferior cone of the monoid in 18 lines, of which one is the tangent line to the curve and 17 are lines common to the cone and the monoid. The monoid has 12 lines on it. The line of kind III counts as two lines of the monoid and as three lines common to cone and monoid. There are thus 10 other lines on the monoid that must be made to count for 14 lines common to cone and monoid. The sextic cone therefore has *4 double edges; these are due to apparent double points*, as the curve can have no actual double points in addition to the two triple points. As the quartic cone cannot have more than 4 double edges in addition to the 2 triple edges, it is evident that all lines of the monoid must lie on the cone. The three branches of the curve at one triple point are cut

out of the monoid at the double point by the three sheets of the cone, and thus do not have their tangents lying in one plane. The three branches of the curve at the other triple point are cut out of the cone by an ordinary sheet of the monoid at the point where the sheet crosses the second triple line; the tangents at this point thus lie in one plane. This curve has 4 apparent double points when viewed from an ordinary point of the curve, or 9 apparent double points when viewed from an arbitrary point. It can be obtained directly from a$'$) above.

6. A septimic curve that has two triple points of the second kind can be obtained as the partial intersection of a quartic cone and a cubic monoid. The quartic cone must have one of the triple points as vertex and the curve as base. The cubic monoid must have the second triple point as a double point and the line from it to the vertex as an ordinary line of kind III. This monoid is determined by 11 points in addition to the vertex and the other double point. In order to make this monoid contain a septimic curve that has each of the double points of the monoid as a triple point, we must make it contain $3 \cdot 7 - 2 \cdot 3 - 2 \cdot 3 + 1$ or 10 additional points of the curve. There is thus one point left at our disposal. The quartic cone meets the inferior cone of the monoid in 8 lines, of which 3 are tangent lines to the curve at the triple point at the vertex and 5 are lines common to the cone and the monoid. The quartic cone can have no double edge in addition to the triple edge to the second triple point. The curve has therefore no apparent double points when viewed from one of the triple points; it therefore has just 9 when viewed from an arbitrary point in space. This curve can be obtained from a septimic of kind a$''$), the two triple points counting as 4 actual and 2 apparent double points. It is to be noticed that when we wish to obtain a curve having a multiple point from a curve that has only double points, we cannot always use the curve that has the minimum number of apparent double points.

7. A quartic cone and a quadric (monoid) that have a line in common intersect in addition in a septimic curve that has a triple point at the vertex of the quartic cone. The tangents to the curve at the triple point lie in the tangent plane to the quadric at this point. The quartic cone may have three double edges, but none due to an apparent double point, for such an edge would count twice as a line common to cone and monoid. The curve therefore has $7 \cdot 3 - 3 \cdot 4$ or 9 apparent double points. This is also evident from our formula: —

$$h + h' + H'' = \frac{\mu\nu(\mu - 1)(\nu - 1)}{2} - \frac{kk'(k - 1)(k' - 1)}{2}.$$

Here $\mu = 4$, $\nu = 2$, $k = 4$, $k' = 1$, $h' = 0$, $H'' = 3$; therefore $h = 9$. This septimic may have as many as three actual double points in addition to the triple point.

A septimic curve with a triple point at which the tangents lie in one plane can be obtained also as the partial intersection of a quartic cone and a quartic monoid. We can then obtain the following species: —

(*1*). Those having 9 apparent double points; these may have as many as three actual double points in addition.

(*2*). Those having 10 apparent double points; these may have as many as two actual double points in addition.

(*3*). Those having 11 apparent double points; these may have one actual double point in addition.

(*4*). Those having 12 apparent double points.

These cases can be obtained directly from d), c), b), a), c′), b′), a′), b″), a″), a‴), respectively.

8. A septimic curve having a triple point at which the tangents do not lie in one plane can be obtained as the partial intersection of a quartic cone and a quartic monoid. We shall then obtain species similar to the four above. These different septimics can be derived directly from d′), c′), b′), a′), c″), b″), a″), b‴), a‴), a$^{\text{iv}}$), respectively.

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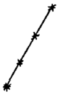

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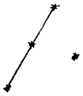

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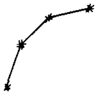

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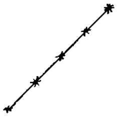

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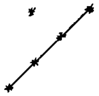

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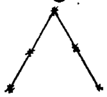

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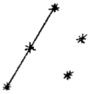

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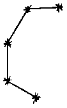

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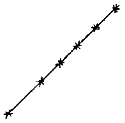

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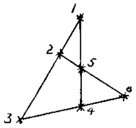

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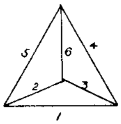

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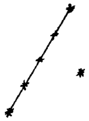

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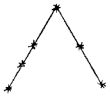

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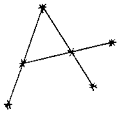

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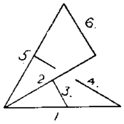

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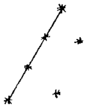

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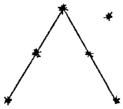

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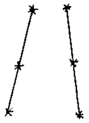

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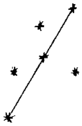

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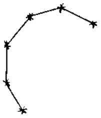

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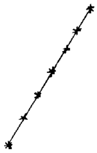

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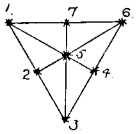

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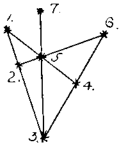

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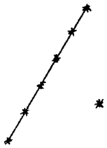

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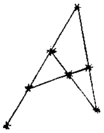

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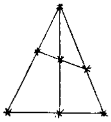

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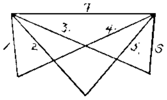

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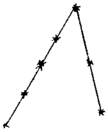

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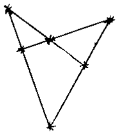

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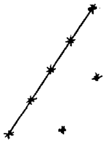

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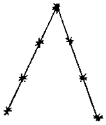

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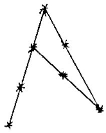

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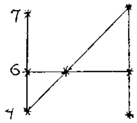

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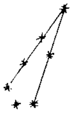

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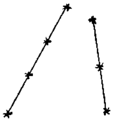

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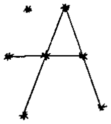

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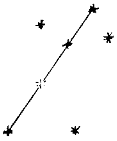

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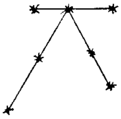

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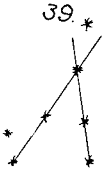

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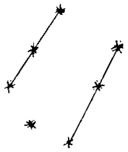

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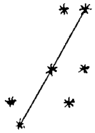

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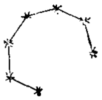

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE. E. L. MARK, DIRECTOR. — No. 136.

MENDEL'S LAW OF HEREDITY.

BY W. E. CASTLE.

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MENDEL'S LAW OF HEREDITY.

By W. E. Castle.

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What will doubtless rank as one of the great discoveries in biology, and in the study of heredity perhaps the greatest, was made by Gregor Mendel, an Austrian monk, in the garden of his cloister, some forty years ago. The discovery was announced in the proceedings of a fairly well-known scientific society, but seems to have attracted little attention and to have been soon forgotten. The Darwinian theory then occupied the centre of the scientific stage and Mendel's brilliant discovery was all but unnoticed for a third of a century. Meanwhile the discussion aroused by Weismann's germ-plasm theory, in particular the idea of the non-inheritance of acquired characters, had put the scientific public into a more receptive frame of mind. Mendel's law was rediscovered independently by three different botanists engaged in the study of plant-hybrids, — de Vries, Correns, and Tschermak, — in the year 1900. It remained, however, for Bateson, two years later, to point out the full importance and the wide applicability of the law. This he has done in two recent publications with an enthusiasm which can hardly fail to prove contagious. There is little danger, I think, of Mendel's discovery being again forgotten.

1. *The law of dominance.* When mating occurs between two animals or plants differing in some character, the offspring frequently all exhibit the character of one parent only, in which case that character is said to be "*dominant.*" Thus, when white mice are crossed with gray mice, all the offspring are gray, that color character being dominant. The character which is not seen in the immediate offspring is called "*recessive,*" for though unseen it is still present in the young, as we shall see. White, in the instance given, is the recessive character. The principle of heredity just stated may be called *the law of dominance.* The first instance of it discovered by Mendel, related to the cotyledon-color in peas

obtained by crossing different garden varieties. Yellow color of cotyledons was found to be dominant over green; likewise, round smooth form of seed was found to be dominant over angular wrinkled form; and violet color of blossoms, over white color. Other illustrations might be mentioned both among animals and among plants, but these will suffice.

2. *Peculiar hybrid forms.* The law of dominance is not of universal applicability; Mendel does not so declare, though some of his critics have thus interpreted him. In many cases the cross-bred offspring possess a character intermediate between those of the parents. This Mendel found to be true when varieties of peas differing in height were crossed.

Again, the cross-breds may possess what appears to be *an intensification of the character of one parent*, as when in crossing dwarf with tall peas the hybrid plant is *taller than either parent*, or as when, in crossing a brown-seeded with a white-seeded variety of bean, the offspring bear beans of a darker brown than that of the brown-seeded parent.

Thirdly, the cross-bred may have a character entirely different from that of either parent. Thus a cross between spotted, black-and-white mice, and albino mice, produces commonly mice entirely gray in color, like the house-mouse. Again, in crossing beans, a variety having yellowish-brown seeds crossed with a white-seeded variety yields sometimes black mottled seed, a character possessed by neither parent.

These three conditions may be grouped together by saying — the hybrid often possesses *a character of its own*, instead of the pure character of one parent, as is true in cases of complete dominance. The hybrid character may approximate that of one parent or the other, or it may be different from both. There is no way of predicting what the hybrid character in a given cross will be. It can be determined only by experiment, but it is always the same for the same cross, provided the parents are pure. Often the hybrid form resembles a supposed ancestral condition, in which case it is commonly designated a reversion. Illustrations are the gray hybrid mice, which are indistinguishable in appearance from the house-mouse, and slate-colored pigeons resulting from crossing white with buff pigeons.

3. *Purity of the germ-cells.* The great discovery of Mendel is this: *The hybrid, whatever its own character, produces ripe germ-cells which bear only the pure character of one parent or the other.* Thus, when one parent has the character A, and the other the character B, the hybrid will have the character AB, or in cases of simple dominance, A(B)* or

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* The parenthesis is used to indicate a recessive character not visible in the individual.

B(A). But whatever the character of the hybrid may be, its germ-cells, when mature, will bear *either the character A or the character B, but not both* ; and As and Bs will be produced *in equal numbers.* This perfectly simple principle is known as the law of " *segregation,*" or the law of the " purity of the germ-cells." It bids fair to prove as fundamental to a right understanding of the facts of heredity as is the law of definite proportions in chemistry. From it follow many important consequences.

A first consequence of the law of purity of the germ-cells is polymorphism of the second and later hybrid generations. The individuals of the first hybrid generation are all of one type, provided the parent races are pure. Each has a character resulting from the combination of an A with a B, let us say AB. [In cases of dominance it would more properly be expressed by A (B) or B (A).] But in the next generation three sorts of combination are possible, since each parent will furnish As and Bs in equal numbers. The possible combinations are AA, AB, and BB. The first sort will consist of pure As and will breed true to that character ever afterward, unless crossed with individuals having a different character. Similarly the third sort will be pure Bs and will breed true to that character. But the second sort, AB, will consist of hybrid individuals, like those of which the first hybrid generation consisted. If, as supposed, germ-cells, A and B, are produced in equal numbers by hybrids of both sexes, and unite at random, combinations AA, AB, and BB should occur in the frequencies, 1 : 2 : 1. For in unions between two sets of gametes, each A + B, there is one chance each for the combinations AA and BB, but two chances for the combination AB.

TABLE I.

Characters, →	A.	AB.	B.
Plants bearing Flowers in Color,	Magenta Red.	Lavender.	White.
1901, Lot 1	19	27	14
" " 2	9	20	9
1902, " 1	12	23	11
" " 2	14	26	11
Totals	54	96	45
Per cent of whole	29	49+	22

If the three forms AA (or simply A), AB, and B are all different in appearance, it will be a very simple matter to count those of each class and determine whether they occur in the theoretical proportions, 1 : 2 : 1. One such case has been observed by Bateson (:02, p. 183) among Chinese primroses (Primula sinensis). An unfixable hybrid variety known as "Giant Lavender," bearing flowers of a lavender color, was produced by crossing a magenta red with a white flowering variety tinged faintly with pink. By seed the hybrid constantly produces plants bearing magenta-red and white flowers respectively, as well as other plants bearing lavender flowers. The numerical proportions observed in two successive seasons are shown in Table I.

The observed numbers, it will be seen, are quite close to the theoretical, 1 : 2 : 1.

In cases wherein the hybrid is indistinguishable from one of the parent forms (i. e. in cases of complete dominance of one parental character), only two categories of offspring will be recognizable and these will be numerically as 3 : 1. But further breeding will allow the separation of the larger group into two subordinate classes, — first, individuals bearing only the dominant character; secondly, hybrids; that is, into groups A and A(B), which will be numerically as 1 : 2.

TABLE II.

Heredity of Cotyledon-color among Cross-bred Peas.

Parents crossed.	Offspring.			
	Gen. I.	Gen. II.	Gen. III.	Gen. IV.
G Y	Y(G)	1G	. G	. . G
			1G	. . G
		3 { 2Y(G)	3 { 2Y(G)	
			1Y . . .	. . Y
		1Y	. . . Y . . .	. . Y

Observed results are in this case very close to theory. Mendel, by crossing yellow with green peas, obtained, as we have seen, only yellow (hybrid) seed. Plants raised from this seed bore in the same pods both yellow seed and green seed in the ratio, 3 : 1. (See Table II.) The green seed produced in later generations green seed only. It bore only the recessive character. Of the yellow seeds, one in three produced only yellow offspring, i. e. contained only the dominant character; but two out of three proved to be hybrid, producing both green and yellow seed, as did the hybrids of the preceding generation. These are precisely the theoretical proportions, A + 2 A(B) + B.

The experiment has been repeated and confirmed by several different observers.

In mice, my friend and pupil, Mr. G. M. Allen, finds the second hybrid generation, obtained by crossing gray with white mice, to consist of gray mice and white mice approximately in the ratio, 3 : 1. (See Table III.) The white are pure recessives, producing only white offspring, when bred *inter se.* What portion of the grays are pure dominants has not yet been determined, but we may confidently expect that it will prove to be not far from 1 in 3.

TABLE III.

Heredity of Coat-color among Cross-bred Mice obtained by Mating White Mice (W) with Gray Mice (G).

Parents crossed	Offspring.		
	Gen. I.	Gen. II.	Gen. III.
W G	G(W)	1W	. . W
		3 { 2(?)G(W)	
		3 { 1(?)G . . .	. . G

A further test of the correctness of Mendel's hypothesis of the purity of the germ-cells and of their production in equal numbers, is afforded by back-crossing of a hybrid with one of the parental forms. For example, take a case of simple dominance, as of cotyledon-color in peas or coat-color in mice. We have here characters D (dominant) and R (recessive). The first generation hybrids will all be D(R). Any one of them back-crossed with the recessive parent will produce fifty per cent of pure recessive offspring and fifty per cent of hybrids.

For the hybrid produces germ-cells . . . . . D + R
The recessive parent produces germ-cells . . R + R
The possible combinations are . . . . . . 2D(R) + 2R

This case has been tested for peas and for mice and found to be substantially as stated.

We have thus far considered only cases of cross-breeding between parents differing in a single character. We have seen that in such cases, no new forms, except the unstable hybrid form, are produced. But when the parent forms crossed differ in two or more characters, there will be produced in the second and later hybrid generations individuals possessing *new combinations* of the characters found in the parents; indeed *all possible combinations* of those characters will be formed, and in the proportions demanded by chance. Thus when parents are crossed which differ in *two* characters, A and B, let us designate the dominant phase of these characters by A, B, the recessive phase by a, b. The immediate offspring resulting from the cross will all be alike, AB(ab),* but the second and later generations of hybrids will contain the stable (i. e. pure) classes, AB, Ab, aB, ab, in addition to other (unstable, or still hybrid) forms, namely AB(ab), A(a)b, and aB(b). In every sixteen second-generation offspring there will be, on the average, one of each of the stable combinations. Two of these combinations will be identical with the parent forms, the other two will be new.

But the difficulty of establishing a stable (i. e. pure) race is greater in this case than in that of one variable character. Only the individual which possesses *both recessive characters* can at once be set aside as pure. For to each of the stable individuals possessing one dominant and one recessive character, there will be *two other individuals, exactly like it in appearance*, but hybrid in one of the two characters. The one pure individual can be distinguished from the two impure individuals only by breeding tests.

Again, nine out of every sixteen of the second-generation hybrids will possess the *two dominant characters*, and so will be in appearance exactly like their parents, the first-generation hybrids. But *only one* of the nine will be *pure* with reference to those characters. Of the remaining eight, four will be hybrid in one character, and four will be hybrid in *both characters* exactly like the entire first generation of offspring.

The greater the number of separately variable characters involved in a cross, the greater will be the number of new combinations obtainable; the greater, too, will be the number of individuals which it will be necessary to raise in order to secure *all* the possible combinations; and the greater, again, will be the difficulty of isolating the pure (i. e.

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* This is Mendel's use of lower-case letters to designate recessive characters, with which I have combined the use of a parenthesis when a character by nature recessive is not visible in the individual.

stable) forms from such as are similar to them in appearance but still hybrid in one or more characters. Mendel has generalized these statements substantially as follows: In cases of complete dominance, when the number of differences between the parents is n, the number of different classes into which the second generation of offspring fall will be 3^n, of which 2^n will be *pure* (stable); the remainder will be hybrid, though indistinguishable from pure individuals. The smallest number of individuals which in the second hybrid generation will allow of *one pure individual* to each visibly different class, will be 4^n. See Table IV.

TABLE IV.

Number of Differences between Parents.	Visibly Different Classes, each containg one Pure Individual.	Total Classes, Pure and Hybrid.	Smallest Number of Offspring allowing one Individual to each Class.	
n	2^n	3^n	4^n	
1	2	3	4	Tested by Mendel for peas and found correct.
2	4	9	16	
3	8	27	64	
4	16	81	256	Calculated.
5	32	243	1024	
6	64	729	4096	

The law of Mendel reduces to an exact science the art of breeding in the case most carefully studied by him, that of entire dominance. It gives to the breeder a new conception of "purity." No animal or plant is "pure" simply because it is descended from a long line of ancestors possessing a desired combination of characters; but *any* animal or plant is pure if it produces gametes of only one sort, even though its grandparents may among themselves have possessed *opposite* characters. The existence of purity can be established with certainty only by suitable breeding tests (especially by crossing with recessives), but it may be safely assumed for any animal or plant descended from parents which were like each other and had been shown by breeding tests to be pure.

Special cases under the law of Mendel. It remains to speak of some special cases under the law of Mendel, which apparently are exceptions to one or another of the principles already stated, and which probably result from exceptional conditions known to us only in part. These special cases have come to light in part through Mendel's own work, in part through that of others.

1. *Mosaic inheritance.* It occasionally happens that in crosses which bring together a pair of characters commonly related as dominant and recessive, the two characters appear in the offspring in patches side by side, as in piebald animals and parti-colored flowers and fruits. The normal dominance apparently gives place in such cases to a balanced relationship between the alternative characters. What conditions give rise to such relationships is unknown, but when they are once secured they often prove to possess great stability, breeding true *inter se.* This, for example, is the case in spotted mice, which usually produce a large majority of spotted offspring. The balanced relationship of characters possessed by the parents is transmitted to the germ-cells, which are, not as in ordinary hybrid individuals D or R, but $\frac{1}{2}$ DR. This has been shown to be the case in spotted mice by G. M. Allen and myself, in a paper published elsewhere. The balanced condition of D and R, which exists in the gamete, is upset when that gamete unites with a pure R (and probably also when it unites with a pure D); for spotted mice bred to white mice regularly gave only uniformly gray or black individuals, after the formula $\frac{1}{2}\,DR + R = D(R)$.* But an exceptional spotted male, own brother to those which gave the described result, apparently produced gametes D and R as well as others $\frac{1}{2}$ DR, for by white females he had pure white offspring as well as those which were gray or black in color. This result can be expressed by the formulæ: —

$$\begin{array}{rcccll} \text{Sperm.} & & \text{Ovum.} & & \text{Offspring.} & \\ \frac{1}{2}\,DR & + & R & = & D(R)^* & \big\rbrace\ \text{ordinary gray or} \\ D & + & R & = & D(R) & \big\rbrace\ \quad\text{black hybrids.} \\ R & + & R & = & R, & \quad\text{white (pure).} \end{array}$$

2. *Stable hybrid forms.* This is a case, in some respects similar to the last, which was familiar to Mendel (:70) himself. It sometimes happens, as we have seen, that the hybrid has a form of its own different from that of either parent. To such cases the law of dominance evidently does not apply. In a few cases — Hieracium hybrids (Mendel), Salix hybrids (Wichura) — it has been found that the hybrid form does not break up in the second generation and produce individuals like the grandparents, but breeds true to its own hybrid character. This can be explained only

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* Observations made since the foregoing was written indicate that the offspring in this case are, sometimes at least, $\frac{1}{2}D(R)\cdot(R)$. For when an individual of this sort forms gametes, they apparently are, not pure D and R, but $\frac{1}{2}$ DR and R respectively. This hypothesis accounts for the *reappearance* of spotted mice after their disappearance for a generation in consequence of crossing.

on one of two assumptions. Either the germ-cells bear the two characters in the balanced relationship, ½ AB, as do spotted mice ordinarily, or, of the two gametes which unite in fertilization, one invariably bears the character A, the other the character B.

3. *Coupled characters.* This is the phenomenon of correlation of characters in heredity. It is sometimes found that, in cross-breeding, two characters cannot be separated. When one is inherited, the other is inherited also. Thus, in crossing different sorts of Datura (the Jamestown weed) it has been found that purple color of stem invariably goes with blue color of flowers, whereas green stems are constantly associated with white flowers. Again in mice, rabbits, and most other mammals, white hair and pink eyes occur together and may not be separated in heredity. Very rarely, however, as I have observed, an otherwise perfectly white guinea-pig has dark eyes; further the ordinary albino guinea-pig with pink eyes has usually smutty (brown-pigmented) ears, nose, and feet. These exceptional conditions probably represent stable couplings of a *part only* of the dominant character (pigmented coat) with the recessive character (white coat), and are similar in kind to the ½ DR character of the spotted mice. For guinea-pigs do occur entirely devoid of the D character, i. e. without dark nose, ears, and feet, and with pink eyes. These doubtless represent the *pure* recessive condition.

Further, coupling may occur between a number of characters greater than two, so that they form, to all intents and purposes, in heredity, one indissoluble compound character. Thus, Correns ('00) observed that in crosses between two species of stocks (Mathiola incana DC. and M. glabra DC.) the second generation hybrids showed reversion to one or the other of the parental forms in *all three of the principal differential characters* studied, viz., hairy or glabrous stems, violet or yellow-white flowers, and blue or yellow seed. A blue seed always produced a hoary plant bearing violet flowers; a yellow seed always produced a glabrous plant bearing yellow or white flowers.

4. *Disintegration of characters.* This is the converse of the foregoing process. Not only may characters apparently simple be coupled together in heredity to form composite units of a higher order, but characters which ordinarily behave as units may as a result of crossing undergo disintegration into elements separately transmissible. Thus the gray coat-color of the house-mouse is always transmitted as a dominant unit in primary crosses with its white variety; but in the second cross-bred generation a certain number of *black* mice appear, some or all of which are probably hybrids. For similar black mice obtained by crossing black-

white with white mice have been shown, by breeding tests, to be hybrids, since on crossing with white mice they produce white mice, black mice, and, in one or two cases, gray mice also. Accordingly black mice clearly belong with grays in the category of dominant individuals [D or D (R)], but they have visibly *only the black constituent* of the gray coat, the remaining constituent, a rufous tint, having been separated from the black in consequence of cross-breeding. There is reason to believe that the rufous constituent may become recessive (i. e. latent) either in the black individuals or in the reverted whites, or in both. It is seen separated from both the black and the white characters, in the chocolate-brown and reddish-yellow individuals obtained in cross-breeding.

A fancier of rabbits tells me that there occurs a similar disintegration of the composite coat-color of the "Belgian hare," when that animal is crossed with ordinary white rabbits, the result being the production of black, yellow, and mottled individuals, in addition to ordinary gray-browns.

The various distinct colors or color patches of the guinea-pig have doubtless originated in a similar way, — by resolution of the composite coat-color of the wild Cavia, upon crossing with an albino sport. This subject is now undergoing investigation.

Correns (:00) mentions a case in plants, which probably belongs in this same category. In crossing the blue-flowered (dominant) Mathiola incana with the yellowish-white flowered (recessive) M. glabra, the second generation recessives produced in some cases pure white flowers, in others yellow flowers. In this case the recessive character, rather than the dominant, underwent disintegration.

5. *Departures from the theoretical ratios of dominants to recessives.* Considerable departures are to be expected when the number of offspring taken into consideration is small, but with increase in the number of offspring examined, the departures should grow less. This is usually found to be true. Mendel's numbers are shown by Weldon (:02) to be well within the limits of probable error. But certain cases have been observed in which departures of a particular sort persist even with considerable numbers of offspring. Thus Allen and I have found the recessive character, white, in mice to be inherited in about three per cent more than the calculated number of cases, while the equally recessive dancing character is inherited in about thirty-three per cent less than the calculated number of cases. These fairly uniform departures indicate, to my mind, a vitality, on the part of the recessive gamete, in one case somewhat superior, in the other much inferior to that of the dominant

gamete. Inferior vitality of gametes of either sort would result in greater mortality and so in a diminished number of individuals derived from such gametes.

Of course other explanations are possible, as, that the two sorts of gametes are *not* produced in equal numbers. More extended investigations of such cases can alone make their meaning clear.

6. *Reversal of Dominance.* Exceptional cases are on record in which crossing of a dominant with a recessive has resulted in the production of *pure* dominants, or recessives, instead of hybrids. Such cases are, I believe, correctly referred by Bateson to the category of "false hybridization" as described by Millardet, a phenomenon akin to parthenogenesis, in which sexual union has served merely to *stimulate one gamete to development* without bringing about its union with the other gamete.

It is possible, however, that there are cases in which one of a pair of characters is sometimes dominant, sometimes recessive. Tschermak (:**01**) believes that he has found a few such cases among cross-bred beans. Sex and certain other dimorphic conditions found in the higher animals and plants may prove to be cases of this sort.

Acceptance of Mendel's principles of heredity as correct must lead one to regard discontinuous (or sport) variation as of the highest importance in bringing about polymorphism of species and ultimately of the formation of new species.

A sport having once arisen affecting some one character of a species, may by crossing with the parent form be the cause of no end of disintegration on the part of any or all of the characters of the species, and the disintegrated characters may, indeed *must*, form a great variety of new combinations of characters, some of which will prove stable and self-perpetuating. Even if a particular combination of characters is uniformly eliminated by natural selection under one set of conditions, it may reappear again and again, and finally meet with conditions which insure its success.

We now have an explanation of the long-recognized principle that new types of organisms are extremely variable, whereas old types vary little. A new type which has arisen as a sport will cross with the parent form. The offspring will then inherit some characters dominant, others latent, and polymorphism of the race results. Only selection continued through long periods of time will serve to eliminate completely the latent recessives, and so to cause the disappearance of certain aberrant variations.

Bateson makes the pregnant suggestion that even cases of continuous variation may possibly prove conformable with Mendelian principles. Take, for example, the height of peas. It has been found in certain crosses of a tall with a dwarf variety of pea, that the hybrid has an intermediate height. Now, if the hybrid produces pure germ-cells, dwarf and tall respectively, in equal numbers, the next generation will consist of three classes of individuals, dwarf, intermediate, and tall, in the proportions, 1 : 2 : 1. But if each of the original characters should undergo disintegration, we might get a dozen classes, instead of three, resulting in a practically continuous frequency-of-error curve.

SUMMARY.

1. The basic principle in Mendel's discoveries is that of the purity of the germ-cells; in accordance with this a cross-bred animal or plant produces germ-cells bearing *only one* of each pair of characters in which its parents differ. From it follow the occurrence in the second and later hybrid generations of a definite number of forms in definite numerical proportions.

2. Mendel's principle of dominance is realized in the heredity of a considerable number of characters among both animals and plants. In accordance with this principle, hybrid offspring have visibly the character of only one parent or the other, though they transmit those of both parents.

3. In other cases the hybrid has a distinctive character of its own. This may approximate more or less closely the character of one parent or the other, or it may be entirely different from both. Frequently the distinctive hybrid character resembles a lost ancestral character. In some cases of this sort, as in coat-color of mammals, the hybrid character probably results from a recombination of the characters seen in one or both parents, with certain other characters latent (that is, recessive) in one parent or the other.

4. There have been observed the following exceptions to the principle of dominance, or to the principle of purity of the germ-cells, or to both: —

(*a*) Mosaic inheritance, in which a pair of characters ordinarily related as dominant and recessive occur in a balanced relationship, side by side in the hybrid individual and, frequently, but not always, in its germ-cells also. This balanced condition, once obtained, is usually stable under close breeding, but is readily disturbed by cross-breeding, giving place then to the normal dominance.

(*b*) Stable (self-perpetuating) hybrid forms result from certain crosses. These constitute an exception to both the law of dominance and to that of purity of the germ-cells. For the hybrid is like neither parent, but the characters of *both* parents exist in a stable union in the mature germ-cells produced by the hybrid.

(*c*) Coupling (i. e., complete correlation) may exist between two or more characters, so that they form a compound unit not separable in heredity, at least in certain crosses.

(*d*) Disintegration of characters apparently simple may take place in consequence of cross-breeding.

(*e*) Departures from the expected ratios of dominants to recessives may be explained in some cases as due to inferior vigor, and so greater mortality, on the part of dominants or recessives respectively.

(*f*) Cases of apparent reversal of dominance may arise from "false hybridization" (induced parthenogenesis). Possibly in other cases the determination of dominance rests with circumstances as yet unknown.

5. Mendel's principles strengthen the view that species arise by discontinuous variation. They explain why new types are especially variable, how one variation causes others, and why certain variations are so peristent in their occurrence.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

ON THE TEMPERATURE COEFFICIENTS OF MAGNETS MADE OF CHILLED CAST IRON.

BY B. O. PEIRCE.

INVESTIGATIONS ON LIGHT AND HEAT, MADE AND PUBLISHED WHOLLY OR IN PART WITH APPROPRIATIONS FROM THE RUMFORD FUND.

ON THE TEMPERATURE COEFFICIENTS OF MAGNETS MADE OF CHILLED CAST IRON.

By B. O. Peirce.

Presented January 14, 1903. Received January 14, 1903.

Besides a number of d'Arsonval galvanometers, furnished with hardened forged steel magnets, from the shops of well known makers in America and in Europe, there are in the Physical Laboratories of Harvard University about thirty similar instruments in which the permanent fields are due to chilled and seasoned cast iron magnets. These latter have proved very satisfactory, and, after a trial of three years, we are about to add to their number.

Although chilled cast iron magnets were used years ago in some forms of telephones, straight magnets are most conveniently made of steel; indeed if they are to be employed in measuring the intensity of the earth's field, the best tool steel, ground slowly into shape under water after the hardening, is not too homogeneous for the purpose. Steel for permanent magnets, however, needs special skill in the handling, if the results are to be satisfactory, and not every successful tool maker knows how to forge and to harden, well and quickly, even a horseshoe magnet, unless it be of very simple form. Two straight, hollow bar magnets were made and ground most carefully for use in the Jefferson Physical Laboratory, by a firm which manufactures machine tools of the highest grade. These were supposed to be as nearly alike as possible, but they proved to be magnetically very different, for the permanent moment of one was twice that of the other.

Of late I have been using magnets made of soft iron castings, subsequently chilled, to furnish the artificial field in an oil damped amperemeter, and in a similar voltmeter firmly set up in the laboratory, and, since it was desirable that the indications of these instruments should be trustworthy within one part in a thousand of their larger deflections, it became necessary to test the permanency of the magnets, and to determine their temperature coefficients. This paper gives the results of measurements made on a number of magnets of this kind.

Most of the magnets experimented upon were made of castings chilled by Mr. G. W. Thompson, the mechanician of the Jefferson Physical Laboratory, who has had a great deal of experience with the process. They were first heated to a bright red in a gas furnace under a power blast, and then plunged into a cold acid bath kept in violent agitation. The castings thus hardened were relaxed by long exposure to boiling water or steam, then magnetized to saturation, and finally seasoned, after prolonged boiling, by being alternately heated in steam and cooled in tap water. The whole seasoning process reduced the magnetic moment of each specimen by perhaps twenty per cent of the value it had just after the magnetization. If, after a magnet has been seasoned, its temperature be suddenly raised from 0° C. to 100° C. and then as suddenly lowered again, it may not wholly recover its original strength until after the lapse of an hour or two: if, however, the range be only 40° or 50° C., I have been unable to detect any lag in the attainment of the whole of the original moment after the heating.

Although there is no advantage in using cast iron for straight magnets, I had a number made for comparison with fine steel magnets of the same dimensions. The cast iron magnets looked rough in comparison with the others, but the moments of a large number of them seemed to differ less among themselves than the moments of the same number of the steel magnets. The strongest steel magnet that I tested had a moment about four per cent greater than that of the strongest cast iron magnet, but the average moment of the cast iron magnets was practically the same as (in fact two per cent greater than) the average of the seasoned steel magnets.

In determining the temperature coefficients, the straight magnet to be experimented upon was fixed firmly in a non-magnetic holder inside a non-magnetic tube so as to be in Gauss's A Position east of a mirror magnetometer. By the help of a system of pipes and cocks, tap water, steam, or a stream from a bath water heater at almost any desired temperature, between 15° C. and 100° C. could be sent through the tube containing the magnet. On the west of the magnetometer, so placed in Gauss's A Position as to bring the needle back exactly into the meridian, was a short, seasoned, compensating magnet, fixed wholly within a wooden holder and completely shielded from sudden temperature changes. If a_0 is the needle deflection which the compensating magnet would cause if the magnet to be tested were removed, M_0, the moment of the last named magnet at the temperature t_0 at which the adjustments have been made, and M, the moment of this magnet when, its temperature having become raised to t, the needle is deflected through the angle a,

$$\frac{M_0 - M}{M_0} = \frac{\tan a}{\tan a_0}.$$

Since the temperature coefficients of seasoned bar magnets of a given length and of given material are in general larger the greater the cross section of the bar, it is necessary in comparing materials to take magnets of nearly the same dimensions. Besides a number of chilled cast iron magnets 18 centimeters long and about 0.95 centimeters in diameter, I had many carefully made steel magnets of the same area of cross section and of almost the same length. In the case of all these, the rate of loss of moment per degree of rise of temperature was greater at higher temperatures than at low; we may, however, for the purpose of comparison, use the mean loss, per degree, of the magnetic moment, when the magnet is heated from about 10° C. to 100° C., expressed in terms of the moment at the lower temperature. These mean losses were found to be

0.00042 in the case of the seasoned chilled iron magnets.

0.00046 in the case of the seasoned magnets made of "Crescent Steel Drill Rod."

0.00046 in the case of the seasoned magnets made of Jessop's Round Black Tool Steel.

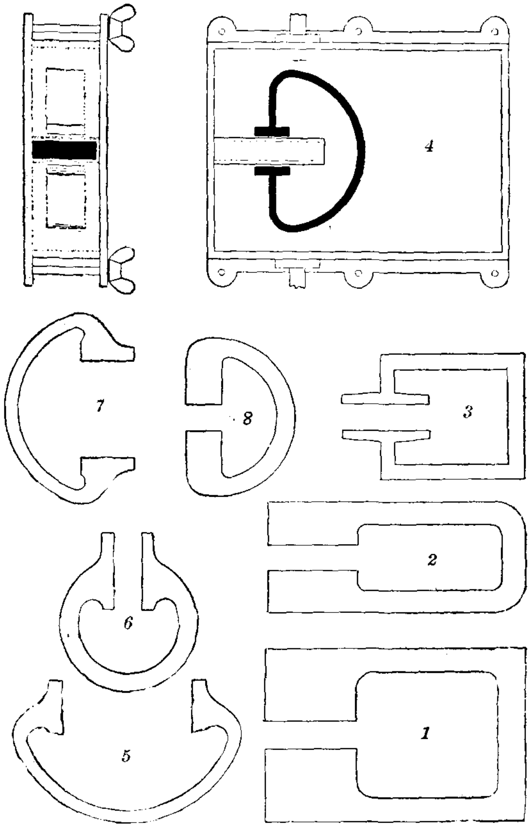
0.00070 in the case of the seasoned magnets made of Jessop's Square Tool Steel.

I had bar magnets made of many other materials, for instance of Jessop's and Mushet's self-hardening steels, but none of exactly the dimensions of the cast iron magnets. No kind of steel that I tested had, however, when proper allowance was made for dimensions, quite so small a temperature coefficient as the chilled iron.

The mean temperature coefficient of chilled cast iron magnets 18 centimeters long and 1.25 centimeters in diameter, as obtained from a number of specimens, was 0.00056, which is very low.

The forms of some of the magnets which we have used (either singly or with others of the same kind), in various instruments, are shown in the subjoined figure. The shapes marked 1, 2, 3, 6, are employed, with the long way of the opening between the poles vertical, in d'Arsonval galvanometers; two or three castings of the shape marked 4, and a number of thin plates of the shape marked 8, are used together in other instruments of the same kind. Magnets of the shapes marked 5 and 7 produce the artificial fields in laboratory amperemeters and voltmeters.

For our present purpose we may define the temperature coefficients of one of these magnets as the rate of change of the whole magnetic induction across a given surface between the poles, when the temperature of the magnet is raised by one degree. This can be measured with sufficient accuracy, by pulling out, from a definite position between the jaws, a coil of suitable shape made of manganine wire and connected with a ballistic galvanometer. In order to be able to make the determinations conveniently I had a brass box made of the shape indicated in the figure. The box itself was first cast in one piece, and then a slot for the coil was



cut on a milling machine, and a rectangular cavity, open to the outside air but closed to the inside of the box, was constructed by soldering two thin pieces of brass into the end and top of the slot. Into this cavity a set of forms carrying thin coils of the shapes needed, fitted exactly. The box itself, and the cover, were mounted on the face plate of a lathe and turned off smooth, so that when a piece of rubber packing was inserted between the two, and the whole was screwed together, the case thus made was water-tight. The box was mounted on a wooden frame which had sliders for the forms which carried the coils. The magnet to be tested was fastened firmly in place by a holder not shown in the figure, and the box was connected with a set of pipes so that cold water, warm water, or steam could be sent through it at pleasure.

The temperature coefficient of a bent cast iron magnet, as defined above, generally increases with the temperature, but for purposes of comparison, we may use the mean value K of this coefficient between 10° C. and 100° C.

Three magnets of the form marked 1, chilled by Mr. Thompson and weighing as much as 1250 grams each (nearly three pounds), gave for K the values 0.00036, 0.00037, and 0.00034 respectively; another magnet of the same pattern treated by a maker of hardened cast iron machinery, yielded the value 0.00082. Whatever the secret process employed in this last case, the resulting magnet was by no means so useful as those made from castings chilled in the manner described above.

Unchilled castings make very undesirable magnets, for the temperature coefficients are usually five or six times as large as in the case of chilled magnets, and it seems impossible to get their magnetic moments really permanent. Curiously enough the chilling process makes a casting less brittle than before, and causes the grain of a fracture to be finer and more uniform.

The values of K seem to indicate that the whole interior of the casting is affected by the chilling, whereas it is extremely difficult to harden a thick piece of steel uniformly. It did not appear that a magnet made up of a lot of thin plates chilled separately had a smaller temperature coefficient than a solid magnet of the same dimensions.

Castings of the shapes marked 3, 4, and 6 weighed about 260 grams, 160 grams, and 500 grams, respectively, and yielded for K the values 0.00040, 0.00040, 0.00031. The actual temperature coefficients at low temperatures are always less than these mean values, and in the case of the last mentioned form the coefficient is not greater than 0.00013 between 10° C. and 40° C. I have myself never found a value quite so small as this for a massive steel magnet, though several observers have obtained extremely low coefficients for very slender steel wires, and even negative coefficients for comparatively weak magnets made of some alloys.

Using such chilled magnets as I have described, and employing composite galvanometer coils of manganine and copper, with permanent manganine shunts, it is not difficult to make a cheap fixed amperemeter, the indications of which shall be almost wholly independent of the room temperature. In the case of a d'Arsonval galvanometer of the usual form, slight temperature changes in the torsional rigidity of the suspension wire have to be taken into account.

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December, 1902.

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CONTRIBUTIONS FROM THE WILDER PHYSICAL LABORATORY
OF DARTMOUTH COLLEGE.

THE PRESSURE DUE TO RADIATION.

By E. F. Nichols and G. F. Hull.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

THE PRESSURE DUE TO RADIATION.

BY E. F. NICHOLS AND G. F. HULL.

Presented December 10, 1902. Received February 5, 1903.

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As early as 1619 Kepler [*] announced his belief that the solar repulsion of the finely divided matter of comets' tails was due to the outward pressure of light. On the corpuscular theory of light Newton [†] considered Kepler's idea as plausible enough, but he was of the opinion that the phenomenon was analogous to the rising of smoke in our own atmosphere. In the first half of the eighteenth century DeMairan and DuFay [‡] contrived elaborate experiments to test this pressure of light theory in the laboratory, but, because of the disturbing action of the gases surrounding the illuminated bodies employed in the measurements, they obtained wholly confusing and contradictory results. Later in the same century the Rev. A. Bennet* performed further experiments, but could find no repulsive force not traceable to convection currents in the gas surrounding the body upon which the light was projected, due in his opinion to the heating effect of the rays. Finding no pressure due to radiation, he made the following unique suggestion in support of the wave theory of light: "Perhaps sensible heat and light may not be caused by the influx or rectilinear projection of fine particles, but by the vibrations made in the universally diffused caloric or matter of heat or fluid of light. I think modern discoveries, especially those of electricity, favor the latter hypothesis." In the meantime Euler,† accepting Kepler's theory attributing the phenomenon of comets' tails to light pressure, had hastened to the support of the wave theory by showing theoretically that a longitudinal wave motion might produce a pressure in the direction of its propagation upon a body which checked its progress. In 1825 Fresnel‡ made a series of experiments, but arrived at no more definite conclusion than that the repulsive and attractive forces observed were not of magnetic nor electric origin.

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[*] DeMairan, Traité physique et historique de l'Aurore Boréale (Seconde Edition), pp. 357–358. Paris, 1754.

[†] Isaaci Newtoni Opera quae Existant Omnia. Samuel Horsley, LL.D., R. S. S. Tom. III., pag. 156, Londinium, 1782.

[‡] DeMarian, l. c., p. 371. This treatise contains also the accounts of still earlier experiments by Hartsoeker, p. 368, and Homberg, p. 369. The later experiments are of more historic than intrinsic interest.

Crookes§ believed in 1873 that he had found the true radiation pressure in his newly invented radiometer and cautiously suggested that his experiments might have some bearing on the prevailing theory of the nature of light. Crookes' later experiments and Zöllner's‖ measurements of radiometric repulsions showed that the radiometric forces were in some cases 100,000 times greater than the light pressure forces with which they had been temporarily confused. Zöllner's experiments are among the most ingenious ever tried in this field of work, and he missed the discovery of the true radiation pressure by only the narrowest margin. An excellent bibliography of the whole radiometric literature is given by Graetz,¶ and an account of some of the older experiments not mentioned above is given by Crookes.**

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* A. Bennet, Phil. Trans., p. 81 (1792).

† L. Euler, Histoire de l'Academie Royale de Berlin (2), p. 121 (1746).

‡ A. Fresnel, Ann. Chem. et Phys., XXIX. 57, 107 (1825).

§ W. Crookes, Phil. Trans., p. 501 (1873).

‖ F. Zöllner, Pogg. Ann., CLX. 156, 296, 459 (1877).

¶ L. Graetz, Winckelmann's Handbuch der Physik, 2 b, p. 262. Breslau, 1896.

** W. Crookes, l. c., p. 501.

In 1873 Maxwell,[*] on the basis of the electromagnetic theory, showed, that if light were an electromagnetic phenomenon, pressure should result from the absorption or reflection of a beam of light. After a discussion of the equations involved, he says: "Hence in a medium in which waves are propagated there is a pressure in the direction normal to the waves and numerically equal to the energy in unit volume." Maxwell computed the pressure exerted by the sun on the illuminated surface of the earth and added: "It is probable that a much greater energy of radiation might be obtained by means of the concentrated rays from an electric lamp. Such rays falling on a thin metallic disc, delicately suspended in a vacuum, might perhaps produce an observable mechanical effect."

Apparently independent of Maxwell, Bartoli[†] announced in 1876 that the Second Law of Thermodynamics required the existence of a pressure due to radiation numerically equal in amount to that derived by Maxwell. Bartoli's reasoning holds for all forms of energy streams in space and is of more general application than Maxwell's equations. Bartoli contrived elaborate experiments to verify this theory, but was balked in the search, as all before him had been, by the complicated character of the gas action which he found no way of eliminating from his experiments.

After Bartoli's work the subject was dealt with theoretically by Boltzmann,[‡] Galitzine,[§] Guillaume,[‖] Heaviside,[¶] and more recently by Goldhammer.[**] Fitzgerald,[††] Lebedew,[‡‡] and Hull[§§] have discussed the bearing of radiation pressure upon the Newtonian law of gravitation with special reference to the repulsion of comets' tails by the sun. Arrhenius[‖‖] has recently discussed the cosmical consequences of radiation pressure not only concerning comets' tails, but, by combining radiation pressure with the known properties of negative ions, has endeavored also to account for the aurora borealis. Swartzschild* computed from radiation pressure on small spherical conductors the size of bodies of unit density for which the ratio of radiation pressure to gravitational attraction would be a maximum.

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[*] J. C. Maxwell, A Treatise on Electricity and Magnetism (1st Edition), II. 391. Oxford, 1873.

[†] A. Bartoli, Sopra i movementi prodotti della luce e dal calorie, Florence, Le Monnier (1876), also Nuovo Cimento, XV. 193 (1884).

[‡] L. Boltzmann, Wied. Ann., XXII. 31, 291 (1884).

[§] B. Galitzine, Wied. Ann., XLVII. 479 (1892).

[‖] Ch. Ed. Guillaume, Arch. de Gen. (3), XXXI. 121 (1894).

[¶] O. Heaviside, Electromagnetic Theory, I. 334. London, 1893.

[**] D. A. Goldhammer, Ann. Phys., IV. 834 (1901).

[††] G. F. Fitzgerald, Proc. Roy. Soc. Dub. (1884).

[‡‡] P. Lebedew, Wied. Ann., XLV. 292 (1892). Astrophys. Jour., XIV. 155 (1902).

[§§] G. F. Hull, Trans. Astron. Soc. Toronto, p. 123 (1901).

[‖‖] S. Arrhenius, Konigl, Vetanskaps. Akademiens Fördhandlingar, p. 545 (1900).

Before the Congrès International de Physique in 1900, Professor Lebedew† of the University of Moscow described an arrangement of apparatus which he was using at that time for the measurement of light pressure. He summarizes the results already obtained as follows: "Les résultats des mesures que j'ai faites jusqu'ici peuvent se résumer ainsi: L'expérience montre qu'un faisceau lumineux incident exerce sur les surfaces planes absorbantes et réfléchissantes des pressions qui, aux erreurs près d'observation, sont égales aux valeurs calculées par Maxwell et Bartoli." No estimate of the "errors of observation" was given in the paper nor other numerical data. Unfortunately the proceedings of the Paris Congress did not reach the writers nor any intimation of the methods or results of Professor Lebedew's work until after the publication of their own preliminary experiments.

The writers‡ presented the results they had obtained by measurements of radiation pressure at eight different gas pressures, in a preliminary communication to the American Physical Society, meeting with Section B of the American Association at Denver, August 29, 1901. A condensed abstract of this paper follows.

In the experiments of earlier investigators every approach to the experimental solution of the problem of radiation pressure had been balked by the disturbing action of gases which it is impossible to remove entirely from the space surrounding the body upon which the radiation falls. The forces of attraction or repulsion, due to the action of gas molecules, are functions, first, of the temperature difference between the body and its surroundings, caused by the absorption by the body of a portion of the rays which fall upon it; and second, of the pressure of the gas surrounding the illuminated body. In the particular form of apparatus used in the present study the latter function appears very complicated, and certain peculiarities of the gas action remain inexplicable upon the basis of any simple group of assumptions which the writers have so far been able to make.

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* K. Swartschild, Kgl. Bayer. Akademie d' Wissenschaften, XXXI. 293 (1901).

† P. Lebedew, Rapports présentés au Congrès International de Physique (2), p. 133. Paris, 1900.

‡ E. F. Nichols and G. F. Hull, Science, XIV. 588 (Oct. 18, 1901), and Phys. Rev., XIII. 293 (Nov., 1901).

Since we can neither do away entirely with the gas nor calculate its effect under varying conditions, the only hopeful approach which remains is to devise apparatus and methods of observation which will reduce the errors due to gas action to a minimum. The following considerations led to a method by which the elimination of the gas action was practically accomplished in the present experiments.

1. The surfaces which receive the radiation, the pressure of which is to be measured, should be as perfect reflectors as possible. This will reduce the gas action by making the rise of temperature due to absorption small, while the radiation pressure will be increased; the theory requiring that a beam, totally reflected, exert twice the pressure of an equal beam, completely absorbed.

2. By studying the action of a beam of constant intensity upon the same surface surrounded by air at different pressures, certain pressures may be found where the gas action is less than at others.

3. The apparatus — some sort of torsion balance — should carry two surfaces symmetrically placed with reference to the rotation axis, and the surfaces on the two arms should be as nearly equal as possible in every respect. The surfaces or vanes should be so constructed that if the forces due to gas action (whether suction or pressure on the warmer surface) and radiation pressure have the same sign in one case, a reversal of the suspension should reverse the gas action and bring the two forces into opposition. In this way a mean of the forces on the two faces of the suspension should be, in part at least, free from gas action.

4. Radiation pressure, from its nature, must reach its maximum value instantly, while observation has shown that gas action begins at zero and increases with length of exposure, rising rapidly at first, then more slowly to its maximum effect, which, in many of the cases observed, was not reached until the exposure had lasted from two and a half to three minutes. For large gas pressures, an even longer exposure was necessary to reach stationary conditions. The gas action may be thus still further reduced by a ballistic or semi-ballistic method of measurement.

The results of ballistic observations of radiation pressure at different gas pressures are given below in Table I, in which p indicates the pressure of the surrounding gas in millimeters of mercury, and d the static equivalent of the ballistic throws of the torsion balance. The results were obtained with substantially the same apparatus and method described on page 568 et seq.: —

TABLE I.

p	d	p	d
96.3	19.7	33.4	21.1
67.7	21.0	1.2	20.9
37.9	21.6	0.13	26.8
36.5	22.1	0.06	23.2
Mean 22.5			

Taking the product of the average deflection in centimeters by the constant of the torsion balance, the radiation pressure was: —

$$2.25 \times 4.65 \times 10^{-5} = 1.05 \times 10^{-4} \text{ dynes.}$$

THE BOLOMETER.

To compare the theoretical value of radiation pressure with the above value, it was necessary to measure the energy of the radiation causing the pressure. This was attempted with the aid of a bolometer constructed as follows: —

On a sheet of platinum 0.001 mm. thick, rolled in silver (by the firm Sy & Wagner, Berlin), a circle *P* (Fig. 1), 11.25 mm. in diameter, was drawn. The sheet was cut from the edges inward to the circumference of the circle, in such a way as to leave five principal strips *A*, *B*, *C*, *D*, *E*, connected to the circle in the manner shown. Other narrower strips, as *e*, *m*, *n*, *o*, etc., were left to give the disc additional support. The disc, by means of the connecting arms, was mounted with asphalt varnish centrally over a hole, 14 mm. in diameter, bored through a slab *S* of thin slate. Portions of the silver not to be removed by the acid were carefully covered by asphalt varnish. Thus on the strips *A* and *B*, the silver was protected to the very edge of the circle, while on all the other arms, the silver was left exposed back to the edge of the boring in the slate. The whole system was then plunged into warm nitric acid, and the silver eaten away from all unvarnished surfaces, leaving only the thin platinum sheet which was blackened by electric deposition of platinum by Kurlbaum's* method. At *A*, *B*, *C*, *D*, *E*, holes were bored extending through the slate. Copper washers were soldered to the silver strips and binding posts were attached.

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* Kurlbaum, Wied. Ann., LXVII. 848 (1899).

The torsion balance was removed from under the bell-jar, the bolometer was put in the place of one of the vanes and was covered by the bell-jar. Connections to the bolometer were made as schematically shown in Fig. 1. The disc P was the exact size of the light image thrown on the vanes in the pressure measurements. The intention was to heat the disc by allowing the image to fall on it, and then, with the light turned off, to heat it to the *same temperature* by sending a current through it from A to B. If r be the resistance from A to B in ohms, when exposed to the lamp, and i be the current in ampères which gives the same temperature in P as that given by the absorbed radiation, then $i^2r \times 10^7$ will be the activity of the beam in erg-seconds. The temperature of the disc, whether exposed to the radiation or heated by the current, was shown by the resistance, C to D-E, which was made one arm of a Wheatstone Bridge. The relation of the heating current to the bridge was adjusted as follows: With the key K open, so that no current flowed through the bridge, the heating current from six storage cells B_2 was turned on, and the sliding contact at F so set that the bridge galvanometer zero was not changed by reversing the heating current. The point F, equipotential to c, was found very near the middle of the wire $a\,b$, which showed the current distribution of P to be symmetrical with respect to a diameter at right angles to $A\,B$. The key K was then closed making the bridge current, and the bridge was balanced. The bolometer was next exposed to the radiation, and simultaneous observations of the intensity of the beam were made on galvanometer G_1 (Fig. 1), and the lamp galvanometer G_2. The deflection of galvanometer G_1 was reduced to standard lamp (a deflection of 100 divisions), as was done in the pressure observations. After shutting off the light the heating current was turned on. It was regulated by means of the variable resistance R_1 (Fig. 1), so that nearly the same throw was obtained from the galvanometer G_1 as when the bolometer was exposed to the lamp. All deflections of the galvanometer G_1 were taken with the

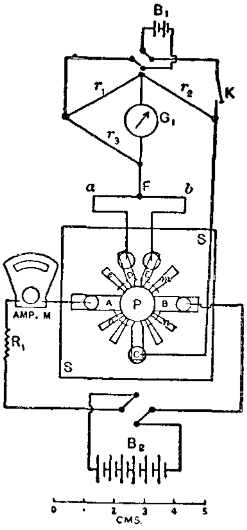

FIGURE 1.

bridge current both direct and reversed to eliminate any local disturbances in the bridge and also with the heating current both direct and reversed. A Siemens & Halske direct-reading precision milliampèremeter was used to measure this current. From repeated observations the current which gave the same heating effect as the light beam was $i = 0.865$ amp. The resistance between the binding posts A and B was measured with the lamp on and gave $r = 0.278$ ohm. The intensity of the beam in erg-seconds was thus: $r\, i^2 \times 10^7$, or $0.278 \times 0.75 \times 10^7$.

Using 0.92 as the reflection coefficient of silver, the pressure computed from the energy measurements was $p = 1.34 \times 10^{-4}$ dynes. The observed pressure was only 78 per cent of this value. No correction for the diffuse reflection of the blackened bolometer face nor for the difference in reflecting power between the two faces of the silver coating, discovered later, was made. The two corrections, however, nearly balance, so no considerable change in the result would have been effected by using them.

It was later discovered that, in dissolving the silver from the platinum when the bolometer was made, the acid had eaten away the silver from the strips A and B for a distance of nearly a millimeter under the asphalt. The resistance 0.278 ohm given for the disc was thus too high. It was impossible to redetermine the resistance by the direct method because of an accident to the bolometer by which the disc was nearly severed from the strip B. The disc was therefore carefully torn away from its supports, mounted on a glass plate and cut on a dividing engine into strips, 1 and 2 mms. wide, parallel to $A\,B$. The resistance along these strips was measured by the fall of potential method. The resistance was found to vary slightly in different parts of the disc due to lack of uniformity in the thickness of the metal. After many measurements, an average value was reached and the resistance of the disc computed theoretically as follows: —

The resistance of a conducting sheet of infinite extent, when the current enters and leaves the sheet by electrodes * of relatively great conductivity, is $\frac{\sigma}{4\,\pi\,C}$, where σ is the resistance of any square of the sheet, and C is the electrostatic capacity of the two electrodes. If the electrodes are cylinders, the lines of flow are circles orthogonal to them. When the sheet, in place of being infinite, is bounded by one of these circular lines of flow, the resistance is $\frac{\sigma}{2\,\pi\,C}$. In particular, if the elec-

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* J. J. Thomson, Electricity and Magnetism, 2d Edition, p. 314. Cambridge, 1897.

trodes of radii r are on a diameter of this circular sheet of radius R, then the resistance can be shown to be

$$\frac{\sigma}{\pi} \log_e \frac{2\,r^2 + R^2 + R\sqrt{4\,r^2 + R^2}}{2\,r^2}.$$

Assuming for the moment that the leading-in strips of the bolometer (Fig. 1) were of great conductivity compared to that of the thin platinum sheet and that they terminated in circular arcs orthogonal to this circular sheet, the resistance would be $0.922 \times \sigma$, giving to r the value of 2.79 mms. and to R the value 11.25 mms. But the leading-in strips terminated on the boundary of the large circle. The resistance was therefore altered by two facts, — the lines of flow were changed and the distance between the electrodes was increased. The latter is the important item. It is necessary therefore to find approximately the resistance of these gibbous portions of the large disc previously considered as electrodes. This may be done by estimating the area of these parts and by considering the average equipotential line as midway between the chord and arc of the cylindrical electrode. It results that the amount to be added on account of this calculation is $0.471 \times \sigma$. Hence the resistance between the electrodes is now $(0.922 + 0.471)\ \sigma = 1.393 \times \sigma$. The value of σ as found by the fall of potential method was 0.148 at 19° C. When corrected for the temperature of the disc exposed to the lamp, σ becomes 0.160. Hence the resistance of the disc when hot was $1.393 \times 0.160 = 0.221$ ohm.* Substituting this computed value of the resistance in place of the one used, the energy of the standard beam becomes $0.221 \times 0.75 \times 10^7$ ergs-seconds and

$$p = \frac{1.92 \times 0.221 \times 0.75 \times 10^7}{3 \times 10^{10}} = 1.05 \times 10^{-4} \text{ dynes}.$$

This result is in accidental agreement with the observed pressure. If necessary corrections, determined by later experiment, had been applied, the difference between the observed pressure and the pressure computed from the energy measurements would have been about three per cent. Moreover the probable error of the final result was roughly double this amount.

In the November number of the Annalen der Physik for 1901 Professor Lebedew † published the results of a more varied series of measurements of radiation pressure than the early measurements of the present writers. The principal difference between the methods employed by him and by the writers for determining the pressure was that he used very thin metallic vanes surrounded by gas at extremely low pressures, thus following Maxwell's suggestion literally, while the writers used silvered glass vanes and worked at large gas pressures for which the gas action had been carefully and exhaustively studied and found to be negligibly small for short exposures. From our knowledge of the variation of gas action in different vacua, we feel sure that our method would not have been successful in high vacua because of the relatively large gas action. Professor Lebedew's own results, with blackened vanes of lower heat conductivity, show that his success in eliminating gas disturbance was due to the high heat conductivity of thin vanes rather than to the high vacua employed.

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* The resistance of a trial disc was measured experimentally with the result that the experimental value differed from the theoretical by about one per cent.

† P. Lebedew, Ann. Phys., VI. 433 (1901).

Professor Lebedew's * estimate of the accuracy of his work is such as to admit of possible errors of twenty per cent in his final results. An analysis of Professor Lebedew's paper and comparison with our preliminary experiments seems to show that his accidental errors were larger than ours, but through the undiscovered false resistance in the bolometer our final results were somewhat further from the theory than his. Either of the above researches would have been sufficient to establish the *existence* of a pressure due to radiation, but neither research offered, in our judgment, a satisfactory *quantitative* confirmation of the Maxwell-Bartoli theory.

Later Pressure Measurements.

Description of Apparatus. — The Torsion Balance.

The form of suspension of the torsion balance, used to measure radiation pressure in the present study, is seen in Fig. 2. The rotation axis *a b* was a fine rod of drawn glass. A drawn glass cross-arm *c*, bent down at either end into a small hook, was attached to the axis. The surfaces *C* and *D*, which received the light beam, were circular microscope cover-glasses, 12.8 mm. in diameter and 0.17 mm. thick, weighing approximately 51 mgs. each. To distinguish the two vanes from each other, in case individual differences should appear in the measurements, and also to mark the two faces of each vane for subsequent recognition, a letter *C* was marked on one, and *D* on the other by diamond scratches. Through each glass, a hole 0.5 mm. or less in diameter, was drilled near the edge, by means of which the glasses could be hung on the hooks on the cross-arm *c*. On opposite sides of the rotation axis at *d* two other drawn-glass cross-arms were attached. The cover glassses slipped easily between these, and were thus held securely in one plane. Further down on *a b*, a small silvered plane mirror m_1 was made fast at right

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* P. Lebedew, Ann. Phys., VI. 457 (1901).

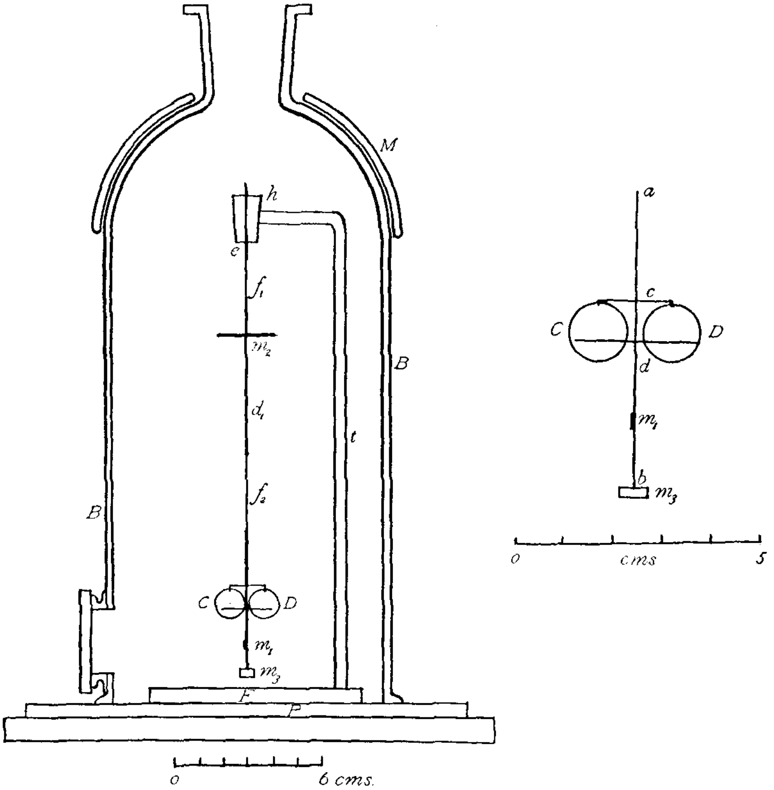

FIGURE 2.

angles to the plane of *C* and *D*. This mirror was polished bright on the silver side, so that the scale at S_5 (Fig. 3) could be read in either face. A small brass weight m_3 (Fig. 2), of 452 mgs. mass and of known dimensions, was attached at the lower end of *a b*. The cover-glasses which served as vanes were silvered and brilliantly polished on the silvered sides, and so hung on the small hooks that both silver faces or both glass faces were presented to the light. A quartz fiber f_2, 3 cms. long, was made fast to the upper end of *a b*, and to the lower end of a fine glass rod d_1 which carried a horizontal magnet m_2. The rod d_1 was in turn suspended by a short fiber to a steel pin *e*, which could be raised or lowered in the bearing *h*. The whole was carried by a bent glass tube *t*, firmly fastened to a solid brass foot *F*, resting on a plane ground-glass plate *P*, cemented to a brass platform mounted on three levelling screws not shown. A bell-jar *B*, 25 cms. high and 11 cms. in diameter, covered the balance. The flange of the bell-jar was ground to fit the plate *P*. A ground-in hollow glass stopper fitted the neck of the bell-jar, which could thus be put in connection with a system of glass tubes leading to a Geissler mercury pump, a MacLeod pressure gauge, and a vertical glass tube dipping into a mercury cup and serving as a rough manometer for measuring the larger gas pressures employed during the observations. The low pressures were measured on the MacLeod gauge in the usual way. A semicircular magnet *M*, fitted to the vertical curvature of the bell-jar, was used to direct the suspended magnet m_2 and thus to control the zero position of the torsion balance. By turning *M* through 180°, the opposite faces of the vanes *C* and *D* could be presented to the light.

THE ARRANGEMENT OF APPARATUS.

A horizontal section of the apparatus through the axis of the light beam is shown in Fig. 3. The white-hot end of the horizontal carbon S_1, of an A. T. Thompson 90° arc-lamp, fed by alternating current, served as a source. The arc played against the end of the horizontal carbon from the vertical carbon which was screened from the lenses L_1 and L_2 by an asbestos diaphragm d_2. A lens, not shown, projected an enlarged image of the arc and carbons on an adjacent wall, so that the position of the carbons and the condition of the arc could be seen at all times by both observers.

The cone of rays passing through the small diaphragm d_2 fell upon the glass condensing lenses L_1, L_2. At d_3 a diaphragm, 11.25 mm. in diametér, was interposed, which permitted only the central portion of the cone of rays to pass. Just beyond d_3, the beam passed to a shutter at S_2. This shutter was worked by a magnetic escapement, operated by the seconds contact of a standard clock. The observer at T_1 might choose the second for opening or closing the shutter, but the shutter's motion always took place at the time of the seconds contact in the clock. Any exposure was thus of some whole number of seconds' duration. The opening in the shutter was such as to let through, at the time of exposure, all of the direct beam which passed through d_3, but to shut out stray light. Just beyond the shutter and attached to the diaphragm d_5 was a 45° glass plate which reflected a part of the beam to the lens L_5, by means of which an image of d_3 was projected upon one arm of a bolometer at R. The glass lens L_3 focused a sharp image of the aperture d_3 in the plane of the vanes of the torsion balance B_1 under the bell-jar. The bell-jar was provided with three plate glass windows W_1, W_2, W_3. The first two gave a circular opening 42 mm. in diameter, and through the third, deflections of the balance were read by a telescope and scale. The lens L_3 was arranged to move horizontally between the stops S_3 and S_4. These were so adjusted that when the lens was against S_3 the sharp image of the aperture d_3 fell centrally upon one vane; and when against S_4 the image fell centrally upon the other. This adjustment, which was a very important one, was made by the aid of a telescope T_2, mounted on the carriage of a dividing engine. This was used to observe and measure the position

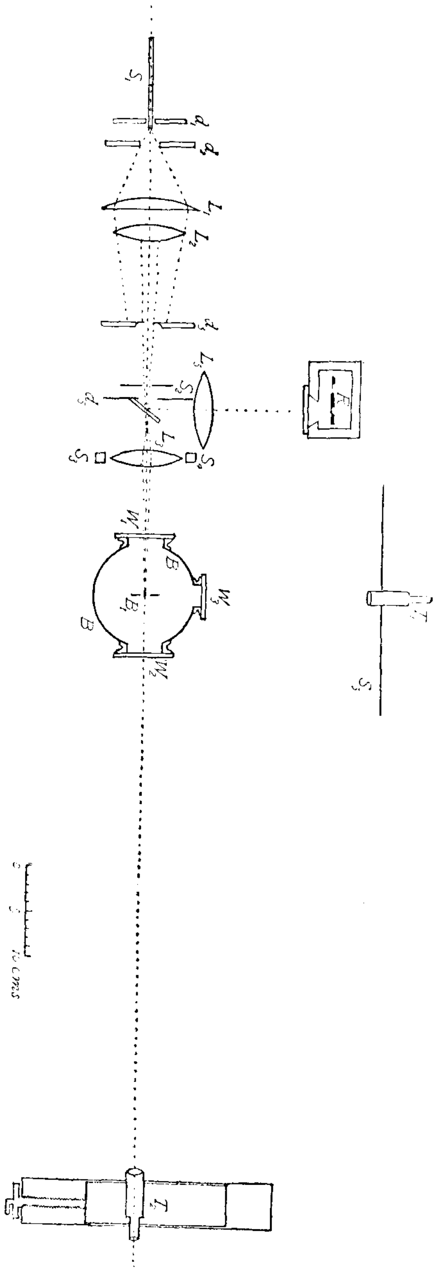

FIGURE 3.

of the rotation axis, as well as the positions of the images of d_3, when the lens L_3 was against the stops. For the latter measurements, the vanes could be moved out of the way by turning the suspension through 90° by the control magnet M (Fig. 2).

To make sure that the balance as used was entirely free from any magnetic moment or disturbance, the small magnet m_2 was clamped in one position to maintain a constant zero, and the period of the balance was accurately measured with the axis of the large magnet M in the vertical plane of the vanes and again when the axis was at right angles to the plane of the vanes. Several series of this sort failed to show a difference of 0.1 second in the period of the balance for the two positions of the magnet.

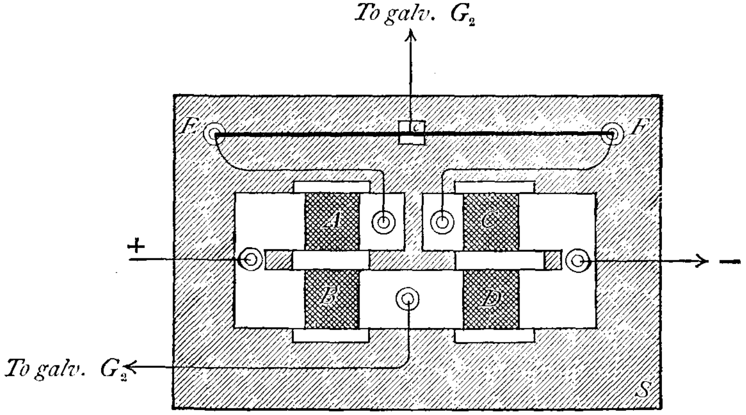

FIGURE 4.

The bolometer at R (Fig. 3) was of sheet platinum 0.001 mm. thick, rolled in silver. The strip was cut out in the form shown in Fig. 4, and mounted on a thin sheet of slate S. Two windows had been cut in the slate behind the strips at $ABCD$ where the silver had been removed leaving the thin platinum. The platinum surfaces were blackened by Kurlbaum's process. The image from L_5 (Fig. 3), fell at D. The silver ends between A and C were connected with E and F respectively. On the heavy wire EF a sliding contact c served to balance the bridge, all four arms of which are shown in the figure.

METHODS OF OBSERVATION.

The observations leading to the results given later were of three different kinds: (1) The calibration of the torsion balance; (2) the measurement of the pressure of radiation in terms of the constant of the balance; and (3) the measurement of the energy of the same beam in erg-seconds by the rate of temperature rise of a blackened silver disc, of known mass and specific heat.

1. The determination of the constant of the torsion balance was made by removing the vanes C and D and accurately measuring the period of vibration. Its moment of inertia was easily computed from the masses and distribution of the various parts about the axis of rotation. The moment of torsion for 1 mm. deflection on a scale 105 cm. distant was 0.363×10^{-5} dyne × cm. This value divided by one-half the distance between the centres of the light spots on the two vanes gave the force in dynes per scale division deflection. As the light spots were circles 11.25 mm. in diameter the area of the image was very nearly 1 $(\text{cm.})^2$, hence the above procedure gave roughly the pressure in dynes per square centimeter.

2. In the measurements of radiation pressure, it was easier to refer the intensity of the beam at each exposure to some arbitrary standard which could be kept constant than to try to hold the lamp as steady as would otherwise have been necessary. For this purpose, the bolometer at R (Fig. 3) was introduced, and simultaneous observations were made of the relative intensity of the reflected beam by the deflection of the galvanometer G_2, and the pressure due to the transmitted beam by the deflection of the torsion balance. The actual deflection of the balance was then reduced to a deflection corresponding to a galvanometer deflection of 100 scale divisions. The galvanometer sensitiveness was carefully tested at the beginning and end of each evening's work. All observations of pressure were thus reduced to the pressure due to a beam of fixed intensity.

At each series of radiation pressure measurements, two sets of observations were made. In one of these sets, static conditions were observed, and in the other, the deflections of the balance due to short exposures were measured. In the static observations, each vane of the balance was exposed in turn to the beam from the lamp, the exposures lasting until the turning points of the swings showed that stationary conditions had been reached. The moment of pressure of radiation and gas action combined would thus be equal to the product of the static deflection and the constant of the balance. The torsion system was then turned through 180° by rotating the outside magnet, and similar observations were made on the reverse side of the vanes. All turning points of the swinging balance in these observations were recorded. From the data thus obtained the resultant of the combined radiation and gas forces could be determined for the time of every turning point. Every value was divided by the deflection at standard sensitiveness of the galvanometer G_2 read at the same time and was thus reduced to a standard lamp. Results thus obtained, together with the ballistic measurements, showed the direction and extent of the gas action as well as its variation with length of exposure.

The reasons for reversing the suspension follow: The beam from the lamp, before reaching the balance, passed through three thick glass lenses and two glass plates. All wave-lengths destructively absorbed by the glass were thus sifted out of the beam by the time it reached the balance vanes. The silver coatings on the vanes absorbed therefore more than the glass. The radiation pressure was always away from the source irrespective of the way the vanes were turned, while the gas action would be exerted mainly on the silvered sides of the vanes.

At the close of the pressure and energy measurements, when the reflecting power of the silver faces of the vanes was compared with that of the glass-silver faces, the reflection from the silver faces was found very much higher than that for the glass faces backed by silver. This result was the more surprising because the absorption of the unsilvered vanes was found by measurement to be negligibly small.* This unexpected difference in reflecting power of the two faces of the mirrors prevented the elimination of the gas action, by the method described, from being as complete as had been hoped for. But by choosing a gas pressure where the gas action after long exposure is small, the whole gas effect during the time of a ballistic exposure may be so reduced as to be of little consequence in any case.

By exposing each of the vanes in turn and by reversing the suspension and averaging results, nearly all errors due to lack of symmetry in the balance or in the position of the light images with reference to the rotation axis, or errors due to lack of uniformity in the distribution of intensity in different parts of the image, could be eliminated.

The changing character of the gas action, both with time of exposure and gas pressure surrounding the balance vanes, is well illustrated in eight series of static observations in which the glass faces of both vanes were exposed.† The results obtained on the two vanes were averaged

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* Lord Rayleigh records a similar difference between the reflection from air-silver and glass-silver surfaces. Scientific Papers, Cambridge, II. 538–539 (1900).

† Observations were also made on the silver faces, but the gas action when the glass faces were exposed was nearly double that for the silver faces, so the least *favorable case is shown.*

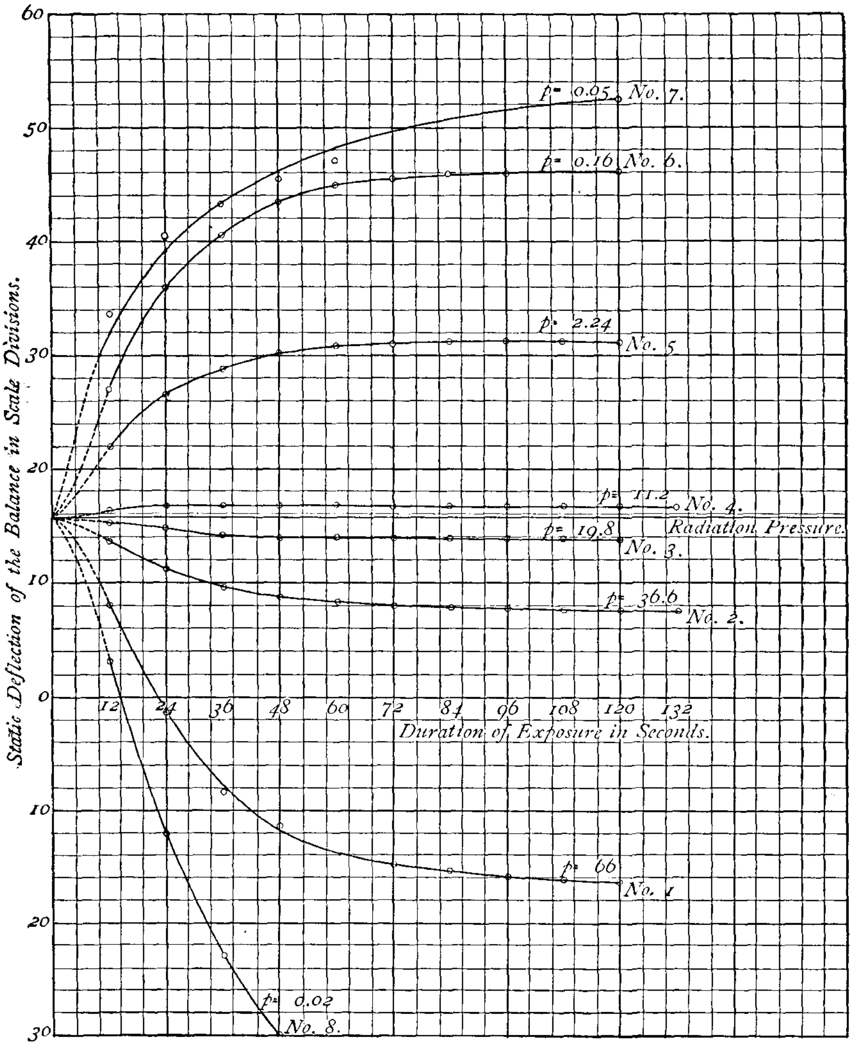

FIGURE 5.

and plotted as curves in Fig. 5, where static deflections due to combined radiation pressure and gas action are shown as ordinates and duration of exposure, in seconds, as abscissæ.* A horizontal line through the diagram gives the mean value of the moment of radiation pressure computed from the data in Table II. Decrease of the deflection with time indicates gas repulsion on the warmed silver faces and increase in deflection, gas suction. It will be seen from the curves that beginning at a gas pressure of 66 mm. of mercury, the gas action was repulsion changing to suction in passing from 19.8 to 11.2 mm. In the last two cases the total gas action is small. For lower pressures the suction increases to 0.05 mm. At a gas pressure of 0.02 mm. the gas action is again a strong repulsion.

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* Ordinates of the curves are proportional to moments.

The curves indicate the existence of two gas pressures, at which the gas action in our arrangement of apparatus should be zero, one between 19.8 and 11.2 mm. and the other between 0.05 and 0.02 mm.* The former region was chosen for the ballistic measurements and nearly all of the observations were made at a gas pressure of approximately 16 mm. Even for the two pressures where the decrease in the static deflection was most rapid, i. e. at gas pressures of 66 and 0.02 mm., the first throw was always in the direction of radiation pressure. The gas action is strongly influenced by very slight changes in the inclination of the plane of the vanes to the vertical and also by any object introduced under the bell-jar anywhere near the vanes. For instance, a very considerable effect was observed when a small vessel of phosphoric anhydride was placed under the jar behind the vanes, though the nearest wall of the vessel was separated from the vanes by a distance of at least 3 cms.

During the observations, the polished silver coatings on the vanes deteriorated rapidly; new coatings rarely lasted for more than two evenings' work. As the balance had to be removed and the mirrors taken from the hooks, silvered, polished, and replaced a great number of times during the entire series of measurements, although great care was taken in setting the plane of the vanes vertical, it is not likely that precisely the same conditions for gas action were ever repeated. The principal value of the static results was in indicating favorable gas pressures for work, rather than affording quantitative estimates of the gas action in short exposures. The dotted parts of the curves are not based on results of observation and might perhaps have been omitted without loss.

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* Crookes in his work with the radiometer discovered certain gas pressures for which the combined gas and radiation forces neutralized, but as he did not discriminate between forces due to radiation and gas forces his results were apparently capricious and his reasoning somewhat confused. See Phil. Trans., p. 519 (1875).

It was plain, therefore, that further elimination of the gas action must be sought in exposures so short that the gas action would not have time to reach more than a small fraction of its stationary value. This led to the method of ballistic observations.

THE BALLISTIC OBSERVATIONS.

In passing from the static to the ballistic observations it must always be possible to compute the static equivalent of the ballistic swings. Furthermore the exposures should be made as short as possible without reducing the size of the swing below a value which can be accurately measured.

If the exposure lasts for one-half the period of the balance, the deflection, if the gas action be small and the damping zero, is equal to $2\,\theta$, where θ is the angle at which the torsion of the fibre will balance the moment produced by the radiation pressure. If the duration of the exposure be one-quarter of the period of the balance, the angle of deflection is $\theta\sqrt{2}$. The deflection is thus reduced by 30 per cent, but the effect of the gas action is reduced in greater proportion. It was decided therefore to expose for six seconds, one-quarter of the balance period. Neglecting the gas action, the equation * of motion of the balance is given by

$$\kappa\frac{\partial^2\theta}{\partial t^2}+2\,\epsilon\frac{\partial\theta}{\partial t}=-\,G\,\theta+L$$

where κ = the moment of inertia of the torsion balance,
ϵ = the damping constant,
G = the moment of torsion of the fibre for $\theta = 1$ radian,
and L = the moment of the radiation force.

The solution of this equation is

$$\theta=\frac{L}{G}\left\{1-e^{-\frac{\epsilon}{\kappa}t}\cos\sqrt{\frac{G}{\kappa}-\frac{\epsilon^2}{\kappa^2}}\,t\right\}$$

$$=\frac{L}{G}\left\{1-e^{-\frac{\epsilon}{\kappa}t}\cos 2\,\pi\frac{t}{T}\right\} \qquad (1)$$

the constants of integration having been determined from the condition that $\theta=\frac{\partial\theta}{\partial t}=0$ when $t=0$.

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* We are justified in using quantitatively this equation, containing a damping term proportional to the velocity, because the amplitudes of the successive swings of the torsion balance, when no energy fell upon the vanes, were found experimentally to follow accurately the exponential law.

When

$$t = \frac{T}{4},\ \theta = \frac{L}{G} \text{ and } \frac{\partial \theta}{\partial t} = \frac{L}{G}\left(\frac{\epsilon}{\kappa}\, e^{-\frac{\epsilon}{\kappa} t} \cos 2\,\pi \frac{t}{T} + e^{-\frac{\epsilon}{\kappa} t}\, \frac{2\,\pi}{T} \sin 2\,\pi \frac{t}{T}\right) \qquad (2)$$

The light being cut off when $t = \frac{T}{4}$, the equation of motion becomes

$$\kappa \frac{\partial^2 \theta}{\partial t^2} + 2\,\epsilon \frac{\partial \theta}{\partial t} = -\ G\,\theta \qquad (3)$$

the solution of which is $\theta = A\, e^{-\frac{\epsilon}{\kappa} t} \cos\left(2\,\pi \frac{t}{T} + \alpha\right)$ where A and α can be determined by the conditions imposed by equation (2). Neglecting very small quantities, the value of the amplitude A is expressed by the equation

$$A = \frac{L}{G}\left\{1 + r + \frac{2}{\pi} r^{\frac{1}{2}} \log\left(\frac{1}{r}\right)\right\}^{\frac{1}{2}}, \qquad (4)$$

where r is the ratio of successive amplitudes of the damped vibrations. If $r = 1$, that is if the motion is undamped, $A = \frac{L}{G}\sqrt{2}$. In the partial vacuum used in the experiments (16 mms. of mercury, a value chosen from the curves in Fig. 5), r was found to be equal to 0.783; consequently $A = 1.357 \frac{L}{G}$. (5)

From this it is seen that the total angle of deflection of the torsion balance in the ballistic measurements is equal to 1.357 times the angle at which the moment of the torsion of the fibre balances the moment of the radiation pressure.

The duration of exposure was always six seconds without appreciable error, but the period of the balance on account of slight accidental shifting of small additional masses upon the counterpoise weight m_3 (Fig. 2), differed from twenty-four seconds sometimes by one per cent. It is necessary therefore to find the error in the deflection due to this variation in the period. This is done by making $t = \frac{T}{4} + \delta$ in equation (2) and in introducing the new conditions in equation (3). But it is simpler and sufficiently accurate to assume the motion as undamped. For this condition, the amplitude

$$A = \frac{L}{G}\left\{2 + 2 \sin 2\,\pi \frac{\delta}{T}\right\}^{\frac{1}{2}} = \sqrt{2}\,\frac{L}{G}\left(1 + \pi \frac{\delta}{T}\right) \text{ nearly.}$$

For $T = 23.75$ seconds $\frac{T}{4} = 5.94$ and $\delta = 0.06$. Hence

$$A = \sqrt{2}\frac{L}{G}(1.008).$$

If $\delta = 0$, $A = \sqrt{2}\frac{L}{G}$, consequently an error of 1 per cent in T causes an error of 0.8 per cent in A.

To make sure that the observed radiation pressures depended only on the intensity of the beam, and were uninfluenced by the wave length of the incident energy, the ballistic observations of pressure, the thermal measurements of intensity, and the determination of the reflection coefficients, were carried out for three entirely different wave-groups of the incident radiation. In the measurements designated "through air," no absorbing medium was introduced in the path of the beam between the lamp and the balance except the glass lenses and plates already mentioned. In the measurements "through red glass," a plate of ruby glass was put in the path of the beam between L_2 and d_3 (Fig. 3). For the observations "through water cell," a 9 mm. layer of distilled water in a glass cell was placed in the path of the beam at the same point.

The separate observations entering into a single series of ballistic measurements and their treatment will appear from Table II, which is copied direct from the laboratory notebook and represents an average ballistic series. The designations EVC_s, WVD_s, EVD_g, and WVC_g mean that the vane C in the first case was on the east side of the rotation axis with its silver face toward the light. The subscript g signifies that the glass face of the vane was toward the light. The second column of the table gives the zero reading of the balance before opening the shutter; the third, the end of the swing produced by a six-second exposure; the fourth, the deflection of the balance; the fifth, the ballistic deflection of the lamp galvanometer G_2. Columns six and seven give the balance deflection reduced to standard lamp.

The results of all the ballistic pressure measurements "through air" are collected in Table III. In the fourth and fifth columns two values are given for the constant of the lamp galvanometer G_2; since reversing the magnet on the balance bell-jar to reverse the suspension within affected the constant of the galvanometer slightly the values for the silver and glass faces forward were never the same. The subscripts show to which series, silver or glass, the constant belongs. The values of the lever-arm l of the balance, in the sixth column, are obtained by mea-

TABLE II.

August 28. Light Pressure. Ballistic Measurements. Air.

Surface.	Zero.	Throw	Deflection.	Lamp.	Deflection (Lamp 100)	
					E. V.	W. V.
			mms.	mms.	mms.	mms.
E V C_s	281.4	248.5	32.9	164.3	20.0	
W V D_s	281.5	313.9	32.4	164.5		19.7
E V C_s	281.4	249.8	31.6	157.9	20.0	
W V D_s	281.5	310.5	29.0	147.0		19.8
E V C_s	281.5	252.6	28.9	144.8	20.0	
W V D_s	281.5	309.6	28.1	141.8		19.8
E V C_s	281.5	252.9	28.6	143.5	19.9	
W V D_s	281.5	309.3	27.8	140.4		19.8
				Average . .	19.97	19.77
				Average, $\frac{C_s + D_s}{2} = 19.87$		
		Magnet reversed.				
E V D_g	280.1	246.0	34.1	180.4	18.92	
W V C_g	280.0	317.8	37.8	187.3		20.20
E V D_g	279.8	247.2	32.6	170.8	19.20	
W V C_g	279.4	313.7	34.3	169.4		20.25
E V D_g	279.1	248.9	30.2	161.1	18.80	
W V C_g	279.0	311.9	32.9	161.6		20.35
E V D_g	279.0	249.1	30.0	158.9	18.90	
W V C_g	278.9	311.2	33.2	164.4		20.20
				Average . .	18.97	20.25
				Average, $\frac{C_g + D_g}{2} = 19.61$		

suring the distance between the centres of the images when on the east and west vanes (by the dividing engine T_2, Fig. 3) and dividing by two. The columns headed $\frac{C_s + D_s}{2} = P_s$ and $\frac{C_g + D_g}{2} = P_g$ are the average

TABLE III.— RADIATION PRESSURE. BALLISTIC MEASUREMENTS, THROUGH AIR.

Date.	Air Pressure in mms. of Hg.	Balance Period T.	Sens. of Galvan'r.		l = Lever Arm. cms.	$\frac{C_s+D_s}{2}=P_s$.	$\frac{C_g+D_g}{2}=P_g$.	P_s corrected for $T=24''$.	P_g corrected for $T=24''$.	$\frac{P_s \times G_s}{l}$.	$\frac{P_g \times G_g}{l}$.	Average.
			G_s (Silver).	G_g (Glass.)								
June 19	32.5	23.75	[734]		.814	Average = 18.88		Average = 18.73		Average = 16.89		16.89
" 20	32.5	23.75	756	768	.814	19.67	16.94	19.51	16.81	18.12	15.86	17.00
" 23	37.0	23.75	700	716	.814							
July 23	16.0	23.75	682	707	.831	21.16	20.42	21.00	20.26	17.25	17.25	17.25
" 25	16.6	23.75	684	684	.815							
" 26	16.6	23.75	720	710	.815	19.34	19.98	19.18	19.82	16.93	17.28	17.10
Aug. 27	16.8	23.82	724	710	.823	20.16	19.40	20.00	19.25	17.60	16.61	17.10
" 28	15.7	23.82	721	716	.824	19.87	19.61	19.73	19.48	17.26	16.97	17.10
" 29	13.7	23.82	712	701	.824	19.68	20.07	19.53	19.92	16.90	16.97	16.94
" 31	14.0	24.00	718	713	.810	18.55	18.94	18.55	18.94	16.44	16.60	16.52
Sept. 1	16.6	24.00	692	672	.808	19.14	20.17	19.14	20.17	16.40	16.78	16.59
" 20	16.4	23.78	670	676	.812	20.96	20.02	20.81	19.87	17.17	16.54	16.86
" 23	16.4	23.78	666	684	.816	21.32	20.27	21.16	20.12	17.27	16.87	17.07
" 24	16.2	23.78	667	669	.816	20.76	19.80	20.60	19.65	16.84	16.11	16.47
								Average		17.11	16.71	16.91 ± 0.053

moments due to pressures for the silver and glass sides of the vanes respectively toward the light. The next two columns contain these moments corrected for a period of 24 seconds of the torsion balance. The columns headed $\frac{P_s \times G_s}{l}$ and $\frac{P_g \times G_g}{l}$ are the corresponding forces reduced to standard sensitiveness, $G = 1000$. The final column contains the averages of the two columns which precede it. Table IV exhibits corresponding data for "red glass" and "water cell." The air pressure, period of the balance, lever arm and galvanometer constants are those given in Table III for the same date.

In these ballistic measurements the lamp reading was the throw due to an exposure of the light upon the bolometer for six seconds, but in the energy measurements the lamp reading was a stationary deflection due to prolonged exposure. To bring the pressure values into comparison with the energy measurements it is necessary to reduce the average of the quantities in the last column to pressures in dynes by multiplying by 0.363×10^{-5}, the torsion coefficient of the quartz fibre, and to reduce not only to a static deflection of the torsion balance but also to a static deflection of the lamp galvanometer G_2. The ratio of a ballistic to a static deflection of the galvanometer G_2 was obtained from a long series of lamp exposures. This ratio was found "through air" to be $= 1.55$; "through red glass" $= 1.535$; "through water cell" $= 1.502$. These differences are probably due not solely to the damping constant of the galvanometer but to the peculiar manner in which the bolometer was warmed up to its stationary conditions by the beam from the lamp. Applying these reduction factors to the averages in Tables III and IV, we obtain the following results. The pressure of the standard light beam which has passed

(*a*) through air $= 16.91 \times \frac{1.55}{1.357} \times 0.363 \times 10^{-5} = (7.01 \pm 0.023) \times 10^{-5}$ dynes;

(*b*) through red glass $= 16.91 \times \frac{1.535}{1.357} \times 0.363 \times 10^{-5} = (6.94 \pm 0.024) \times 10^{-5}$ dynes;

(*c*) through water cell $= 16.20 \times \frac{1.502}{1.357} \times 0.363 \times 10^{-5} = (6.52 \pm 0.028) \times 10^{-5}$ dynes.

TABLE IV.

Radiation Pressure. Ballistic Measurements.

Date.	$\frac{C_s + D_s}{2}$.	$\frac{C_g + D_g}{2}$.	P_s corrected for $T = 24''$.	P_g corrected for $T = 24''$.	$\frac{P_s \times G_s}{l}$.	$\frac{P_g \times G_g}{l}$.	P Average.
Through Water Cell.							
June 20	18.62	17.10	18.46	16.96	17.14	16.00	16.57
July 25	19.00	20.10	18.85	19.94	15.82	16.74	16.28
" 26	18.03	19.39	17.89	19.33	15.80	16.84	16.32
Aug. 27	18.63	18.66	18.50	18.53	16.29	15.99	16.14
" 29	18.25	19.02	18.10	18.87	15.68	16.06	15.87
Sept. 20	20.39	19.14	20.23	19.00	16.69	15.82	16.25
" 23	20.21	19.51	20.05	19.36	16.37	16.23	16.30
" 24	19.84	18.91	19.69	18.77	16.10	15.40	15.70
				Average . . .	16.24	16.15	16.20 ±0.066
Through Red Glass.							
June 23	19.99	18.40	19.83	18.26	17.05	16.06	16.56
July 25	20.70	20.94	20.54	20.77	17.24	17.43	17.33
Aug. 27	19.97	19.25	19.82	19.10	17.46	16.46	16.96
" 28	19.99	19.42	19.84	19.28	17.36	16.75	17.05
" 29	19.99	19.92	19.84	19.77	17.14	16.82	16.98
" 31	18.98	19.14	18.98	19.14	16.82	16.84	16.83
Sept. 20	21.00	19.97	20.83	19.82	17.19	16.50	16.84
" 23	21.48	20.34	21.31	20.18	17.39	16.92	17.15
" 24	21.00	19.68	20.83	19.53	17.03	16.03	16.53
				Average . . .	17.18	16.65	16.91 ±0.051

The Energy Measurements.

Before rejecting the bolometer method used in the preliminary measurements of energy, a second bolometer of slightly different construction was tried; but the lack of uniformity of resistance, already mentioned, made its indications too uncertain for the present work. The radiant intensity of the beam used in the later experiments was determined by directing it upon the blackened face of a silver disc, weighing 4.80 grams, of 13.3 mm. diameter and of 3.58 mm. thickness, and by measuring its rate of temperature rise as it passed through the temperature of its surroundings. The disc was obtained from Messrs. Tiffany & Co. and was said by them to be 99.8 per cent fine silver. Two holes were bored through parallel diameters of the disc, one-fourth of the thickness of the disc from either face. Two iron-constantan thermo-junctions, made by soldering 0.1 mm. wires of the two metals, were drawn through the holes into the centre of the disc. To insulate the wires from the disc, fine drawn glass tubes were slipped over them and thrust into the holes, leaving less than 2 mm. bare wire on either side of the junctions. The wires were sealed into the tubes, and the tubes into the disc by solid shellac. The tubes projected 15 mm. or more from the disc and were bent upward in planes parallel to the faces of

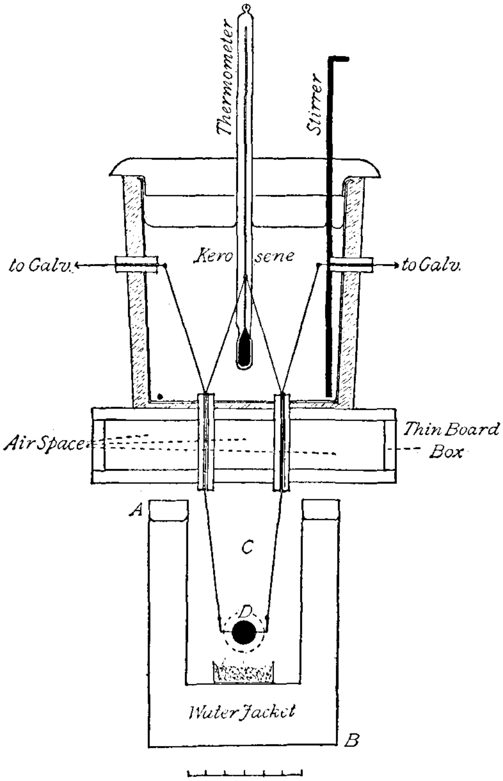

Figure 6.

the disc. The general arrangement will be seen in Fig. 6. The disc was suspended by the four wires some distance below a small flat wooden box. On the box was fastened a calorimeter can swathed in cotton and filled with kerosene in which the constant thermo-junctions were immersed. Copper wires soldered to the two ends of the thermo-electric series were brought out of the calorimeter, and the circuit was closed through 1000 ohms in series with the 500 ohms resistance of galvanometer G_1. The thermo-junctions in the disc were in series, and as each junction was midway between the central plane of the disc and either face, it was assumed that when the disc was slowly warmed by heating one face the electromotive forces obtained corresponded to the mean temperature of the disc. One face of the disc was blackened by spraying it with powdered lampblack in alcohol containing a trace of shellac. This method was suggested by Prof. G. E. Hale and gives very fine and uniform dead black coatings not inferior to good smoke deposits.

For the energy measurements the bell-jar and the torsion balance were removed from the platform P (Fig. 2) and a double walled copper vessel, AB (Fig. 6), which served as a water jacket surrounding a small air chamber C, was mounted in the same place. A tube 2 cm. in diameter was soldered into the front face of the jacket to admit the light beam into the chamber C. This opening was covered by a piece of plate glass similar to the plates forming the larger windows in the bell-jar.

The needle system in G_1, a four-coil du Bois Rubens galvanometer, was suspended in a strong magnetic field so that its period was about four seconds. The system was heavily damped by a mica air-fan of large surface. The disc junctions and galvanometer responded quickly to the radiation, as was shown by the reversal of motion of the magnet system 1.2 seconds after the light was cut off from the disc when the latter was a few degrees above the temperature of the room.

The disc was calibrated for temperature in terms of the deflection for a definite sensitiveness of the galvanometer G_1. For this purpose the disc was immersed in a kerosene bath and the galvanometer deflection measured for two different temperatures of the disc. One of these was about 18° C. above the comparatively steady temperature of the room, or calorimeter containing the standard temperature junctions (see Fig. 6), and the other about the same number of degrees below the room temperature. These two temperatures were measured by a Fuess Standard Thermometer divided into tenths of a degree and calibrated at the Reichsanstalt. Two calibrations of the silver disc were made some days apart. One of these series appears in full in Table V. The first three

TABLE V.

CALIBRATION OF SILVER DISC.

Cold Bath.

G_1 Readings.			Disc T_1°.	Room cal. T_2°.	Deflection of G_1.	Means of alternate Deflec'ns.	G_1 Means.	T_2°—T_1°.
Rev.	Zero.	Direct.						
	221.2 *221.0*	35.2	1°.58	20°.05	185.8			
402.0	220.9 *221.0*		1°.60	20°.07	181.0	185.7	183.4	18.47
	221.2 *221.2*	35.7	1°.57	20°.10	185.5	181.3	183.4	18.53
403.1	221.2 *221.5*		1°.56	20°.14	181.6	185.8	183.7	18.58
	221.9 *222.0*	35.9	1°.54	20°.16	186.1	182.2	184.1	18.62
405.0	222.1 *222.2*		1°.52	20° 20	182.8	186.3	184.5	18.68
	222.4 *222.7*	36.2	1°.57	20°.22	186.5	182.7	184.6	18.65
405.8	223.0 *223.1*		1°.60	20°.26	182.7	186.7	184.7	18.66
	223.2 *223.3*	36.3	1°.59	20°.30	187.0			
	223.5						184.0	18°.60

Correction to $T_1 = 0°.00$.

Warm Bath.

G_1 Readings.			Disc T_1°.	Room cal. T_2°.	Deflection of G_1.	Means of alternate Deflec'ns.	G_1 Means.	T_2°—T_1°.
Rev.	Zero.	Direct.						
	217.3 *218.6*	434.2	41°.45	20°.40	215.6			
2.0	219.9 *220.5*		41°.35	20°.42	218.5	213.7	216.1	20.93
	221.1 *222.4*	434.2	41°.25	20°.44	211.8	216.4	214.1	20.81
10.0	223.7 *224.4*		41°.08	20°.50	214.4	209.1	211.7	20.58
	225.1 *225.7*	432.1	40°.90	20°.55	206.4	212.4	209.4	20.35
16.3	226.3 *226.7*		40°.80	20°.60	210.4	204.8	207.6	20.20
	227.2 *227.8*	431.0	40°.67	20°.61	203.2	208.5	205.8	20.07
21.8	228.4 *228.5*		40°.55	20°.63	206.7	201.8	204.2	19.92
	228.7 *229.0*	429.4	40°.43	20°.65	200.4			
	229 3						209.8	20°.41

Correction to $T_1 = 0°.10$. Corrected, 20.51

columns of the table give the zero, direct and reversed reading of the galvanometer G_1. The fourth column gives the temperature of the bath in which the disc was immersed, and the fifth, that of the constant temperature calorimeter. The sixth column gives the deflections of G_1. The seventh column the means of the alternate deflections. The eighth, the mean of the two columns preceding it. The last column gives the difference in temperature between the two calorimeters in degrees C. For the total temperature range in the table, 39.11°, the deflection of G_1 was 393.8 scale divisions for a sensitiveness of $G_1 = 996$. A range of one degree would thus give a deflection of 10.03 divisions for a sensitiveness of $G_1 = 1000$. The mean of two separate calibrations was 9.96 scale divisions for one degree temperature difference.

Before beginning a series of intensity measurements the disc was suspended in an air-chamber containing phosphoric anhydride and surrounded by a jacket of ice and salt. The disc was thus lowered to a temperature of about zero degrees and was then quickly transferred to the chamber C (Fig. 6), and the beam was directed upon it. When its temperature had risen to within five or six degrees of that of the chamber C, galvanometer readings were made at intervals of five seconds until the disc was heated to a temperature several degrees above its surroundings. The temperature of the chamber C was determined by removing the disc and cooling it to a point near the room temperature, then replacing it and observing its rate of temperature change for several minutes.

The notebook record of one series of observations showing the heating of the disc by the light beam is given in full in Table VI. It will be seen from the table that the temperature of the disc passed that of the chamber thirty seconds after the beginning of the series. The readings of G_1 at equal time intervals on either side of the zero are on horizontal lines. The last column of the table contains the rate at which the galvanometer deflection was changing when the disc and its surroundings were at the same temperature.

Energy series were made "through air," "through red glass," and "through water cell," as in the pressure measurements. During the experiment the black coatings were frequently cleaned off from the disc and new ones deposited. The final result therefore does not correspond to an individual, but to an average coating.

To correct for any inequality between the two disc thermo-junctions or any lack of symmetry in their positions, referred to the central plane of the disc, which might prevent the mean temperature of the two junctions from representing the mean temperature of the mass, series of

TABLE VI.

August 16. Energy Measurements. Through Air. Series 4.

Zero of Galvanometer G_1 (closed circuit) determined by method of cooling $= 216.8 =$ reading at room temperature.

Time.	G_1.	Time.	G_1.	ΔG_1.	Δt.	$\frac{\Delta G_1}{\Delta t}$ (in mm. per sec.)
0 secs.	174.5	60 secs.	253.2	78.7	60 secs.	1.312
5 "	182.0	55 "	247.3	65.3	50 "	1.306
10 "	189.0	50 "	241.3	52.3	40 "	1.308
15 "	196.2	45 "	235.2	39.0	30 "	1.300
20 "	203.0	40 "	229.1	26.1	20 "	1.305
25 "	209.7	35 "	222.8	13.1	10 "	1.310
30 "	216.4			Average	. . . . .	1.307

The lamp reading (G_2) was 924.
The sensitiveness of G_2 was 667, and of G_1 was 996.
$\frac{\Delta G_1}{\Delta t}$ reduced to standard conditions becomes

$$1.307 \times 667 \times 996 \div (924 \times 1000) = 0.943 \text{ mm. per sec.}$$

observations were made on each face of the disc. The black coating was always cleaned off from the face of the disc away from the light. All of the series of energy measurements are gathered together in Tables VII and VIII. In the tables, under the head "through air," the first column contains the observed rate of increase in the galvanometer deflection G_1, when the disc and its surroundings were at the same temperature; the second column, the corresponding mean lamp deflections of galvanometer G_2. The third and fourth columns contain the sensitiveness of galvanometers G_1 and G_2 respectively, and the last column the values of the first column reduced to standard lamp and standard sensitiveness of both instruments. The series on the two faces of the disc are recorded and averaged separately, then combined with their probable errors in the general average at the end of Table VIII.

Tables VII and VIII give the following results. The average increase in the reading of G_1 for standard conditions is 0.966 mm. per second. From the thermal calibration, a deflection of 9.96 divisions corresponds

TABLE VII. — FRONT FACE.

Date.	Through Air.					Through Red Glass.			Through Water Cell.		
	$\frac{\partial G_1}{\partial t}$.	G_2 (Lamp).	S_1.	S_2.	$\frac{\partial G_1}{\partial t}$ reduced to Standard.	$\frac{\partial G_1}{\partial t}$.	G_2.	$\frac{\partial G_1}{\partial t}$. reduced to Standard.	$\frac{\partial G_1}{\partial t}$.	G_2.	$\frac{\partial G_1}{\partial t}$ reduced.
Aug. 10	1.387	980	990	689	.965				.437	345	.864
" "	1.263	920	990	689	.936				.400	311	.877
" "									.369	279	.902
" 11	1.244	866	986	701	.992	.750	546	.950	.412	315	.905
" "	1.455	1010	986	701	.995	.750	546	.950	.510	382	.922
" "	1.505	1047	986	701	.994				.516	381	.935
" 16	1.447	1022	996	669	.942	.736	529	.927	.416	327	.873
" "	1.284	886	996	669	.966	.740	527	.936	.451	352	.853
" "	1.316	925	996	669	.948	.797	550	.965	.502	382	.875
" "	1.307	924	996	669	.943						
" 18	1.598	1110	995	667	.955	.738	515	.952	.449	333	.895
" "	1.550	1047	995	667	.984	.732	518	.940	.445	342	.865
" "	1.548	1031	995	667	.995	.730	518	.938	.451	346	.867
" "	1.410	957	995	667	.977						
" "	1.330	898	995	667	.983						
" 19	1.241	862	1001	675	.975	.760	532	.965	.451	343	.892
" "	1.360	934	1001	675	.985	.728	512	.960	.452	338	.904
" "	1.324	905	1001	675	.990	.738	525	.950	.466	351	.898
" "	1.364	934	1001	675	.988						
			Average . . .		0.973 ±0.003	Average . .		0.948 ±0.002	Average . .		0.888 ±0.004

TABLE VIII. — REAR FACE.

Date.	*Through Air.*					*Through Red Glass.*			*Through Water Cell.*		
	$\frac{\partial G_1}{\partial t}$.	G_2 (Lamp).	S_1.	S_2.	$\frac{\partial G_1}{\partial t}$ reduced to Standard.	$\frac{\partial G_1}{\partial t}$.	G_2.	$\frac{\partial G_1}{\partial t}$ reduced.	$\frac{\partial G_1}{\partial t}$.	G_2.	$\frac{\partial G_1}{\partial t}$ reduced.
Aug. 12	1.374	960	991	684	.970	.808	578	.949	.495	370	.906
" "	1.331	932	991	684	.968	.740	536	.935	.434	320	.919
" "	1.284	900	991	684	.967	.765	542	.957	.489	371	.895
" 15	1.428	992	996	670	.960	.703	506	.926	.490	368	.890
" "	1.428	984	996	670	.968	.742	526	.941	.466	352	.885
" "	1.531	1068	996	670	.962	.765	551	.926	.440	337	.873
" 20	1.477	1047	996	685	.961	.703	522	.918	.458	375	.833
" "	1.520	1090	996	685	.951	.760	537	.965	.497	400	.848
" "	1.576	1130	996	685	.951	.781	570	.935	.507	408	.848
" "	1.568	1124	996	685	.950						
" 21	1.783	1224	995	668	.970	.846	604	.932	.503	393	.852
" "	1.773	1232	995	668	.957	.790	575	.915	.481	377	.850
" "	1.705	1190	995	668	.953	.803	575	.930	.483	373	.862
" "	1.452	1019	995	668	.948						
		Average . . .			0.960 ± 0.0014	Average . .		0.936 ± 0.003	Average . .		0.872 ± 0.005
	Average of front and rear face, 0.966 ± 0.0034							0.942 ± 0.0036			0.880 ± 0.0064

to a temperature difference of 1° C. Consequently the rise in temperature of the silver disc per second when the light passed : —

(*a*) through air $= 0.966 \div 9.96 = (0^\circ.0970 \pm 0^\circ.00034)$ C.;

(*b*) through red glass $= 0.942 \div 9.96 = (0^\circ.0946 \pm 0^\circ.00036)$ C.;

(*c*) through water cell $= 0.880 \div 9.96 = (0^\circ.0884 \pm 0^\circ.00064)$ C.

The mass of the silver disc was 4.80 grams, its specific heat* at 18° C. = 0.0556 ; the mechanical equivalent of heat at 18° C. $= 4.272 \times 10^7$ ergs.† Consequently the energy of the standard radiation is

(*a*) through air, $0.0970 \times 4.80 \times 0.0556 \times 4.272 \times 10^7$

or $E_a = (1.108 \pm 0.004) \times 10^6$ ergs per second.

(*b*) through red glass, $E_g = (1.078 \pm 0.004) \times 10^6$ " " "

(*c*) through water cell, $E_w = (1.008 \pm 0.007) \times 10^6$ " " "

Reflecting Power of the Surfaces Used.

According to Maxwell, the pressure in dynes per square centimeter for normal incidence is equal to the energy in ergs in unit volume of the medium. The energy in unit volume is made up of both the direct and reflected beams. If E is the intensity of the incident beam and ρ the reflection coefficient, the pressure $p = \frac{E\,(1 + \rho)}{V}$, where V is the velocity of light. The methods for measuring p and E have already been described. The determination of ρ for both sides of the vanes C and D was made as follows. The supports of the torsion balance were replaced by the divided circular plate A (Fig. 7), of a force table which could be rotated about a central, vertical axis. The rod about which the plate turned passed up through the plate and at its top the mirror holder *b b* was fastened. The vanes were freshly silvered and mounted on a plate-glass carrier *a a*, which was held by a clamp against the back face of *b b*. The beam was directed on the vanes by the lens L_3 (Figs. 3 and 7) exactly as it had been in the pressure observations. After reflection from the vane the beam fell on a concave mirror M which projected an image of the vane upon a simple sheet bolometer B, forming the unknown resistance of a postoffice-box bridge. The current was supplied from storage cells and the galvanometer was the same used in the energy determinations but fitted with low resistance coils. The bolometer was covered by the bell-jar used earlier. The mirror M, the bell-jar and bolometer were attached to the plate of the force table. The full line diagram shows the arrangement for reflection. The dotted figure shows the position for a measurement of the direct beam. All measurements of direct reflection were made for an angle of incidence of 12°.5.

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* U. Behn, Ann Phys., IV. 266 (1900).

† Mean of Rowland's and Griffith's values, Phil. Trans., V. 184, 496 (1893).

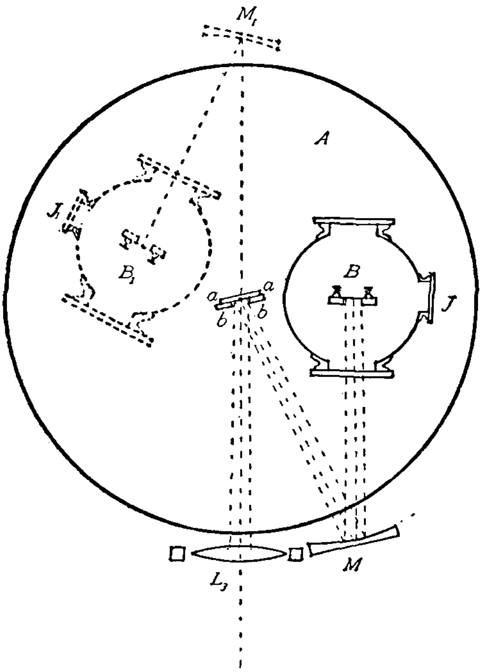

FIGURE 7.

The method of observing will be seen from the notebook record of a single series of measurements given in Table IX. In the table, D and R indicate direct and reflected beams, respectively. The first and second columns contain the zero points and end of swings of the galvanometer G_1, and the third column, the deflection. The remaining columns, in order, contain the lamp galvanometer deflection; the deflection of G_1 reduced to constant lamp; the means of each pair of D or R values; the means of alternate readings; and the final column, the quotients of the two preceding columns which are the reflection coefficients sought. In all, three series of measurements were made on the silver, and two series on the glass-silver faces of each vane. To get average coefficients which would represent the range of condition of the mirrors during the pressure measurements, the vanes were cleaned and new silver coatings deposited between each two series on the same vane. The reflection coefficients are collected in Table X. For each surface studied the diffused reflection for a beam which had traversed air was determined by setting the mirror holder for normal incidence. The diffuse energy reflected at an angle of 25° falling on the full aperture of the mirror M was measured, and the total diffuse energy for the hemisphere computed on the basis of the cosine law. If $I_\theta \, \partial A$ is the amount of diffuse radiation falling normally upon the area ∂A, distant r from the vane and at

TABLE IX.

OCTOBER 31, 1902. REFLECTION COEFFICIENT OF D_g. AIR.

G_1		Deflect. G_1	Lamp.	G_1 reduced to Standard	Averages.	Alternate Averages.	Reflect'n Coeffi't.
Zero.	Turning Point.						
R 350.0	159.5	190.5	132.6	143.8	142.5		
349.0	152.0	197.0	139.3	141.3			
D 349.5	100.5	249.0	136.8	182.1	182.5	142.0	.779
350.0	111.5	238.5	130.5	183.0			
R 346.0	177.0	169.0	119.3	141.2	141.5	182.5	.775
347.0	171.0	176.0	124.4	141.7			
D 348.5	123.0	225.5	124.0	181.8	182.4	141.1	.773
348.5	120.0	228.5	125.0	183.0			
R 345.0	172.0	173.0	122.6	141.0	140.6	182.7	.770
345.0	171.0	174.0	124.0	140.3			
D 346.0	132.0	214.0	115.5 ?		183.0	141.2	.773
346.0	124.0	221.0	120.7	183.0			
R 344.0	173.0	171.0	120.7	141.8	141.7	182.1	.778
344.5	171.0	173.5	122.6	141.6			
D 346.0	119.0	227.0	125.3	181.0	181.2	141.6	.781
346.0	117.5	228.5	126.0	181.3			
R 342.0	174.0	168.0	118.0	142.3	141.5	181.8	.780
342.0	170.5	171.5	122.0	140.8			
D 347.0	130.0	217.0	119.0	182.3	182.5	141.4	.775
347.0	134.0	213.0	116.7	182.7			
R 341.5	174.5	167.0	118.0	141.3	141.2		
341.0	173.0	168.0	119.0	141.1			
					Average		0.776

TABLE X.

Reflection Coefficients in Percentages.

	C_s.				C_g.			
	Through Air.	Red Glass.	Water.	Diffuse.	Through Air.	Red Glass.	Water.	Diffuse.
	92.8	94.5	88.9	0.98	77.8	75.9	80.8	
	89.8	90.8	86.0	0.92	77.6	76.6	80.0	1.6
	90.8			1.23				
Av'ge	91.1	92.7	87.5	1.04	77.7	76.3	80.4	1.6
	D_s.				D_g.			
	95.0	96.3	91.5	2.2	77.6	76.5	81.0	2.8
	92.0	94.0	90.4		76.7	75.2	79.7	2.2
	94.8	95.0	92.3	0.8				
Av'ge	93.9	95.1	91.4	1.5	77.2	75.9	80.4	2.5
	Average Reflection.							
	Air-Silver.				Glass-Silver.			
	92.5	93.9	89.5	1.3	77.5	76.1	80.4	2.0
	Corrected Reflection Coefficients.							
	92.0	93.4	89.0		77.6	76.2	80.5	

Average Coefficients through Air, *84.8;* Red Glass, *84.8;* Water, *84.8.*

an angle θ with the incident radiation, then $I_\theta \, \partial A = I_0 \cos \theta \, \partial A$. The total amount of diffuse radiation $= \int\int I_\theta \cos \theta \, \partial A$, over the surface of the hemisphere $= \int_0^{\frac{\pi}{2}} 2 \pi r^2 \, I_0 \cos \theta \sin \theta \, \partial \theta = \pi \, I_0 \, r^2$. This integral is the amount of the diffuse radiation in Table X. The force, due to radiation of intensity $I_\theta \, \partial A$, normal to the vane is $I_\theta \cos \theta \, \partial A$, and the total is equal to $\int_0^{\frac{\pi}{2}} 2 \pi r^2 \, I_0 \cos^2 \theta \sin \theta \, \partial \theta = \frac{2}{3} \pi \, I_0 \, r^2$. It is thus seen that of the diffuse reflection, two-thirds is effective as light pressure. This increases the air-silver reflection coefficients by 0.9 per cent and the glass-silver values by 1.3 per cent. The small glass rod *d* (Fig. 2), not present in the reflection measurements, decreased the reflecting area of the silvered surfaces in the pressure measurements by 1.54 per cent. The air-silver values are thus decreased by $0.92 \times 1.54 = 1.4$ per cent, and the glass-silver values by $0.78 \times 1.54 = 1.2$ per cent. The application of these two corrections gives the final corrected coefficients in Table X. The diffuse reflection of black coatings deposited by the method used in blackening the silver disc was measured and computed in the same manner as the diffused reflection from the vanes *C* and *D*. The agreement found by Ångström* between the diffuse reflection of matte surfaces for normal incidence and the cosine law was abundantly close for the present purpose. Five determinations of this reflection were made under different conditions and with different coatings. The values in percentages of the incident beam were 4.4 per cent, 4.5 per cent, 4.2 per cent, 4.6 per cent, and 5.2 per cent; average, 4.6 per cent. Thus only 95.4 per cent of the incident beam was absorbed by the black coating on the silver disc in producing the temperature increase observed. Hence the true energy of the beam is equal to the observed energy divided by 0.954.

The silver disc, diameter 13.3 mms., used in the energy measurements, received long waves and scattered radiation which passed round and through the light pressure vanes of diameter 12.8 mms. This amount was experimentally determined for both thin and thick silver coatings in order to approximate to the average condition of the coatings in the light pressure measurements and it was found to average (*a*) through air, 1.40 per cent; (*b*) through red glass, 1.44 per cent; (*c*) through water,

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* K. Ångström, Wied. Ann., XXVI. 271 (1885).

0.46 per cent. On this account the energy E of the standard radiation must be reduced by the above percentages. Applying these corrections[*] and the corrections due to the diffused radiation from the black coating on the silver disc, the energy of the standard radiation becomes

(*a*) through air, $$E_a \times \frac{0.986}{0.954};$$

(*b*) through red glass, $$E_g \times \frac{0.986}{0.954};$$

(*c*) through water, $$E_w \times \frac{0.995}{0.954}.$$

Hence the pressure produced by standard radiation calculated by Maxwell's formula, $p = \frac{E(1+\rho)}{3 \times 10^{10}}$, since $\rho = 0.848$, becomes

(*a*) through air $$= E_a \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954}$$
$$= 1.108 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954} \times 10^6 \text{ dynes}$$
$$= (7.05 \pm 0.03) \times 10^{-5} \text{ dynes};$$

(*b*) through red glass $$= E_g \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954}$$
$$= 1.078 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954} \times 10^6 \text{ dynes}$$
$$= (6.86 \pm 0.03) \times 10^{-5} \text{ dynes};$$

(*c*) through water $$= E_w \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.995}{0.954}$$
$$= 1.008 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.995}{0.954} \times 10^6 \text{ dynes}$$
$$= (6.48 \pm 0.04) \times 10^{-5} \text{ dynes}.$$

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[*] As the average pitch of the cone of the incident beam was about one part in forty, no correction need be applied for inclination. Furthermore, the inside of the bell-jar was blackened and the zero of the balance was so chosen that energy reflected from the window admitting the beam could produce no pressure effects.

A comparison of observed and computed pressures follows: —

	Observed values in 10^{-5} dynes.	Computed values in 10^{-5} dynes.	Obs.-comp. in percentages.
Through air,	$p = 7.01 \pm 0.02$ *	7.05 ± 0.03	-0.6
Through red glass,	$p = 6.94 \pm 0.02$	6.86 ± 0.03	$+1.1$
Through water,	$p = 6.52 \pm 0.03$	6.48 ± 0.04	-0.6

An estimate of the approximate magnitude of the gas action, not eliminated by the ballistic method of observation, may be reached from the following considerations.

When radiation falls upon a vane of the torsion balance, part of it is absorbed by the silver surface. From the amounts directly and diffusely reflected, as given in Table X, the amount transmitted by the average surface (experimentally determined but not given in Table X), the effect of the glass rod and the reflection coefficient of the glass surface, it was found that, when the silver side of the vane was toward the radiation source, the absorption coefficient for radiation through air was 6 per cent, and when the glass surface was forward, it was 18 per cent.

The total force acting on the vane is made up of two parts, that due to radiation pressure and that due to gas action. Let F_r be the force due to the first cause, assuming that all the radiation is absorbed, and F_g the effect due to the second, on the same condition. Then the total effect, when the silver side of the vane is forward and the radiation is "through air," is $1.92\ F_r + 0.06\ F_g$. When the glass side is forward the total effect is $1.776\ F_r - 0.18\ F_g$. Making these expressions equal to the reduced deflections (Table III, columns 11 and 12) on the silver and glass surfaces respectively, we have two equations by means of which the values of F_r and F_g may be obtained. Hence the effect due to gas action on each face of the vane is approximately determinate, as is

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* The pressure and energy measurements for the three different wave groups through air, red glass, and water cell, constitute three independent experiments. In the values for pressure, 7.01, 6.94, and 6.52, equality is not to be looked for. The difference arises from the different reflecting power of the 45° glass plate (Fig. 3) for the different beams and from the fact that the indications of the lamp galvanometer G_2 connected with the bolometer R, were probably not strictly proportional to energy for throws differing as widely as 33, 60, and 100, which, roughly, were the relative intensities of the beams through water cell, red glass, and air. The function of the lamp bolometer and galvanometer was purely to keep a check on the small variations of the lamp which rarely fluctuated more than 10 per cent on either side of the mean value.

also the part (0.06 F_g) not eliminated when we average the two columns to obtain column 13.

Applying this method to all the results of Table III (with the exception of those results taken with poor mirrors as shown by our notes), the gas action present in the ballistic deflections "through air" is 0.8 per cent. Applying the corresponding data and equations to Table IV, the gas action present in the red glass values is 1.1 per cent and in the water cell values, 0.3 per cent. The sign of F_g comes out negative, which means that the gas action was suction.

This reasoning assumes that the glass faces of the vanes during the six seconds exposure are not warmed by absorption nor by the conduction of heat through the thin glass from the silver coating. The effect of any such absorption or conduction would be to diminish the computed gas action. As estimated from the static observations, the gas action in the ballistic measurements is comparable in magnitude with the computed values obtained above, and of the same sign. Both results show that the uneliminated gas action by the most liberal estimate cannot have exceeded 1 per cent of the radiation pressure. Because of its smallness and indefiniteness no correction for gas action has been made to the final pressure values. If corrections were applied its effect would be to slightly reduce the observed pressures.

Aside from the measurements of pressure and energy for which the probable errors are given, the percentage accuracies in the other measurements entering into the computations, and their effects upon the final result follow: —

1. Quantities which affect individual series:

(*a*) Pressure values, —

Period of balance T,	accurate to	0.2% ;	effect on	result	0.0%
Lever arm of balance l,	"	0.1% ;	"	"	0.0%
Constant of galv'meter G_2,	"	0.5% ;	"	"	0.0%
Estimate of possible error due to changing ratio of period of G_2 to length of exposure of bolometer		0.4% ;	"	"	0.1%

(*b*) Energy values, —

Constant of galv'meter G_1,	"	0.1% ;	"	"	0.0%
" " G_2,	"	0.5% ;	"	"	0.0%

2. Quantities which affect final averages:

(*a*) Pressure values, —

Torsion of fibre,	accurate to	0.2% ;	effect on result	0.2%
Reducing factor, 1.357	"	0.1% ;	" "	0.1%
Reducing factor, 1.550 for G_2	"	0.2% ;	" "	0.2%
Reflection of surfaces of vanes	"	0.4% ;	" "	0.2%

(*b*) Energy values, —

Mass of silver disc	"	0.1% ;	" "	0.1%
Thermal calibration of disc	"	0.5% ;	" "	0.5%
Diffuse reflection black coating	"	5.0% ;	" "	0.1%

From the agreement within the probable error of the air, red glass, and water values with the theory, it appears that the radiation pressure depends only upon the intensity of the radiation and is independent of the wave length.

The Maxwell-Bartoli theory is thus quantitatively confirmed within the probable errors of observation.

To Professor J. L. Mann, and Messrs. J. A. Brown, Philip Fox, L. E. Woodman, H. R. Willard, H. E. K. Ruppel, and A. V. Ruggles, the writers are indebted for friendly assistance given at some of the stages in the protracted course of these experiments.

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THE HEREDITY OF ALBINISM.

By W. E. Castle and Glover M. Allen.

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I. INTRODUCTION.

THIS paper contains a preliminary statement of certain results of breeding experiments with mice, guinea-pigs, and rabbits, which have been conducted in the Zoölogical Laboratory of Harvard University during the last two and a half years. The experiments with mice are the work principally of the junior author; those with guinea-pigs and rabbits, of the senior author.

Albinism, or absence of the normal pigmentation of an organism, is a not infrequent phenomenon in both animals and plants, yet its occurrence in nature is sporadic and has usually been interpreted as an indication of organic weakness. But this interpretation is probably erroneous, for albino races of domesticated animals are apparently not inferior in vigor to other races. Such is demonstrably the case with albino mice.

The idea that albinos lack constitutional vigor may have its origin in the observation that, in crosses between albinos and normal individuals, no albino offspring are produced. But the disappearance of albinism in this case is not final. The albino character has not ceased to exist, but has merely become latent in the offspring. It will reappear unimpaired in the next generation if the cross-bred individuals be mated *inter se.*

II. COMPLETE ALBINISM A RECESSIVE CHARACTER.

The disappearance of the albino character for a generation, and its subsequent reappearance under close breeding, show that it is inherited in conformity with Mendel's law of heredity,* and that it is, in the terminology of that law, a *recessive* character.

(*a*) *In Mice.*

In mice this has been conclusively demonstrated by Cuénot (:**02**),† who, on crossing wild gray house-mice with albinos, obtained always gray mice indistinguishable in appearance from the pigmented parent. Yet these gray hybrids, when bred *inter se,* produced both gray and white offspring approximately in the Mendelian ratio, 3 : 1. The exact numbers recorded are 198 gray: 72 white, or 26.6 per cent albinos. According to Mendelian principles the grays of this second filial generation should consist in part of *pure* grays, which would not transmit the albino character, and in part of *hybrid* grays like their parents, — the first filial generation, — which would transmit alike the pigmented and the albino characters. This Cuénot demonstrated to be actually so, for certain pairs formed by random selection of the grays gave only gray offspring (189 individuals); the remaining pairs produced albino as well as gray offspring, and in the expected ratio, 3 grays : 1 albino.

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* A brief statement of Mendel's law has been made by one of us elsewhere (Castle, :**03**ᵃ). For a fuller exposition, see Bateson (:**02**), Bateson and Saunders (:**02**), de Vries (:**02**), or Correns (:**01**).

† In an earlier paper (Castle, :**03**ᵃ) the first published recognition of the recessive nature of albinism in mice is erroneously credited to Bateson (:**02**). The papers of Cuénot (:**02**, :**02**ᵃ), which at that time were unknown to us, apparently antedate Bateson's.

Crampe ('**85**) seems to have been the first to recognize clearly that the first cross between pigmented and unpigmented (albino) varieties gives rise, in the case of rats, to pigmented individuals, *not* to albinos. Crampe crossed gray, black, gray-white, and black rats (*Mus norvegicus*) with albinos, and found that in each cross the albino character disappeared. He noticed also that the albino character might reappear in subsequent generations, but he did not discover the conditions necessary for this reappearance beyond establishing that albino offspring were produced under close breeding by those pigmented rats only in whose immediate ancestry there had been a cross with the albino form.

The precise numbers recorded are 162 grays: 57 albinos, or 26 per cent albinos.

Cuénot's observations are fully substantiated by experiments performed by the junior author of this paper, a full account of which will be published elsewhere. Wild gray mice were crossed with albino mice, and the offspring, sixty-four in number, were all gray like the wild parent, though a single litter of three young, which died without attaining their full growth, were of a somewhat lighter gray than the wild parent. Certain of the cross-breds were paired together, and produced 66 offspring, 42 of which were gray, 24 white. This is a considerable deviation from the expected ratio, 3 : 1, but it should be remembered that the total number is relatively small. The result is of the *nature* expected, in that both gray and albino offspring are produced, and of the former a larger number than of the latter.

To determine whether the grays are, as expected, of two sorts, one hybrid, the other pure, six pigmented individuals have been crossed with the parental white stock. Three of the six have thus far produced only pigmented offspring, indicating that they are pure; the other three have produced both gray offspring and white offspring, showing that they are hybrids. The two sorts of offspring produced in the case last mentioned should, according to Mendelian expectation, be equally numerous.

The numbers thus far recorded are 35 pigmented individuals: 23 albinos, a result agreeing with expectation in that both gray and white offspring are produced, though these are not in the exact proportions demanded by Mendel's laws.

If we *combine* the results of this cross with those obtained by interbreeding hybrids of the first filial generation, we get for the whole a close agreement between expectation and observation. The expectation is 78.5 gray : 45.5 white; the observed result is 77 gray : 47 white.

White mice obtained by one or the other of these crosses have repeatedly been bred together, but without the occurrence of a single exception to the expected Mendelian result, the offspring being invariably albinos.

A further test of the Mendelian hypothesis as applied to albinism in mice was made by Cuénot. By back-crossing hybrid grays with the ancestral white stock he obtained gray as well as white individuals, which in the phraseology of breeders should be $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, etc., white "blood," yet all the grays, irrespective of ancestry, gave precisely similar results in crosses with whites, viz. equal numbers of gray and white offspring.

It is evident, then, that when a pure gray race of mice is crossed with a pure white race, the gray character invariably dominates in the offspring, and in subsequent generations both gray individuals and white individuals occur approximately in the proportions demanded by Mendel's principles of dominance and segregation. No better illustration of Mendel's law has yet been produced than is afforded by the cross between gray and white mice.

(*b*) *In Other Mammals, in Fishes, and in Plants.*

In the case of guinea-pigs, we have many times mated together albinos born of mottled parents, or obtained by mating a mottled with a white animal, but never with any but the expected Mendelian result, all the young being albinos.

In the case of rabbits, the same law appears to hold. Professor R. T. Jackson kindly placed at our disposal last summer three white rabbits, a male and two females, all born in the same litter, of spotted parentage. The two females have borne by their brother, in three litters, seventeen young, all albinos.

In man, Farrabee (:03) and Castle (:03) have recently shown albinism to be in all probability recessive.

As to fishes, Dr. Hugh M. Smith, of the United States Fish Commission, informs us that in one of the State fish-hatcheries of this country there is bred as a curiosity a race of albino trout which "*breed true*," indicating that the albino character is recessive.

In plants, more than two-thirds of the Mendelian cases mentioned by de Vries (:02, p. 146) are cases of "depigmentation" of flowers or fruit, the depigmented condition being invariably recessive in crosses with the normal condition.

It appears, then, that in organisms in general, albinism behaves as a recessive character in heredity.*

III. Partial Albinism a Mosaic of Dominant and Recessive Characters, and a Unit in Heredity.

Darbishire (:02) finds that in crosses between a peculiar race of partial albino mice and true albinos, the albinism does not entirely disappear in the offspring, and he thinks that this weighs heavily against the entire

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* The only exception known to the writers is the dominance of white plumage, in certain crosses of poultry, as recorded by Bateson and Saunders (: 02). Yet the dominant character in this case is one of *partial* albinism only, and its dominance is not invariable. We suspect that the dominance of white plumage results from its *coupling* in the gametes with some other character strongly dominant by nature.

Mendelian hypothesis. In reality Darbishire's observations, when rightly interpreted, afford strong evidence in favor of that hypothesis. It may be well, therefore, to examine them with some care. But before doing so one or two earlier observations should be noticed.

Haacke ('95) crossed spotted blue-[black-]and-white Japanese dancing mice with albino mice, and obtained offspring uniformly gray in color, like the wild house-mouse, or uniformly black. Occasionally, however, one of the gray or black offspring bore a fleck of white on forehead or belly.

Von Guaita ('98, :00) repeated the experiment, crossing spotted black-and-white Japanese dancing mice with an inbred stock of albinos. He obtained twenty-eight young, all uniformly gray like the house-mouse. These gray mice bred *inter se* yielded in subsequent generations gray, gray-white, black, black-white, and white offspring.

Darbishire's experiment consisted in crossing albino mice with a peculiar race of Japanese dancing mice which had *pink eyes* and were uniformly white except for patches of pale fawn-color on the cheeks, shoulders, and rump. The dancing mice had been tested and found to breed true *inter se*. From the cross between these partial albinos and true albinos forty-eight young were obtained, of which all except two were marked more or less extensively with *gray;* those two were *fawn-color all over* except on the belly, where — we infer from Darbishire's likening them to certain of his gray mice — they were either of a *lighter* fawn-color or else white.

Of the forty-six young which were marked with gray, fifteen were gray all over except on the belly and tail, where they are said to have been "nearly white." What is meant by this expression we do not know, unless it be a light shade of gray. If this is the author's meaning, then the fifteen mice were really gray all over. The wild house-mouse itself is often lighter colored on the belly and tail, and these fifteen individuals must be regarded as substantially complete reversions to the gray pigmented type of the wild house-mouse, a type radically different from that of either parent. This result agrees with that obtained by Haacke ('95), von Guaita ('98, :00), and ourselves, upon crossing spotted with albino mice. In thirteen of the forty-six gray-marked mice obtained by Darbishire, the gray covered much more of the body than did the fawn in the Japanese parent; while in eighteen others the gray corresponded roughly in distribution with that of fawn in the Japanese parent. In no case is the gray described as being less extensive than the fawn in the Japanese parent, and *no mention is made of any hybrid* *having pink eyes,* though both parents had unpigmented eyes.* Not only were the offspring *not intermediate* in pigmentation between the two parents, as we should expect on any hypothesis other than the Mendelian one, but *not a single mouse in the forty-eight contained as little pigment as was found in the pigmented parent.* In every case the pigmentation was greatly increased over what it had been in the pigmented parent, either in intensity or in extent, and usually in *both* respects. *Not a single albino was produced,* though the mother had been in every case an albino. Yet Darbishire maintains that the case is not one of Mendelian dominance!

It is hard to imagine a plainer case of Mendelian dominance than this, — a spotted mouse is bred to a white mouse; the offspring are all spotted, none white. The spotted character is plainly dominant, the white, recessive! What would Darbishire require to satisfy him that the case is a Mendelian one? Offspring all like *neither* parent, but like a third form, a supposed ancestral form. But is this simple dominance? No, it is dominance associated with something else, reversion, i. e., the coming again into activity of a character long latent, in this case the formation of black pigment. Because the *reversion* is not in every case complete, Darbishire maintains there is no *dominance,* a conclusion utterly fallacious.

What is the true explanation of cases of heredity like that observed by Darbishire? The explanation has been given elsewhere by one of us (Castle, :03[a]), but it may be well to repeat it here. Albinism is in mice and other mammals a character recessive in relation to the pigmented condition. But the spotted mouse is not a simple dominant (pigmented) individual as contrasted with simple recessive (albino) individuals. The spotted mouse is a peculiar kind of individual, in which are found both the dominant and the recessive characters, yet not in their usual relationship (one latent, the other alone visible), but *both* visible side by side, in distinct areas of the animal's body. Such an individual is called a *mosaic.* In the soma of a mosaic individual the law of dominance is suspended, but cross-breeding serves usually to bring it again into operation in the next generation, for the crossing of a mosaic with a normal, or with a recessive individual, results usually in the production of normally pigmented individuals only. Thus when spotted rats or mice are crossed either with gray individuals (dominants) or with albinos (recessives), the offspring are commonly all gray or black in color, none spotted. Here we find that dominant and recessive characters have abandoned the balanced relationship which they had in the spotted parent, and returned to the ordinary relationship of a dominant to a recessive character.

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* This significant omission was observed by Bateson also, who mentions it in a letter received since the above paragraph was written.

Just as in mosaic individuals the law of dominance is suspended, so, too, in the formation of their gametes the law of segregation is commonly suspended also. The gametes, as well as the soma, of a mosaic individual are commonly mosaic, containing side by side the dominant and the recessive characters. For it is evident from the experiments of Haacke, von Guaita, and Darbishire that the spotted mice employed by them in crosses with albino mice did not form any gametes containing *only* the recessive (albino) character, otherwise albino offspring would have been produced, but *not one* was produced. Hence segregation of the pigment-forming character from the albino character cannot have occurred at the formation of gametes in these several cases, but the gametes themselves must have possessed a mosaic (or else a dominant) character.

A similar explanation must be made of certain results obtained by one of us in crossing black-white (not dancing) mice with albino mice. Two spotted males of a black-white stock which bred true *inter se*, were crossed with albinos. The offspring were like those obtained by Haacke in crossing Japanese dancing mice with albinos, namely, mice uniformly gray or black in color, though sometimes with a fleck of white on the belly, or with one or more white bands on the tail. Of the forty-three young produced by this cross, twenty-eight were gray, and fifteen black. The fact that no albino offspring were produced shows that the spotted males formed no recessive gametes, but only those which were either mosaic, or else purely dominant, in character. But if dominant gametes had been formed by segregation from a dominant-recessive mosaic, we should expect that by a residual process *recessive* gametes would be formed also. The latter not having been formed, it is safe to suppose that the former were not formed either, but that *all* the gametes formed by these spotted males were *mosaic*.

That the albino character entered as a latent constituent into the gray or black hybrids formed by the cross just described, is shown conclusively by the character of their offspring. When bred *inter se* they produced albino as well as pigmented offspring, and approximately in the ratio, 1 : 3; or, when bred to albinos, in the ratio, 1 : 1.

Accordingly, in the original cross described, we have a case of simple dominance of the pigment-forming over the albino character. This dominance is attended, in about two cases out of three, by reversion to the particular form of pigmentation found in the wild house-mouse. The reversion is due to the coming into activity of a capacity (previously latent) to form *yellow* pigment, which with black pigment forms gray. This latent capacity must have been present in one or the other or possibly in *both* of the parents crossed.

Darbishire's results differ from ours only in degree, not in kind. He, too, gets invariably dominance of the pigment-forming over the albino character, and this is associated in all except two out of forty-eight cases with reversion to the ancestral *kind* of pigmentation, gray. The only differences between his results and our own are as follows: —

1. In our experiments *yellow* was the latent constituent of gray which was brought into activity by a cross with albinos; in the experiments of Darbishire *black* was the latent constituent brought into activity.

2. In our experiments white disappeared for the most part from the bodies of the hybrids; but in Darbishire's experiments the disappearance of white was much less complete. There was a strong tendency for the mosaic gamete to *dominate as a unit* without serious disturbance of the balanced relationship of pigmented and unpigmented areas in the mosaic structure. This tendency is observable in thirty-one out of forty-eight cases. The remaining seventeen cases are strictly comparable with our own. In fifteen of them the black character, latent in one or possibly in both parents, has become active and, combining with yellow (fawn), visible in one parent, has formed gray; in the other two offspring, black, if present, has remained latent, leaving the individuals fawn-colored. This result is comparable with the production of *black* hybrids in our own experiments.

In Darbishire's experiments, as in our own, the effect of a cross with albinos is to release the dominant character from the strict localization which it had in the mosaic parent. In Darbishire's mosaic mice the localization of pigment was much more rigid than in our own. His mice bore pigment only on the shoulders and rump, and had *pink* eyes; ours were pigmented over at least half of the body and had *black* eyes. It is not surprising, then, that the pigmentation should be less extensive in Darbishire's hybrid mice than in our own. Yet it is evident that in all his hybrids there occurred release, more or less complete, of the pigment-forming character from its localization in the original mosaic. In every case, apparently, the hybrid had pigmented eyes, though neither parent possessed this character.

IV. Pure and Hybrid Mosaics.

In our experiments in crossing spotted with albino mice, a third black-white male was employed. He was a half-brother[*] to the two spotted males previously mentioned, born of the same mother but by a different sire. The white areas on his body were less extensive than those on his two brothers. He was bred to the same stock of white females as they, but with different results. By him the albinos bore albino as well as pigmented offspring; of the former twenty-one, of the latter twelve, ten being gray and two black. The albino offspring were found in this case, as in all others tested, to breed true *inter se.*

It is evident that the third black-white male differed in nature from his two half-brothers, for he formed *recessive* gametes, whereas they did not.

Examination of other breeding records of spotted mice kept by us during the past two years shows that it is possible in these also to distinguish two different sorts of mosaic individuals. These are: —

1. *Pure mosaics*, spotted and forming only mosaic gametes, DR. They bred true *inter se*, but when crossed with albinos produce only individuals of the following class.

2. *Hybrid mosaics*, usually *more extensively pigmented than pure mosaics*, often pigmented all over. (Compare the results of Haacke, von Guaita, and Darbishire.) They form, in approximately equal numbers, mosaic and recessive gametes, DR and R respectively. Accordingly, when bred *inter se*, they produce offspring of three different sorts, DR, $DR \cdot (R)$ [or $D(R) \cdot (R)$],[†] and R, that is, pure mosaics, hybrid mosaics, and recessives. Theoretically these three classes of offspring should be numerically as 1 : 2 : 1. There is reason to believe that these proportions are approximated in our experiments, but this matter has not yet been fully tested. When bred to albinos, hybrid mosaics produce (in equal numbers?) hybrid mosaic and albino offspring.

It is possible to recognize in the experiments of von Guaita also these two different classes of mosaics. Von Guaita's original stock of dancing mice consisted of *pure* mosaics, for they bred true *inter se* and when bred to albinos produced only pigmented offspring. These offspring, though not themselves spotted, were clearly *hybrid mosaics*, for when bred *inter se* they produced spotted as well as albino offspring, of the former nine, of the latter fourteen, in a total of forty-four young; the expectation on our hypothesis is eleven of each!

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[*] In an earlier paper (Castle, :**03**, p. 542) it is stated erroneously that this mouse was an *own* brother to the other two.

[†] The period is used to indicate the distinctness, in the zygote, of the gametes which united to form that zygote, as well as to show that, when in the hybrid individual segregation of characters takes place at gamete formation, splitting will occur at the point marked by the period, producing gametes DR and R.

The idea that there are, as just explained, two different sorts of mosaic individuals, originated with the junior author of this paper. It constitutes a discovery of no small importance, one which lends strong support to the Mendelian hypothesis of essential gametic purity. It shows that alternative parental characters, when united in fertilization, do not mix, but that each retains its own identity and subsequently separates from the other when gametes are formed. This takes place *even when one of the parental characters is itself a mosaic!* Herein we have a confirmation of the conclusion based upon morphological observations, that the paternal and maternal contributions to the zygote retain each a distinct individuality. Further, the idea of Bateson receives confirmation, that the gray color of mice obtained by crossing black-white with white mice is itself a " heterozygote " character.

Darbishire's premature conclusion, that in his spotted hybrids " albinism is not recessive," will undoubtedly be abandoned by him when he has reared from them a second generation of hybrids. From our own experiments and those of von Guaita we confidently predict that he will obtain approximately one in four of albino mice, and of the pigmented mice obtained, part will be *pure mosaics* (as defined by us), but a larger part will be *hybrid mosaics* like their parents.*

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* This prediction has been fulfilled sooner than we had expected. In a paper just received Darbishire (:03) states that his hybrid mice bred *inter se* have produced 66 young. Of these 13 are albinos, and 17 are (in our terminology) pink-eyed mosaics, while the remaining 36 have dark eyes. The last named class consists clearly of *hybrid* mosaics like their parents. The Mendelian expectation is that half the offspring, or 33, will be of this sort; the observed number is 36. The other two classes (albinos and pure mosaics) should theoretically number 16.5 each, or together 33; they number 13 and 17 respectively, or together 30.

The correctness of this interpretation of Darbishire's results can easily be tested by further breeding of his animals. The pink-eyed individuals, if really pure mosaics, as we suppose, will, when bred to albino individuals, produce only dark-eyed offspring; whereas their dark-eyed brothers and sisters when similarly mated will produce both dark-eyed individuals (like themselves) and albinos, in approximately equal numbers. Further, the pink-eyed individuals will probably breed true *inter se*, whereas dark-eyed individuals will continue to produce in subsequent generations, as in the case already observed by Darbishire, offspring of three sorts,

V. Pure and Impure Recessives.

Although Darbishire's interpretation of his results is clearly unsound, he has made an observation of great importance, both theoretical and practical. In his crosses with dancing mice, he employed two different stocks of white mice, and with results somewhat different in the two cases. The two stocks were (1) "pure-bred" albinos purchased from breeders, and presumably descended from white parents, and (2) "cross-bred" albinos known to be descended from spotted [i. e. hybrid mosaic] parents. The latter, when crossed (four different pairs) with the fawn-white dancing mice, produced only spotted gray-white offspring, nineteen in number. The former, when crossed in the same manner (five different pairs), also produced gray-white mice, twelve in number, but produced in addition fifteen gray mice (with lighter bellies and tail) and two

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namely, albinos, dark-eyed individuals, and pink-eyed individuals, approximately in the proportions, 1 : 2 : 1.

The numbers of first- and second-generation hybrids which have been reared by Darbishire are now considerable. They show conclusively that both albinism and the dancing character are recessive in relation to the normal conditions. The original cross between pink-eyed dancing mice and albinos has yielded 203 offspring, all dark-eyed and with bodies more or less extensively pigmented; none dance. Certain of these hybrids bred to albinos have produced 205 young, of which 111 are albinos; the Mendelian expectation in this case is 102.5 albinos. The remaining (pigmented) offspring of this cross are all dark-eyed like their hybrid parent, and none dance; this is precisely the Mendelian expectation.

First-generation hybrids bred *inter se* have yielded both albinos and dancing individuals. The Mendelian expectation is one in four of either sort. The observed number of albinos is, as already stated, 13 in a total of 66; the number of dancers in the entire 66 is not stated, but we are told that, in 37 mice of this generation, 8 were dancers; the Mendelian expectation is 9.

Darbishire grants that these facts are "in possible accordance with some form of Mendelian hypothesis," but holds that "the behavior of eye-color is in every respect discordant with Mendel's results." The latter conclusion he reaches only by first assuming that "the possession of pink eyes must on Mendel's view depend on a separate embryonic element from that which determines coat-color." This is a wholly unnecessary assumption. The pigmented areas of the eye are morphologically and (as far as heredity is concerned) also physiologically parts of the general integument. A pink eye is simply an eye devoid of pigment; it represents an unpigmented area of the integument, and is no more a distinct element in heredity than is an unpigmented (white) spot on the side or tail of the animal. In an earlier paper (Castle, :**03**^{a}, p. 543) one of us has likened a white animal with dark eyes to a white animal with dark extremities, both are essentially mosaics of the dominant and recessive characters. On this view the observed inheritance of eye-color is in every respect *accordant* with Mendel's results.

fawn-colored mice (likewise lighter below). The two fawn-colored mice were borne by a single "pure-bred" mother. Gray mice were borne by all five "pure-bred" mothers.

This result indicates that *not all albinos breed alike* when crossed with the same pigmented stock, a conclusion which our own experiments fully substantiate. Darbishire's white mice clearly show individual differences in the way in which they breed. These differences are even more striking among the "pure-bred" than among the "cross-bred" mothers, doubtless because the former were obtained from different breeders, whereas the latter all came from one source.

Darbishire is entirely right in concluding that the ancestry of white mice does "make a difference" in their breeding capacity, but he is certainly wrong when he surmises that "the more in-bred an albino is the less power it has of transmitting its whiteness," unless he is willing to limit this statement to the *first hybrid* generation, which, however, he has not done. The truth probably is that in crosses between albino and spotted races, *reversion* to the ancestral form of pigmentation is more complete the *purer* the white stock is. But the ability of the white parent to transmit its whiteness to generations other than the first is certainly not diminished by inbreeding. For, while von Guaita employed a stock of white mice which had been inbred for many generations, and which showed in consequence a considerably diminished fertility, the white stock used by us gave no indications of extensive inbreeding; and yet in the experiments of von Guaita, as in our own, complete albinism is inherited in slightly more than the proportions demanded by Mendelian principles. The experiments of Cuénot also show a slight excess of albinos over expectation.

Inbreeding, then, does not affect the inheritance of complete albinism in crosses; whether it affects the character of the pigmentation of the hybrid individuals formed is an entirely different question, one which can be tested only by the use of albino individuals resulting from a cross between unrelated pure albino stocks. Such a cross should serve to counteract, at least in part, the effect of any previous inbreeding either in one or in both albino stocks. Our present opinion is that *purity* of the albino stock, rather than inbredness, is of consequence in determining the extent of reversion to the primitive gray pigmentation in the primary cross with mosaic individuals; but further experiments are needed to settle this point.

In the foregoing pages we have used such expressions as *impure albino* and *impure recessive,* expressions which seem incompatible with the hypothesis of gametic purity, and which therefore require justification. Construed in the strictest sense, the doctrine of gametic purity is untenable. We cannot accept that interpretation of it which requires that the gametes formed by an individual be the *precise equivalents* in all respects of the respective gametes which united to form that individual. Mendel himself would not have assented to such an interpretation, for in the latter part of his original paper ('**66**) he clearly states the important principle that a composite character may undergo resolution into its elements in consequence of crossing. This allows a *part* of a complex character to pass into one gamete, while the remaining parts pass into another; in other words, it makes possible the formation of *mosaic gametes*, into whose composition the dominant and recessive characters may *both* enter in varying degree. All gametes which contain *any portion* of a dominant or of a recessive character associated with its opposite, are in reality *mosaic*; yet, if a gamete essentially recessive contains only *traces* of the dominant character, it may be convenient to recognize this fact in its designation, which we do by calling it an *impure* recessive.

In guinea-pigs the impurity of recessives tainted with the dominant character is commonly visible. Ordinary white guinea-pigs with pink eyes, though they invariably produce albinos when bred *inter se*, have a greater or less amount of sooty black pigment in the skin and hair of their ears, nose, and feet, showing the presence of a trace of the dominant character. Rarely is it possible to obtain an animal free from this visible taint, and even when obtained, we are informed by breeders, such individuals are likely to produce offspring with a certain amount of pigment on their ears or feet.

The so-called Himalayan rabbit is another illustration of a mosaic with a predominantly recessive (albino) character, in which the dominant (pigment-forming) character is localized precisely as in the impure albino guinea-pig, namely, at the extremities. Himalayan rabbits have brownish-black noses, ears, feet, and tails, being elsewhere snowy white and having pink eyes. They breed true *inter se*, yet, according to Darwin ('**76**, p. 114), may occasionally produce a silver-gray animal, in which the pigment is not restricted to the extremities. This condition must result from *liberation* of the dominant character from the strict localization which it has in ordinary individuals and which it must have also in the gametes that produce them.

In mice, on the other hand, impurity in recessive individuals is not visible, though doubtless sometimes present. So long as the breeder wishes only to obtain white mice, it makes no difference what the ancestry of his breeding animals is. All albinos alike will produce only white offspring when bred to albinos. But if the breeder desires to cross his albinos with colored mice, the pedigree of the former is of consequence. Different albinos will, in crosses with the same pigmented stock, yield different results. This is shown both by Darbishire's experiments and by our own. In October, 1900, we began a breeding experiment in which a family of black-white mice was crossed with two different stocks of albino mice. All three stocks bred true among themselves; but, in crosses with the black-whites, one albino stock produced only gray or black offspring, whereas the other produced *no gray* offspring, but only black or fawn-colored ones, often extensively spotted with white. Manifestly the gametes formed by the two albino stocks, though all predominantly recessive, were not all alike. It is probable that some of them at least were impure, containing traces of a latent pigment-forming character. Such a latent character is apparently not liberated, in the case of mice, by a cross with a different stock of albinos; but this result can be secured, probably, by a cross with dominants. We infer this not only from the observed result in crosses between black-white and albino mice, but also from what has been observed to take place in guinea-pigs. On crossing a "dark-pointed" albino guinea-pig with a stock of red guinea-pigs which for a number of generations had bred true *inter se*, there were obtained offspring which in every instance were predominantly *black* in color, yet with a certain proportion of red hairs mixed with the black, which gave them a "brindle" or finely mottled black-and-red appearance. This result must be attributed to a liberation of the black-pigment-forming character either from its visible, strict localization in the albino parent, or from a possible latent and invisible occurrence in the red parent. We incline at present toward the former explanation, but the matter has not yet been fully tested. The complete disappearance of the albino character in this cross is noteworthy as being parallel to its behavior in the cross between black-white and white mice.

Darbishire's pink-eyed, fawn-white dancing mice were mosaics predominantly recessive, and might with some propriety be designated *impure* recessives, but they differed from impure guinea-pig recessives in that, when crossed with ordinary recessives, they did not produce pink-eyed animals like themselves, but rather animals which were in a majority of cases extensively pigmented. It seems more appropriate, therefore, to designate them *mosaics*.

"Dutch-marked" varieties of guinea-pigs, rabbits, and mice, and the somewhat similarly marked Holstein and Hereford cattle, though they do not breed so true *inter se* as dark-pointed albino guinea-pigs or Himalayan rabbits or, perhaps, as pink-eyed dancing mice, nevertheless indicate a fairly precise localization of the pigment-forming and albino characters within mosaic germs.

If we adopt the Roux-Weismannian idea of the nature of the chromosomes, it is probable that particular chromosomes, or part chromosomes, in the mosaic germ, contain the dominant character, while the remaining chromosomes, or part chromosomes, contain the recessive character.

VI. Cross-breeding, Reversion, and the Doctrine of Gametic Purity.

Union with a recessive gamete usually, though not always, serves to break up this localization, allowing the dominant character to extend its influence throughout the entire body. This is the case, for example, in the cross between spotted and white mice in the experiments of Haacke, von Guaita, and in part of Darbishire, as well as in our own. It is possible to suppose in such cases either (1) that the resolving effect of the cross is restricted to the soma of the cross-bred, or (2) that it extends also to the germ-cells of the cross-bred. If the former hypothesis is correct, the cross-bred should form gametes $D\,R$ and R in equal numbers; if the latter, then only gametes D and R should be formed, and these in equal numbers. A simple test is afforded by the breeding *inter se* of hybrid mice produced by crossing pure mosaics with recessives. On the first hypothesis suggested, the offspring of the hybrids should consist, in at least one case out of four, of *spotted* mice formed by the union of two pure $D\,R$ gametes; on the second hypothesis *no spotted* mice should be produced, but only classes D, $D(R)$, and R, as in breeding together hybrids between wild gray mice and white mice. Von Guaita's experiments show the formation at the second filial generation of *nine spotted mice*, twenty-one uniformly gray or black mice, and fourteen white mice. The expectation on hypothesis (1) is 11 spotted : 22 gray or black : 11 white mice, which approximates closely the observed result; whereas on hypothesis (2) there should be *no* spotted mice, but only such as are pigmented all over or else are albinos. The result is conclusive in favor of hypothesis (1) — that a mosaic gamete, on uniting in fertilization with a recessive gamete, does not lose its own identity nor undergo resolution into its constituent parts.*

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* Compare the results of Darbishire, as stated on p. 612, footnote.

Yet we must not fail to observe that a cross of the sort just described is not without its effects on the nature of the gametes; these do not retain their original character. For whereas the mosaic gametes of the original spotted parents produced, on union in pairs, invariably black-white offspring, the mosaic gametes formed by their hybrid offspring, when similarly combined, formed in von Guaita's experiments eight *gray*-white offspring, but only one *black*-white. Accordingly, though it seems certain that mosaic gametes may in crosses retain their mosaic character, the cross is nevertheless able to bring into activity characters latent in the parents, and to add these to the previous visible total of the mosaic, giving it thus a new character. In the original black-white stock, the mosaic consisted of the active characters, black and white, while the character yellow was latent, either in the mosaic gametes or in the recessive gametes with which they united when the cross was made. The cross brought at once into activity the latent character, yellow; and this combined with black to form the composite dominant character, *gray*, while white, though present in *both* gametes uniting, usually became for the time being altogether latent. But the gametes formed by the gray hybrid were not, as we should expect on the principle of strict gametic purity, *black*-white and white, but *gray*-white and white respectively. The character yellow, latent previous to the cross, having once become active remained so.

By this experiment we are put in possession of a principle of great importance, both theoretical and practical. It modifies essentially the Mendelian doctrine of gametic purity as commonly understood, yet without denying the soundness of that doctrine at core. It allows the breeder (as breeders habitually do) to reap substantial benefit from crosses, for in addition to permitting him to secure new combinations of the elementary characters visible in the parents crossed, it places at his disposal characters latent in the parents, and particularly facilitates the re-acquisition of lost characters.

The gray of hybrid mice obtained as in von Guaita's experiments is a composite character resulting from the combination of visible black with latent yellow. In Darbishire's experiments it results from the combination of visible yellow with latent black. In either case gray is obtained by synthesis (Bateson) of black and yellow. This view is supported by the observation of the reverse of this process, in crossing wild gray with white mice. In the second and later hybrid generations *black* pigmented as well as gray pigmented mice are obtained. These must result from a resolution of gray into its constituent elements, black and yellow, of which the latter then becomes latent.

It is not necessary to suppose, as Mendel apparently did, that the segregated elements of a composite character pass invariably into different gametes. There is reason to believe that yellow is frequently, if not always, latent in black mice, and black in yellow mice, though such an occurrence has not yet been conclusively demonstrated.

We would not, however, deny the possible correctness of Mendel's explanation in other cases. This should be indicated in breeding experiments by the simultaneous appearance in different individuals of the segregated elements of the composite character, and in particular numerical proportions. Each new sort should be incapable of producing the other, under either close or cross breeding. This is a subject well meriting more careful investigation.

VII. SUMMARY.

1. Complete albinism, without a recorded exception, behaves as a recessive character in heredity.

2. Partial albinism is a mosaic condition in which the dominant pigment-forming character and the recessive albino character are visible in different parts of the same individual.

3. Pure mosaic individuals form only gametes which partake of their own mosaic nature. They and their gametes may be designated $D\,R$.

4. Hybrid mosaics result from the union of a mosaic gamete, $D\,R$, with a recessive gamete, R, as in a cross between a pure mosaic and a recessive individual. Such crosses are made when a race of spotted mice which breeds true is crossed with albinos.

5. Hybrid mosaic mice are usually more extensively pigmented than pure mosaics, frequently they are pigmented all over. When they are spotted with white, we may consider this the result of dominance of the mosaic gamete *as a unit* and may designate them accordingly, $D\,R \cdot (R)$. When they are pigmented all over, it is clear that the *dominant element* only of the mosaic gamete is visible in them. They should then be designated $D(R) \cdot (R)$.

6. The gametes formed by a hybrid mosaic are of two sorts, like those which united to produce it, namely, $D\,R$ and R. Accordingly when hybrid mosaic individuals breed together, they produce offspring of three different sorts, — pure mosaics, hybrid mosaics, and recessives. We should expect these three classes to be numerically as 1 : 2 : 1, and this is probably the case. When a hybrid mosaic is bred to a recessive, two sorts of offspring are produced, — hybrid mosaics and recessives. These classes should be of approximately equal size.

7. Albinism apparently complete may in reality conceal traces of the pigment-forming character either in an active or in a latent condition. Albinos thus constituted are in reality mosaics of the contrasted characters, but with the pigment-forming character (ordinarily dominant) occurring in a condition of partial or complete latency. When bred to other albinos they uniformly produce albinos, hence they may for convenience be distinguished as *impure* recessives. In guinea-pigs and rabbits the impurity of recessive individuals is, in certain cases at least, visible; in mice it apparently is not.

8. Cross-breeding is able to bring into activity latent characters or *latent elements* of a complex character. This is probably the true explanation of many cases of reversion. Conversely it is able to cause one or another element of a complex character to *become latent* and to *remain* so under close breeding. This principle probably explains how races of black or yellow mice may be obtained by crossing wild gray mice with albinos.

9. The Mendelian doctrine of gametic purity is fully substantiated by experiments in breeding mice, guinea-pigs, and rabbits, but with the important qualification stated under 8, a qualification which really enhances the practical utility of that doctrine in its every-day application by breeders.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
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DIFFUSION AND SUPERSATURATION IN GELATINE.

By Harry W. Morse and George W. Pierce.

DIFFUSION AND SUPERSATURATION IN GELATINE.

By Harry W. Morse and George W. Pierce.

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I. Introduction.

The German photographer, R. Ed. Liesegang, in the course of a research on chemical reactions in gelatine,* discovered a phenomenon which Ostwald cites as evidence that there exists a definite limit beyond which, at a given temperature and pressure, the supersaturation of a solution cannot be carried. We have undertaken an experimental study of this subject by a method based on Liesegang's discovery.

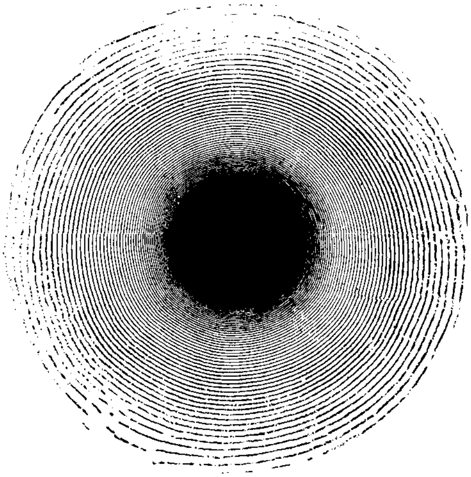
Figure 1.

Liesegang's experiment was as follows: A glass plate was covered with gelatine impregnated with potassium chromate. The plate was laid horizontal, and upon it a drop of silver nitrate was placed. The silver nitrate diffused slowly out into the gelatine, reacted with the potassium chromate, and formed a precipitate of silver chromate. The silver chromate, instead of growing continuously in the gelatine, as diffusion of the reacting substances went on, formed in distinct *rings* about the drop, as shown in Figure 1. The formation of the precipitate in distinct rings is clearly a phenomenon of supersaturation. We have undertaken certain measurements of these rings in the hope of being able to obtain from them some understanding of the conditions that exist in supersaturated solutions. In particular we have set for ourselves the problem to calculate from these rings the rate of diffusion of the reacting substances, and to estimate the limit of supersaturation attained when the solid phase is not present, — the so-called "Metastable Limit" of Ostwald.

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* Chemische Reactionen in Gallerten. Düsseldorf, 1898.

Before proceeding to the experiments we shall give certain definitions and explanatory paragraphs from Ostwald's Lehrbuch.*

"Between a solid substance and its supersaturated solution an equilibrium is formed, which depends on the nature of the substances, the temperature, and the pressure. If we leave out of account the influence of the pressure, for every temperature there is a definite equilibrium. This is evidenced by a definite concentration up to which the solid substance dissolves. . . .

"When the solid phase is not present, the concentration of the dissolved substance is arbitrary. The limit of concentration on one side is zero; on the other side the limit is a concentration difficult to determine, which, however, is greater than that of the equilibrium with the solid phase.

"Solutions whose concentrations exceed the concentration of saturation with a possible solid phase are called supersaturated with reference to this solid phase. . . .

"The simplest way of obtaining a supersaturated solution is to produce by change of temperature from the solid substance and the solvent a solution that is more concentrated than a saturated solution at the temperature of the experiment. In most cases the temperature required to produce supersaturation is higher than the temperature of the experiment, but it can also be lower in cases where the solubility decreases with increase of temperature.

"A better way to obtain supersaturated solutions, since it is thereby easier to exclude germs of the solid phase, is to employ reactions that produce in the solvent concentrations of the substance in question in appropriate amount.

"Among supersaturated solutions there are some which under definite conditions can be kept indefinitely, if germs are excluded, without formation of the solid phase. Such solutions are called Metastable.

"On the other hand, there are some solutions in which, even when germs are excluded, the solid phase appears after a short time. Solutions of this type are called Labile.

"Metastable solutions always show smaller concentration than the labile solutions of the same substances. By increase of concentration, therefore, a metastable solution goes over into a labile condition. The concentration at which the transition occurs is called the metastable limit.

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* Lehrbuch, II. 2, 780–784.

"The metastable limit is, in the first place, dependent on the nature of the substances, on the temperature, and on the pressure. In addition it is influenced by various other circumstances not yet explained. At the present time (1899) it is hardly possible to speak more definitely concerning the value of the metastable limit and the methods of determining it."

It is in connection with this discussion in the Lehrbuch that Ostwald cites the sharpness and regularity of Liesegang's rings as evidence that the metastable limit is something definite. But so far as we are aware, no attempt has hitherto been made to study Liesegang's rings quantitatively, and to obtain from them the actual numerical value of the metastable limit. In fact, so far as we are aware, there has not been published a determination of the metastable limit in any case. We quote further Ostwald's explanation* of the formation of Liesegang's ring-system: —

"By the diffusion of the silver salt into gelatine containing chromate a solution is formed which is supersaturated with respect to silver chromate; precipitation does not take place until the metastable limit has been exceeded. This naturally happens simultaneously in a circle concentric with the drop. Silver chromate, in relation to which the neighborhood of the ring is supersaturated, deposits on the precipitate already formed and strengthens it; this continues until the soluble chromate has been removed from the neighborhood and deposited on the precipitate. The silver salt, diffusing on farther, supersaturates a new circular region, and the process repeats itself. Since the silver nitrate becomes more dilute by diffusion, the critical concentration, at which precipitation begins, is reached later and later, and the rings form farther and farther apart."

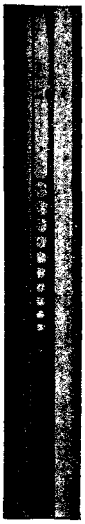

FIGURE 2.

Liesegang has also shown that if the potassium chromate in gelatine is put into a capillary *tube* instead of being spread upon a plate, analogous phenomena occur when one end of the tube is dipped into a solution of silver nitrate. The precipitate in the tube is formed in layers, or discs, perpendicular to the axis of the tube. (Figure 2.)

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* Lehrbuch, II. 2, 778.

This phenomenon of the formation of a precipitate in rings on a plate or in discs in a tube is not confined to the case of a reaction between silver nitrate and potassium chromate. Liesegang found such a formation of a precipitate in the case of mercurous chromate, lead chromate, and Prussian blue, in the familiar reactions which give these precipitates. We have found that with a proper choice of concentrations rings are also formed in the following cases: —

TABLE I.

Diffusing Subs.	Subs. in Gel.	Precip.	Nature of Rings.
$Pb(NO_3)_2$	Na_2SO_4	$PbSO_4$	Fine. White. Close together.
$AgNO_3$	$K_2C_2O_4$	$Ag_2C_2O_4$	“ “ “
$AgNO_3$	Na_2CO_3	Ag_2CO_3	Thick. Far apart.
$AgNO_3$	Na_2HPO_4	$Ag_4P_2O_7$	Fine. White.
$AgNO_3$	NH_4CNS	$AgCNS$	“ “
$AgNO_3$	KBr	$AgBr$	“ “
$Co(NO_3)_2$	$NaOH$	$Co(OH)_2$	Gelatinous. Thick.
$BaCl_2$	$K_2C_2O_4$	BaC_2O_4	Distinctly crystalline.
$HgNO_3$	KBr	$HgBr$	Fine. White.
$FeCl_3$	Na_2CO_3	CO_2	Gas bubbles.

It is our intention to give a more extended description of the various reactions in this table at some future time. At present, however, in order to discuss the quantitative aspect of the problem, we shall confine our attention to a single case, — the formation of supersaturation rings of silver chromate.

The experiment was as follows: A capillary tube of diameter .5 to 1 mm. was filled with a $\frac{1}{75}$ or a $\frac{1}{150}$ normal solution of potassium chromate in gelatine. After the gelatine had set, a piece of the tube about 5 cm. long, obtained by breaking the tube under water so as to expose a fresh moist surface of the gelatine, was plunged vertically into a water solution of 2N, N or $\frac{N}{2}$ $AgNO_3$ contained in a flat-walled glass vessel. By means of a cathetometer carrying a microscope of low power we watched the formation of the precipitate. After a certain period of time, depending on the concentration of the solution, the precipitate near the advancing front of the diffusing substance began to appear in layers, or discs, widely separated in comparison with their thickness. These discs appeared suddenly as a sharp, thin film, so that the *time* of their appearance could be determined with accuracy. The *distance* of each disc from the bottom of the tube was read off on the cathetometer. Measurements so obtained are recorded in the appended tables, IV to XXIV. The experiment was conducted in a constant-temperature room, which, except for the short time necessary for the taking of readings, was kept dark, or dimly lighted, to exclude the possible action of light in hardening the gelatine.

The reaction is in accordance with the formula:

$$2AgNO_3 + K_2CrO_4 \leftrightarrows Ag_2CrO_4 + 2KNO_3$$

The advance of the reaction in the one sense or the other is conditioned upon the concentration of the active substances. For equilibrium the concentrations must satisfy the quantitative relation:

$$\overset{+}{\mathrm{Ag}}{}^2 \times \overset{--}{\mathrm{CrO}_4} = K \, . \, \mathrm{Ag_2CrO_4} \qquad (a)$$

where $\overset{+}{\mathrm{Ag}}$ and $\overset{--}{\mathrm{CrO}_4}$ are the concentrations of the silver ion and the chromate ion respectively, Ag_2CrO_4 the concentration of the undissociated silver chromate in solution, and K the equilibrium constant. When the solid phase is present the undissociated silver chromate in solution must be in equilibrium with the solid phase, and must, therefore, be present in constant amount. In saturated solution, therefore,

$$\overset{+}{\mathrm{Ag}}{}^2 \times \overset{--}{\mathrm{CrO}_4} = k, \qquad (1)$$

where k is the *solubility product*.

Now in supersaturated solution the mass law (Eq. a) is probably still true, and the question arises; is there a relation similar to (1), with, however, a different constant product that defines the limit of supersaturation?

Is there a formula of the form

$$\overset{+}{\mathrm{Ag}}{}^2 \times \overset{--}{\mathrm{CrO}_4} = H \qquad (2)$$

for the condition that the concentrations must satisfy when the precipitate just begins to form in the *absence* of the solid phase?

II. Theoretical Considerations.

We can calculate the value of H in equation (2) provided we can calculate the values of the concentrations of the two ions at the point and at the time at which a ring begins to appear. We propose to attempt to obtain the values of these concentrations by the mathematical theory of diffusion. To avoid possible confusion from the use of general terms we shall take the specific case in which silver chromate is the precipitated substance; the same analysis will, of course, apply to any other case.

Suppose a capillary tube (Figure 3), containing in gelatine a dilute solution of potassium chromate, to be plunged at the time $t = 0$ into a vessel containing a strong solution of silver nitrate; let the distance x be measured from the end of the tube, with its positive direction along the axis of the tube; let u be the concentration of the $\overset{+}{Ag}$ ion at the point x at the time t, v the corresponding value for the $\overset{--}{CrO_4}$ ion. At the instant the tube is plunged in, that is, at the time $t = 0$, u is equal to U_0 for all negative values of x, and 0 for all positive values of x. Let 0 and V_0 be the corresponding initial values of v.

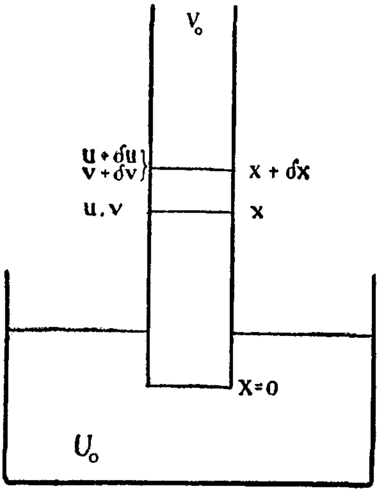

Figure 3.

As a first approximation we may assume that the dissociation of the K_2CrO_4 is complete, since this substance is present in very dilute solution for values of x and t such as we shall need to consider; the concentration of the silver salt in the tube will also be small, and we shall at present consider it to be completely dissociated. Later we can see how to correct the error introduced by this assumption. Thus if we measure concentration in gram-molecules, the concentration of the $\overset{+}{Ag}$ ion is the same as that of the $AgNO_3$, and the concentration of the $\overset{--}{CrO_4}$ ion is that of the K_2CrO_4. We are to calculate the two concentrations from the diffusion of $AgNO_3$ and K_2CrO_4.

The diffusion constant is defined as the number of units of mass that will go through a unit cross-section in a unit of time under head of unit gradient of concentration. Let a^2 (essentially positive) be the diffusion constant of silver nitrate; then the quantity of silver nitrate that will cross a section s of the tube at a distance x from the origin in a time δt is the diffusion constant × gradient × time × area of cross-section. Therefore

$$Q = - a^2 \frac{\partial u}{\partial x} \cdot \delta t \cdot S \tag{3}$$

The quantity crossing a section at distance $x + \delta x$ is

$$Q + \frac{\partial Q}{\partial x} \delta x = - a^2 \frac{\partial u}{\partial x} \delta t \, S - a^2 \frac{\partial^2 u}{\partial x^2} \cdot \delta t \cdot \delta x \cdot S \tag{4}$$

The accumulation of silver nitrate in the region between x and $x + \delta x$ is the difference of these quantities,

$$a^2 \frac{\partial^2 u}{\partial x^2} \cdot \delta t \cdot \delta x \cdot S$$

This accumulation may also be expressed as change of concentration multiplied by the volume, $= \delta u \cdot \delta x \cdot S$;

$$\therefore a^2 \frac{\partial^2 u}{\partial x^2} \delta t \cdot \delta x \cdot S = \delta u \cdot \delta x \cdot S;$$

whence, dividing by $\delta t \cdot \delta x \cdot S$

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \tag{5}$$

This is Fick's Law.*

Analogous considerations give us an exactly similar equation for the concentration of the $\overline{\overline{\mathrm{CrO_4}}}$ ion at the point x and time t; namely,

$$b^2 \frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t} \tag{6}$$

where b^2 is the diffusion constant for potassium chromate.

In deriving these equations we have made the assumption that the formation of earlier precipitates does not materially affect the head under which the ions accumulate for the formation of new precipitates. If we make this assumption we can solve the problem completely.

Equations (5) and (6) are exactly alike, but their solutions will be different because they have to satisfy different initial conditions in the two cases. The lower end of the tube was kept constantly, by occasional stirring, at the concentration U_0, so that equation (5) must be solved subject to the condition $u = U_0$ when $x = 0$ for all values of t, and $u = 0$ for all positive values of x at the time $t = 0$. Equation (6) has for initial values $v = V_0$ for all positive values and $v = 0$ for all negative values of x when $t = 0$. The theory of differential equations of this type teaches us that if we can obtain any one solution for each of these equations that satisfies its boundary conditions it is the only solution. The following are unique solutions of (5) and (6) subject to these conditions:

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* Pogg. Ann., XCIV. 59 (1855).

$$u = \frac{2\,U_0}{\sqrt{\pi}} \int_{\frac{x}{2a\sqrt{t}}}^{\infty} e^{-\beta^2}\, d\beta, {}^{*} \tag{7}$$

$$v = \frac{V_0}{\sqrt{\pi}} \int_{\frac{-x}{2b\sqrt{t}}}^{\infty} e^{-\beta^2}\, d\beta, {}^{\dagger} \tag{8}$$

in which β is merely a variable of integration.

These may be seen to be solutions of (5) and (6) respectively if we differentiate them according to the rules for the differentiation of a definite integral and substitute the results in (5) and (6). Equation (7) is also seen to satisfy the initial conditions, for if $t = 0$ and x is positive, the lower limit of integration becomes $+\infty$, which is the same as the upper limit; the integral is therefore 0 and $u = 0$. If $x = 0$ the lower limit is 0 and the value of the integral is $\frac{\sqrt{\pi}}{2}$

$$\therefore u = U_0$$

Similar reasoning shows equation (8) to satisfy initial conditions for v.

The *metastable product* H, as we have defined it on page 8 (Eq. 2), should be given by the following equation:

$$H = u^2 v = \frac{4\,U_0^2\,V_0}{\pi^{\frac{3}{2}}} \left[\int_{\frac{x}{2a\sqrt{t}}}^{\infty} e^{-\beta^2}\, d\beta \right]^2 \cdot \int_{\frac{-x}{2b\sqrt{t}}}^{\infty} e^{-\beta^2}\, d\beta \tag{9}$$

It is our problem to ascertain whether H is a constant for several initial concentrations of U_0 and V_0.

If we knew the diffusion constants a^2 and b^2 we could expand the integrals in (7) and (8), integrate term by term for a given value of x and t, and thus obtain u and v and from them H. This would necessitate a previous independent determination of the diffusion constants, and would beside be very laborious, as the series do not begin to converge until after the sixth or seventh term. The difficulty of the series expansion might, however, be obviated by the employment of tables that have been computed for the definite integral.

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* Fourier, Th. an. de Chaleur, § 366 (1822).

† Fourier, loc. cit.; Stefan, Wiener Sitzungsber., 79, II. 176 (1879).

We have employed a method in which a previous independent determination of the diffusion constants was not necessary. Before passing to the solution we shall call attention to an important and easy deduction from equation (9). In order for H to be a constant the lower limits of integration

$$\frac{x}{2\,a\sqrt{t}} \text{ and } \frac{-x}{2\,b\sqrt{t}}$$

should be constant for all rings in a given tube with given initial values of U_0 and V_0, and since a and b are constants $\frac{x}{\sqrt{t}}$ should be a constant. That is, for example, if we plunge a tube containing potassium chromate in gelatine into a solution of silver nitrate and observe with a cathetometer the distance from the bottom of the tube at which each thin disc of precipitate appears, and note, at the same time, the number of seconds that have elapsed since plunging in the tube, the distance divided by the square root of the time is a constant for all the discs in this tube, and for all tubes with the same initial concentrations of the reacting substances. The sample set of results given in Table II and all the tables, IV to XXIV (pp. 643–648), show with what consistency this conclusion is verified.

III. DETERMINATION OF THE DIFFUSION CONSTANT AND THE METASTABLE LIMIT.

This constancy of $\frac{x}{\sqrt{t}}$ for any one set of observations under given conditions does not show, when taken alone, that there is a constant metastable solubility product H, but does show that, for a given concentration of one of the ions, there is a definite concentration of the other ion that will cause precipitation.

In order to examine into the constancy of H, the product of ionic concentrations as defined by equation (9), we shall need to employ the data furnished by different initial concentrations of the reacting substances, as collected in Table III, p. 635.

Every two sets of initial concentrations, together with the corresponding values of $\frac{x}{\sqrt{t}}$, will give the data for calculating the diffusion constant

TABLE II.

TEMP. 15.7. SILVER NITRATE, N. POTASSIUM CHROMATE, $\frac{N}{75}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1245	.537	.01522
1420	.575	.01526
1607	.611	.01524
1825	.649	.01519
2068	.694	.01526
2345	.738	.01524
2658	.785	.01523
3000	.834	.01523
3395	.888	.01524
3823	.940	.01520
4305	.998	.01521
4842	1.056	.01518
5443	1.125	.01524
6102	1.188	.01520
6870	1.260	.01520
		.01523
Mean error		.00002

a^2 and the value of H, as defined by equation (9). For example, if we substitute for U_0, V_0, and $\frac{x}{\sqrt{t}}$, the respective values of these quantities in sections (a) and (b) of Table III., we obtain the two following equations: —

$$H = \frac{4 \cdot 2^2 \cdot \frac{1}{75}}{\pi^{\frac{3}{2}}} \left[\int_{\frac{.0167}{2a}}^{\infty} e^{-\beta^2} \, d\beta \right]^2 \cdot \int_{\frac{-.0167}{2b}}^{\infty} e^{-\beta^2} \, d\beta. \qquad (10)$$

$$H = \frac{4 \cdot 1^2 \cdot \frac{1}{75}}{\pi^{\frac{3}{2}}} \left[\int_{\frac{.0153}{2a}}^{\infty} e^{-\beta^2} \, d\beta \right]^2 \cdot \int_{\frac{-.0153}{2b}}^{\infty} e^{-\beta^2} \, d\beta. \qquad (11)$$

TABLE III.

Recapitulation from Tables IV–XXIV.

Silver Nitrate 2N *diffusing into*

(a) Potassium Chromate, $\frac{N}{75}$.

Table.	Temp.	$\frac{x}{\sqrt{t}}$
IV.	16.0	.01656
V.	16.3	.01663
VI.	16.3	.01671
VII.	16.1	.01700
Mean		.01670

Silver Nitrate N *diffusing into*

(b) Potassium Chromate, $\frac{N}{75}$.

Table.	Temp.	$\frac{x}{\sqrt{t}}$
II.	15.7	.01523
VIII.	15.5	.01524
IX.	17.0	.01549
X.	16.7	.01533
Mean . . .		.01530

(d) Potassium Chromate, $\frac{N}{150}$.

Table.	Temp.	$\frac{x}{\sqrt{t}}$
XI.	17.3	.01462
XII.	16.3	.01466
XIII.	16.7	.01467
XIV.	16.5	.01462
XV.	15.5	.01453
XVI.	16.6	.01477
XVII.	16.6	.01468
Mean . . .		.01465

Silver Nitrate $\frac{N}{2}$ *diffusing into*

(c) Potassium Chromate, $\frac{N}{75}$.

Table.	Temp.	$\frac{x}{\sqrt{t}}$
XVIII.	16.6	.01384
XIX.	16.8	.01377
Mean . . .		.01380

(e) Potassium Chromate, $\frac{N}{150}$.

Table.	Temp.	$\frac{x}{\sqrt{t}}$
XX.	17.7	.01311
XXI.	16.9	.01292
XXII.	16.0	.01335
XXIII.	15.5	.01313
XXIV.	15.5	.01299
Mean . . .		.01310

These two equations must be solved as simultaneous. We have been able to do this by the aid of a set of definite integrals given in the appendix of Kramp's *Analyse des Refractions Astronomiques et Terrestres*, published at Strassburg in 1799. Kramp's table contains values of the integral

$$\int_{a}^{\infty} e^{-\beta^2} d\beta$$

for 301 values of a from 0 to 3.00. This table shows, in the first place, that various values of

$$\int_{\frac{-x}{2b\sqrt{t}}}^{\infty} e^{-\beta^2} d\beta,$$

obtained on the assumption that b^2 is of the order of magnitude of the diffusion constant of an electrolyte, and that $\frac{x}{\sqrt{t}}$ is of the order of magnitude given by our experimental data, differ from each other and from $\sqrt{\pi}$ by less than 2 per cent. This means merely that the change of concentration, v, of the substance originally in the gelatine can be neglected.

Dividing equation (11) by equation (10), and extracting the square root, we have

$$\frac{\int_{\frac{.0153}{2a}}^{\infty} e^{-\beta^2} d\beta}{\int_{\frac{.0167}{2a}}^{\infty} e^{-\beta^2} d\beta} = 2. \qquad (12)$$

To solve this equation for a it is only necessary to find two values in Kramp's table for which the lower limits of integration are in the ratio 0.0167 to 0.0153, and for which the value of the integral corresponding to the second is double the value of the integral corresponding to the first.

To give an idea of the procedure in such a calculation we shall give on the next page a few values from Kramp's table in the neighborhood of the solution, of which the *starred* value is seen to be the one satisfying equation (12).

The ratio of 0.0167 to 0.0153 is 1.091.

a and a_1 are the lower limits of integration, A and A_1 the corresponding values of the integral. The correct solution is singled out by the relation $\frac{A}{A_1} = 2$,

a.	a_1.	$\frac{a_1}{a}$.	A.	A_1.	$\frac{A}{A_1}$.
1.77	1.932	1.091	.010909	.005677	1.921
1.79	1.954	1.091	.010067	.005077	1.983
1.80*	1.965	1.091	.009668	.004837	1.999
1.81	1.975	1.091	.009284	.004632	2.005
1.83	1.999	1.091	.008555	.004164	2.055

from which the value of a that most nearly satisfies equation (12) is seen to be 1.80.

$$\therefore \frac{.0153}{2\,a} = 1.80$$

$$a = 4.25 \times 10^{-3}.$$

The corresponding value A of the integral is .009668.
Therefore from equation (11)

$$H = \frac{4}{75\,\pi}\,(.009668)^2 = 1.6 \times 10^{-6}\left(\frac{\text{Gm. Mol.}}{\text{liter}}\right)^3.$$

The diffusion constant is not a, but a^2, which, reduced to the usual units with the day as unit of time by multiplying a^2 by 8.64×10^4, becomes

$$a^2 = 1.56\left(\frac{\text{cm}^2}{\text{day}}\right).$$

In this calculation of H the reacting substances have been assumed to be completely dissociated. The result of the calculation shows that the concentration of the silver ion was about $\frac{1}{200}$ Normal and that of the chromate ion $\frac{1}{75}$ Normal when precipitation occurred. The dissociation of $\frac{N}{200}$ silver nitrate differs so little from that of $\frac{N}{400}$, which enters into the calculation of subsequent cases, that the assumption of its complete dissociation introduces no appreciable inaccuracy. On the other hand, the dissociation of $\frac{N}{150}$ potassium chromate is about 1.05 times the dissociation of $\frac{N}{75}$, and this factor representing the ratio of dissociation must be introduced into calculations involving different concentrations of potassium chromate in the gelatine.

Observing this precaution we obtain the following values for a^2 and H from other sets of data of Table III.

FOR THE DIFFUSION CONSTANT OF SILVER NITRATE AND THE METASTABLE SOLUBILITY PRODUCT OF SILVER CHROMATE IN GELATINE AT 16° C.

From Table III. Data.	Diffusion Constant. $\frac{\text{cm}^2}{\text{day}}$.	Metastable Product. $\left(\frac{\text{Gm. Mol.}}{\text{Liter}}\right)^3$.
(a) and (b)	1.56	1.6×10^{-6}
(b) and (c)	1.56	1.6×10^{-6}
(a) and (c)	1.56	1.4×10^{-6}
(d) and (e)	1.53	1.3×10^{-6}
(b) and (d)	1.50	1.1×10^{-6}
(c) and (e)	1.53	1.3×10^{-6}
Mean	1.54	1.4×10^{-6}

IV. RESULTS.

Discussion of Results. — The values obtained for the diffusion constant and for the metastable solubility product are constant within the limit of error of the method, as will be seen by a reference to the recapitulation of Table III. While the value of $\frac{x}{\sqrt{t}}$ for any particular tube is constant within one or two tenths of one per cent, the same good agreement is not obtained when several series of observations are made upon different tubes under as nearly as possible the same conditions. The mean deviation of different tubes from the average is as great as one per cent. We think this deviation can be accounted for by lack of uniformity in the bore of some of the tubes, so that the diffusion takes place in a conical cavity instead of in a truly cylindrical cavity, as the theory requires. That an error of one per cent in the determination of $\frac{x}{\sqrt{t}}$ will account for the variations in the values of a^2 and H may be seen by a glance at the result of a recalculation of these quantities from the data (d) and (e) of Table III, on the assumption that the correct value of $\frac{x}{\sqrt{t}}$ in (e) is .0130 instead of .0131; a^2 comes out to be 1.63 $\frac{\text{cm}^2}{\text{day}}$, and H becomes 1.9×10^{-6}. Or, assuming $\frac{x}{\sqrt{t}}$ to be .0132 instead of .0131, a^2 becomes 1.42 and H becomes 0.7×10^{-6}.

In view of this sensitive dependence of the diffusion constant and the metastable product on the value of $\frac{x}{\sqrt{t}}$, we consider the results to be satisfactorily consistent.

We have not been able to find elsewhere any determination of the value of the diffusion constant of silver nitrate in gelatine. F. Voigtlaender* has found that the diffusion constants of salts in agar from his own measurements are in some cases smaller, in others equal to or larger than the corresponding constants for water solutions, the solutions employed having varying concentrations. He says that the question whether the diffusion in agar and water is identical can only be answered by further observations on pure water and other gelatinous solvents, such as water glass. Voigtlaender's measurements point to the conclusion, however, that the diffusion in agar and that in water are not very different, which makes our result for silver nitrate, 1.54, seem rather high, as the values obtained by Kawalki † would be for silver nitrate at 16° C. 1.13 to 1.20. Calculation of the diffusion constant for silver nitrate by Nernst's formula

$$a^2 = 0.0477 \times 10^7 \frac{u\,v}{u + v} [1 + 0.0034\,(t^\circ - 18)]$$

gives the value 1.29.

It should be noted that the value we have obtained is for diffusion through gelatine containing a rather heavy solid precipitate of silver chromate, and should not, therefore, be expected to agree with diffusion in water or pure gelatine. The important fact for this discussion is that a^2 is a constant and that there is hence no inconsistency in the conclusion that the metastable solubility product of silver chromate in gelatine is a definite constant quantity satisfying the relation

$$H = \overset{+}{\mathrm{A}}\mathrm{g}^2 \times \mathrm{Cr}\overset{-\,-}{\mathrm{O}_4}.$$

As to the value of H, there are no other determinations of this quantity with which we might compare our results. As far as we know there have not been published any numerical data as to the value of the metastable product in any case. The ordinary solubility product of silver chromate in saturated water solution in the presence of the solid phase is 5.1×10^{-13} as determined by Kohlrausch.‡ The concentrations of Ag_2CrO_4 in the two cases are in the ratio $\frac{\sqrt[3]{1.4 \times 10^{-6}}}{\sqrt[3]{5.1 \times 10^{-13}}}$. This means that the supersaturated gelatine at the concentration of precipitation held in solution 145 times the amount of silver chromate required to saturate it in the presence of the solid phase.

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* Zeit. Phys. Chem., III. 317 (1889).

† W. A., LII. 300 (1894).

‡ Leitver. d. Elektrolyten, p. 202.

Lobry de Bruyn * and Liesegang † have shown that in the case of a large number of substances, which, like the silver haloids, form amorphous precipitates, gelatine inhibits precipitation. This phenomenon is especially easy to observe and demonstrate in the case of colored substances like silver chromate and lead iodide. It is well known that electrolytes, whether or not they act chemically on the colloid, may cause the precipitation of a colloid from solution, and it might be supposed that the case we are discussing can be explained as such an action of the diffusing silver nitrate in throwing down colloidal silver chromate. We are of the opinion that this is not the case, because, first, precipitation is here sudden and not slow, as is the case when colloids are precipitated by electrolytes, and, second, in the several cases of different concentrations of the reacting substances, the precipitate is formed, not for a constant concentration of the diffusing electrolyte, but for a constant value of the product

$$\overset{+}{\mathrm{Ag}}{}^2 \times \overset{--}{\mathrm{CrO}_4}.$$

Purification of the Gelatine. The best commercial gelatine contains usually about two per cent of ash, consisting of phosphates and carbonates, both of which form insoluble compounds with silver. Although the silver salts of these radicals are more soluble than silver chromate, it was thought necessary to remove them and other salts as thoroughly as possible from the gelatine to avoid their possible influence on the precipitation of the silver chromate. This removal was effected by electrolyzing the gelatine between membranes of parchment paper at five hundred volts continually for a week, during which time the membranes and electrodes were frequently washed by a stream of distilled water. Specimens of gelatine so prepared gave but a trace of ash on ignition.

Liesegang in his original research on reactions in gelatine solutions in capillary tubes found two classes of rings: the heavy widely separated red deposits of silver chromate, and beyond these and between them another series, consisting of fine, microscopic white lines. According to Liesegang's description, when the time came for the formation of a new red line one of the fine white lines suddenly became yellow and grew broader, until it encompassed four of the white lines and developed into the dark red of silver chromate. Liesegang observed, however, that these white lines are not necessary for the formation of the red ones and are not present when chromate diffuses into gelatine impregnated with silver. We are of the opinion that these secondary white lines are due to the presence of impurities in the gelatine, since they appear with the same distinctness when silver nitrate diffuses into commercial gelatine to which nothing has been added. They did not appear in our purest electrolyzed gelatine. The addition of soluble chlorides, bromides, or iodides in very small quantities causes the white rings to appear. Figure 4 and Figure 5 show them between the larger lines of silver

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* Rec. trav. chim. Pays-bas, XIX. 236.

† Phot. Wochenblatt, p. 229 (1894).

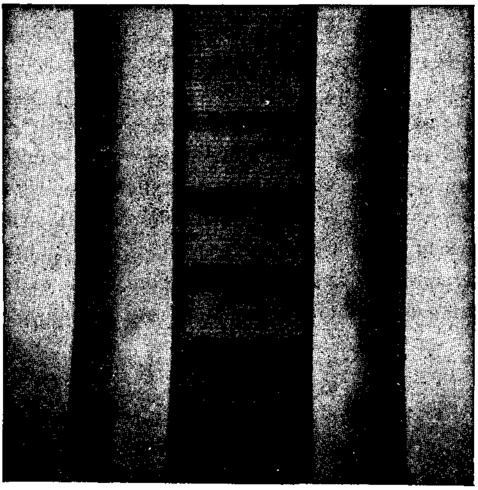

FIGURE 4.

Precipitation in commercial gelatine, showing fine white lines between the heavy dark deposits of $Ag_2 CrO_4$. This is a section of a tube like Figure 2, magnified 10 diameters.

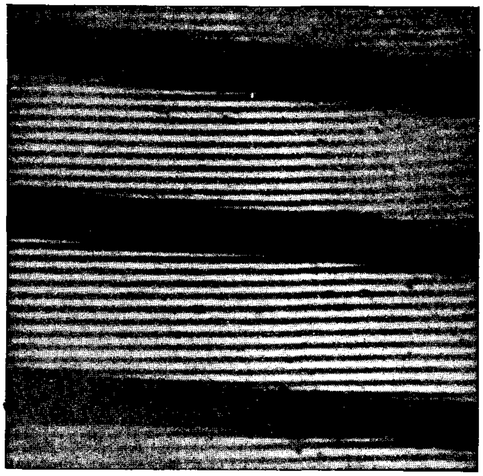

FIGURE 5.

Section of a plate like Figure 1, magnified 20 times, showing two sets of rings in commercial gelatine. The three heavy dark bands are Ag_2CrO_4.

chromate. It is our intention to investigate further the possible influence of impurities on the formation of these precipitates.

It is of interest to know that gelatine is not necessary for the production of rings, for when a very fine capillary tube filled with a water solution of potassium chromate is carefully plunged into a silver nitrate solution precisely similar phenomena may be observed. In this case, however, the layer of precipitate exists for only a short time after its formation. It breaks up under the influence of gravity and convection and mingles with the mass of silver chromate below. Three or four layers may exist at the same time, those of later formation being sharp, and the older ones gradually disintegrating into a mass of precipitate.

V. CONCLUSION.

We have shown by quantitative measurement that there exists in the case of the formation of silver chromate in gelatine solution a definite constant product

$$\overset{+}{\mathrm{Ag}}{}^{2} \times \overset{--}{\mathrm{CrO}_4} = H$$

which determines the limit of supersaturation with respect to silver chromate in the absence of the solid phase.

Incidentally we have obtained a value for the diffusion constant of silver nitrate diffusing through gelatine containing solid silver chromate.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY, CAMBRIDGE, MASS., U. S. A.,
March 1, 1903.

TABLE IV.

Temp. 16.0. Silver Nitrate, 2N.
Potassium Chromate, $\frac{N}{75}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1732	.687	.01650
1920	.723	.01650
2350	.801	.01652
2596	.845	.01659
2865	.888	.01659
3170	.934	.01659
3505	.982	.01659
3870	1.032	.01662
4315	1.083	.01652
4700	1.139	.01662
		.01656

TABLE V.

Temp. 16.3. Silver Nitrate, 2N.
Potassium Chromate, $\frac{N}{75}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1625	.672	.01667
1805	.705	.01660
2000	.742	.01660
2215	.782	.01662
2450	.822	.01661
2710	.866	.01663
2995	.910	.01663
3300	.955	.01663
3645	1.003	.01662
4020	1.051	.01658
4430	1.107	.01663
4885	1.162	.01663
5365	1.220	.01666
5895	1.280	.01667
		.01663

TABLE VI.

Temp. 16.3. Silver Nitrate, 2N.
Potassium Chromate, $\frac{N}{75}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1705	.684	.01657
1885	.719	.01655
2083	.760	.01662
2300	.798	.01665
2540	.842	.01671
3090	.930	.01673
3405	.976	.01673
3740	1.023	.01673
4110	1.077	.01680
4525	1.128	.01679
		.01671

TABLE VII.

Temp. 16.1. Silver Nitrate, 2N.
Potassium Chromate, $\frac{N}{75}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1580	.674	.01696
1750	.711	.01700
1940	.747	.01694
2150	.788	.01700
2385	.830	.01700
2625	.870	.01702
2910	.920	.01703
3215	.963	.01699
3550	1.015	.01704
3920	1.065	.01701
4320	1.117	.01700
4755	1.171	.01699
		.01700

TABLE VIII.

Temp. 15.5. Silver Nitrate, N.
Potassium Chromate, $\frac{N}{75}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1530	.598	.01529
1745	.638	.01527
1985	.680	.01526
2564	.771	.01523
2903	.819	.01521
3290	.872	.01521
3720	.927	.01521
4210	.986	.01524
		.01524

TABLE IX.

Temp. 16.7. Silver Nitrate, N.
Potassium Chromate, $\frac{N}{75}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1440	.585	.01535
1605	.619	.01545
1795	.655	.01546
2010	.695	.01551
2240	.734	.01551
2785	.819	.01551
3450	.911	.01551
3845	.964	.01555
		.01549

TABLE X.

Temp. 16.7. Silver Nitrate, N.
Potassium Chromate, $\frac{N}{75}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1073	.498	.01520
1165	.523	.01532
1270	.546	.01532
1500	.594	.01534
1635	.622	.01538
1778	.649	.01538
1935	.675	.01535
2105	.706	.01539
		.01533

TABLE XI.

Temp. 17.3. Silver Nitrate, N.
Potassium Chromate, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1332	.529	.01450
1535	.575	.01468
1790	.619	.01460
2090	.670	.01465
2422	.721	.01465
2830	.780	.01466
3280	.838	.01463
		.01462

TABLE XII.

Temp. 16.3. Silver Nitrate, N.
Potassium Chromate, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
800	.413	.01461
920	.443	.01461
1050	.473	.01459
1215	.508	.01456
1400	.548	.01465
1605	.586	.01463
1850	.631	.01466
2120	.675	.01466
2425	.724	.01470
2780	.775	.01470
3180	.830	.01471
3630	.884	.01466
4130	.949	.01475
4710	1.013	.01474
		.01466

TABLE XIV.

Temp. 16.5. Silver Nitrate, N.
Potassium Chromate, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1225	.511	.01460
1410	.551	.01468
1653	.595	.01464
1930	.644	.01466
2243	.692	.01461
2606	.746	.01461
3028	.803	.01460
3500	.862	.01457
		.01462

TABLE XIII.

Temp. 16.7. Silver Nitrate, N.
Potassium Chromate, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1270	.520	.01459
1465	.557	.01454
1685	.602	.01466
1935	.645	.01466
2225	.694	.01471
2555	.743	.01470
2910	.794	.01471
3345	.847	.01465
3810	.908	.01471
4340	.970	.01472
		.01467

TABLE XV.

Temp. 15.5. Silver Nitrate, N.
Potassium Chromate, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
965	.451	.01452
1108	.484	.01454
1295	.526	.01462
1505	.564	.01454
1745	.609	.01458
2025	.654	.01453
2345	.703	.01452
2702	.755	.01453
3115	.811	.01453
3595	.872	.01455
4165	.937	.01452
4770	1.002	.01451
5500	1.077	.01452
6380	1.155	.01448
		.01453

TABLE XVI.

Temp. 16.6. Silver Nitrate, N.
Potassium Chromate, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
945	.453	.01474
1090	.485	.01469
1260	.525	.01479
1460	.564	.01475
1685	.608	.01481
1950	.654	.01481
2250	.702	.01478
2605	.756	.01481
2995	.809	.01478
3440	.867	.01478
		.01477

TABLE XVII.

Temp. 16.6. Silver Nitrate, N.
Potassium Chromate, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1080	.479	.01458
1245	.515	.01462
1425	.554	.01468
1630	.592	.01466
1865	.637	.01475
2130	.679	.01467
2460	.725	.01468
2790	.780	.01477
3170	.833	.01479
. . .	. . .	. . .
10050	1.467	.01463
		.01468

TABLE XVIII.

Temp. 16.6. Silver Nitrate, $\frac{N}{2}$.
Potassium Chromate, $\frac{N}{75}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
905	.421	.01399
1105	.461	.01387
1210	.485	.01394
1333	.506	.01386
1468	.530	.01384
1765	.586	.01395
1935	.606	.01378
2120	.636	.01382
2316	.661	.01374
1543	.693	01374
2780	.726	.01377
		.01384

TABLE XIX.

Temp. 16.8. Silver Nitrate, $\frac{N}{2}$.
Potassium Chromate, $\frac{N}{75}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1625	.555	.01377
1870	.596	.01378
2145	.640	.01382
2460	.683	.01376
2820	.734	.01382
3294	.784	.01366
3705	.839	.01379
		.01377

TABLE XX.

TEMP. 17.7. SILVER NITRATE, $\frac{N}{2}$.
POTASSIUM CHROMATE, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1026	.420	.01311
1244	.460	.01304
1510	.508	.01307
1819	.558	.01308
2181	.614	.01315
2625	.672	.01312
3185	.740	.01311
3850	.819	.01320
4700	.901	.01314
5720	.988	.01307
		.01311

TABLE XXI.

TEMP. 16.9. SILVER NITRATE, $\frac{N}{2}$.
POTASSIUM CHROMATE, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
705	.346	.01274
860	.380	.01296
1048	.419	.01294
1282	.465	.01298
1535	.508	.01297
1862	.559	.01295
2250	.614	.01295
2715	.674	.01293
3250	.734	.01288
		.01292

TABLE XXII.

TEMP. 16.0. SILVER NITRATE, $\frac{N}{2}$.
POTASSIUM CHROMATE, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
1305	.434	.01340
1560	.529	.01340
1880	.580	.01337
2270	.633	.01329
2730	.697	.01334
3270	.761	.01331
		.01335

TABLE XXIII.

TEMP. 15.5. SILVER NITRATE, $\frac{N}{2}$.
POTASSIUM CHROMATE, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
933	.400	.01310
1120	.440	.01315
1340	.481	.01314
1590	.525	.01317
1900	.572	.01312
2265	.625	.01310
2690	.681	.01313
3190	.742	.01314
		.01313

TABLE XXIV.

TEMP. 15.5. SILVER NITRATE, $\frac{N}{2}$.
POTASSIUM CHROMATE, $\frac{N}{150}$.

t sec.	x cm.	$\frac{x}{\sqrt{t}}$
900	.387	.01290
1080	.427	.01299
1300	.465	.01290
1565	.513	.01297
1875	.564	.01303
2235	.615	.01301
2670	.674	.01305
3180	.738	.01306
		.01299

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

ON THE THERMAL CONDUCTIVITIES OF CERTAIN PIECES OF ROCK FROM THE CALUMET AND HECLA MINE.

By B. O. Peirce.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD COLLEGE.

ON THE THERMAL CONDUCTIVITIES OF CERTAIN PIECES OF ROCK FROM THE CALUMET AND HECLA MINE.

By B. O. Peirce.

Presented April 8, 1903. Received April 9, 1903.

Several years ago Dr. R. W. Willson and I presented to the Academy an account of some measurements of the conductivity for heat of several kinds of marble, and we discussed at that time* the theory of our method and described the apparatus we employed in our work. This present paper gives the results of determinations, by the same method, of the thermal conductivities of six representative slabs of rock from the Calumet and Hecla mine, very kindly lent to me by Professor Alexander Agassiz. These conductivities are interesting in view of the fact that the rate of increase of mean annual temperature with the depth seems to be very unusually small near the shaft where Dr. Agassiz's measurements were made.

Each of the slabs is a rectangular parallelopiped about four inches thick and two feet square: their average weight is a little over two hundred pounds. There are two specimens of trap, two of amygdaloid, and two of conglomerate. Each of the pieces of trap seems very homogeneous, and each piece of amygdaloid sufficiently so to make the result of each measurement an excellent determination of the mean conductivity of the slab in question: in the case of conglomerate, some variations in conductivity are, of course, to be expected in different parts of any large piece; I hope, however, that the figures given below represent with some accuracy the average conductivities of the slabs examined. The determinations involved steady work for several months.

The size of the blocks to be experimented on made some slight modi-

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* These Proceedings, August, 1898. Vol. XXXIV. 1.

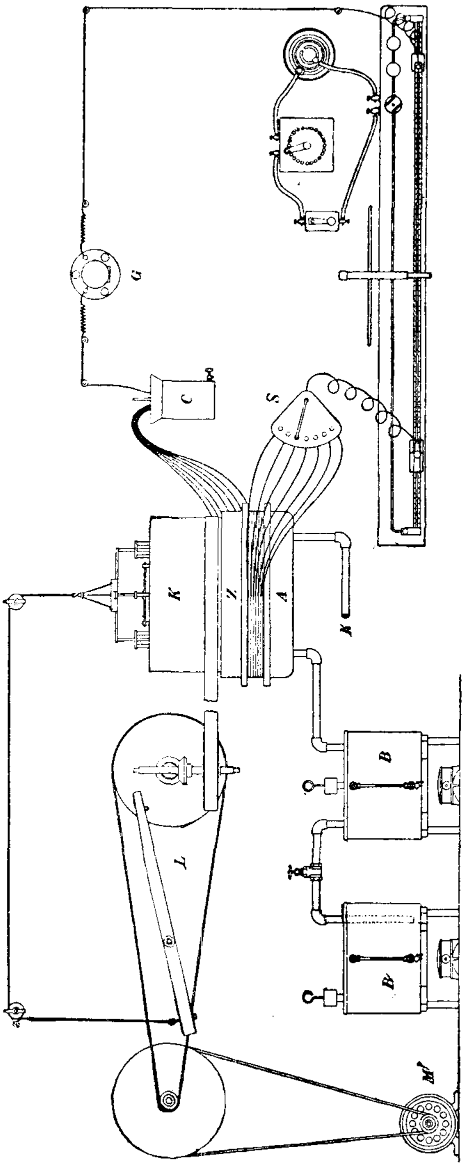

FIGURE 1.

fications necessary in the apparatus used for the work on marble, but the procedure was much the same. The square, plane-faced slab of the material to be tested, enclosed between two other thin slabs of similar material, formed a rectangular parallelopiped or prism, which was clamped and left for many hours, between the steam chest *A* and the ice-box *Z* of the apparatus, represented without any of its elaborate system of jackets, by Figure 1. The final temperatures at the centres of the faces of the slab to be tested were determined by the aid of thermal elements, and the flux of heat through a definite central portion of the colder base of the prism was measured.

The hot chamber *A*, which weighed about two hundred kilograms, rested in a thick jacket on a heavy table or stand made to hold it: it was connected directly with one (*B*) of two stout-walled copper boilers, *B* and *B'*, each of which held about forty litres of water. A light cup-shaped weight inverted and laid on a large tube with smooth end which projected above the top of the boiler, acted as a sensitive safety valve and prevented any appreciable rise in temperature within. *B* could be refilled when necessary with boiling water from *B′*, without stopping the constant flow of steam through *A* by means of the siphon *f* which was provided with a valve. The steam, after passing through the hot chamber, was led to the outer air by a jacketed pipe *h* descending from the bottom of *A*.

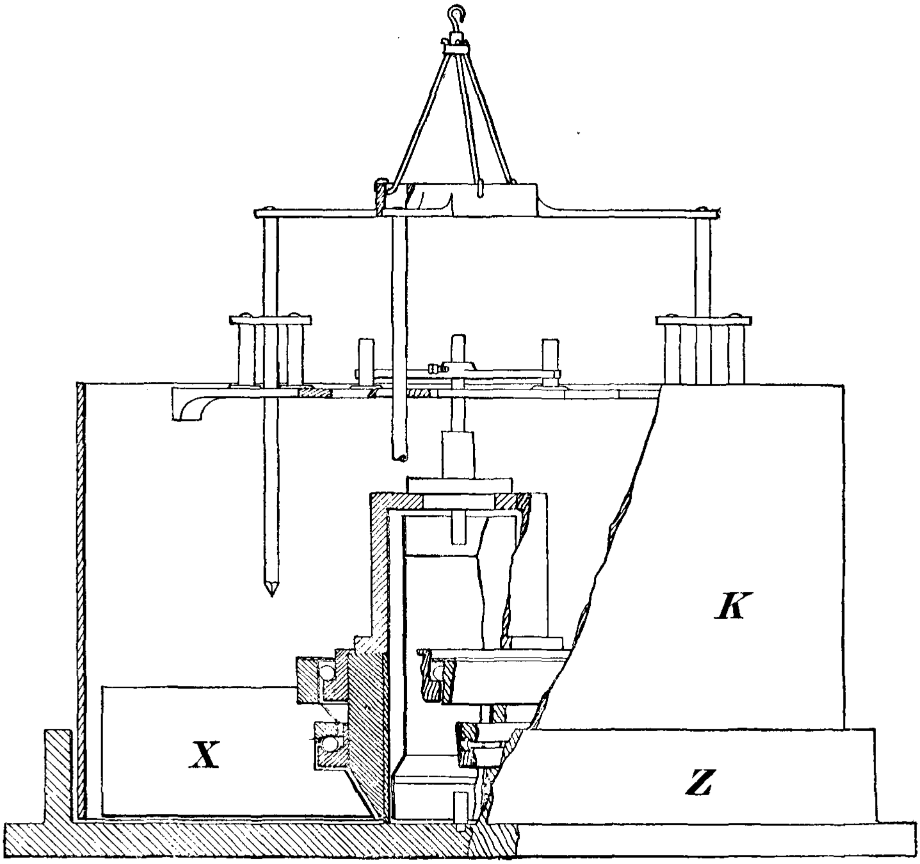

FIGURE 2.

The connections of the thermal elements were led out of the sides of the prism shut in by *Z* and were held between slabs of wood, which acted as a sort of guard-ring jacket to the prism, for about 40 cm. before they emerged. The platinoid or german-silver leads of these thermal junctions within the prism were soldered together, and to a copper wire leading to the (copper) wire of a potentiometer. The copper ends of the couples led to a mercury switch by which any one of them, or any pair pitted against each other, might be quickly connected with the potentiometer. On its way from this switch to the cold junctions in *C* through the potentiometer wire the current encountered only copper. By means of a somewhat elaborate standard potentiometer, not shown in the diagram, the resistance, *R*, in the potentiometer circuit could be so adjusted that a millimeter on the potentiometer wire corresponded to any desired small potential difference, such as one micro-volt or one-tenth of a micro-volt. The potentiometer wire, which was 0.25 mm. in diameter, could be changed in a few seconds for new wire if the old became dented or stretched. An iron casting *Z* (Figures 2 and 3), accurately planed below and turned true above, formed the bottom of the ice-box: it could be

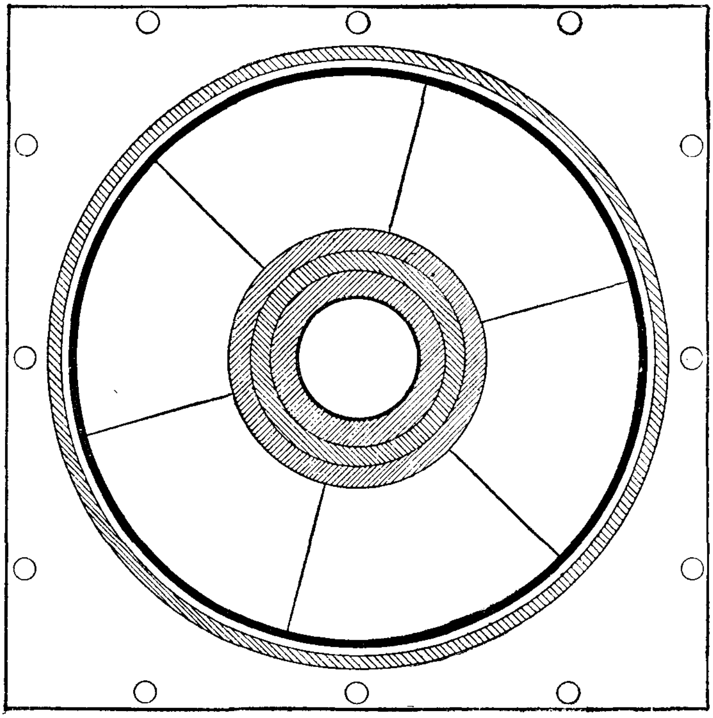

FIGURE 3.

bolted securely to *A* with the prism to be examined between the two. While *Z* was in the lathe, a small hole, *H*, about 3 mm. in diameter and 4 mm. deep, was drilled exactly in the centre of its upper face. Subsequently a piece of solid drawn brass tube 12.3 cm. in outside diameter and 13.5 cm. high, with carefully squared ends, was held centrally in *Z* by means of a disc turned to fit it, and a central pin inserted in *H*, and was then soldered firmly to *Z* with the help of white pitch as a flux. The walls of the pot thus formed were jacketed on the outside except for a height of about 2 mm. at the bottom (Figure 4) by an inch-thick casting of hard rubber made for the purpose in the form of a cylindrical shell. This casting, which was cut off square at the top of the pot, tapered to nothing near the bottom, but did not rest upon the floor. Upon the top of this jacket was fastened a hard-rubber cover shaped somewhat like a cylindrical hat. This had an opening at the top which could be closed by an accurately fitting ebonite plug. In the box, *P*, thus made, was placed a thin-walled copper ice-holder, *Q*, open at top and bottom, of the same outside diameter below as the inside of the brass pot, but some-

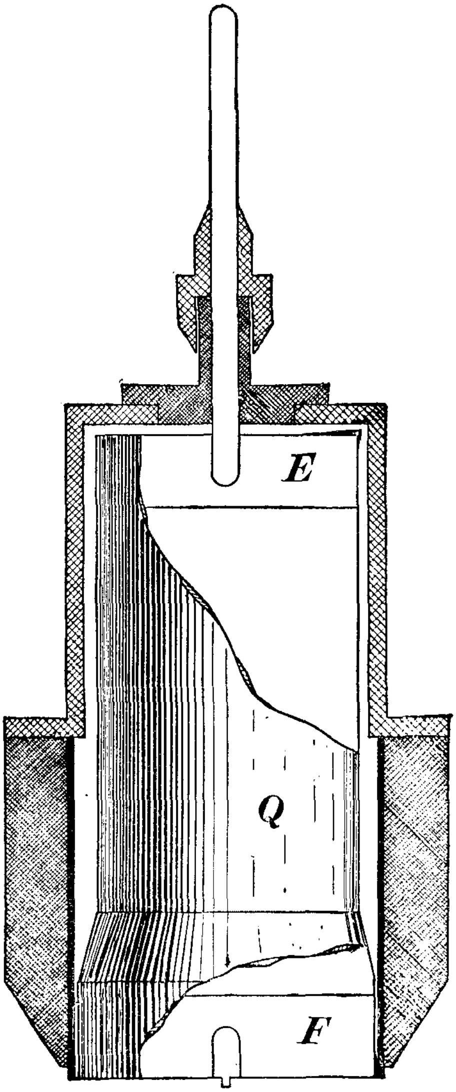

FIGURE 4.

what smaller above, so as to leave an air-space between it and the walls of the pot.

In order that the holder might be easily rotated, a pin soldered to a thin diametral web *F* which ran across the bottom of the holder was inserted in *H*, and a vertical brass rod soldered to a similar web, *E*, at the top of the holder passed through a hole in the cover of the pot which it fitted closely. An ebonite thimble fitting tightly on the rod and turning with it, permitted the slow entrance of ice-cold air into the pot without allowing any water to leak in. The rod could be clamped at pleasure to a brass yoke which is turned slowly by the electric motor. In order to prevent the introduction of heat into the pot by conduction down the rod, the exposed portion was buried in cracked ice. When the holder was filled with ice and turned by the motor, the web at the bottom compelled the ice to rub over the floor of the casting, since the holder itself had no bottom; and as a result of this, the lower surface of the ice quickly acquired and kept a mirror-like surface. The drip from the pot came out of the edge of the casting *Z* through a straight hole about 26 cm. long and 0.6 cm. in diameter drilled horizontally in the plate and ending just inside the pot. The whole apparatus was very slightly tipped to insure the steady outflow of the drip.

A large cylinder *K*, 35 cm. high, made of rolled brass 4 mm. thick and open at the top and bottom, was mounted on brass ball-bearings placed on the outside of the hard rubber jacket of the pot, *P*, by means of six vanes, one of which, *X*, is shown in Figure 2. *K* weighed about twenty kilograms when empty, and rested upon 144 brass balls each 12 mm. in diameter. When set in motion by a slight push *K* continued to rotate for about a minute before coming to rest; it was so truly hung that the outside could be used as a pulley and the whole rotated by means of the belt shown in Figure 1. The vanes reached to within about 2 mm. of the floor of the casting, and when the whole was filled with cracked ice and then rotated, the ice at the bottom which rubbed on *Z* soon got and held a very smooth surface.

A hole in the bottom of *Z* carried away the drip and prevented any accumulation of water on the floor of the ice-box. To prevent irregularities arising from honeycombing of the ice in the box, a suitably loaded brass tripod was used to pack the ice by light blows delivered at intervals of about twenty-one seconds by aid of the lever *L*. A train of wheels was necessary to reduce the speed of *K* to one revolution in twenty seconds, though only two wheels are shown in the drawing. The tripod slid in guides which revolved with *K*, and a swivel at the top prevented the cord from twisting.

The rotation of K and of the inside ice-holder, Q, which is connected with K at the top by means of a yoke, were matters of much importance. The continual rubbing of the ice over the flat surface of the casting seems to be necessary if the upper face of the prism is to be kept at a uniform constant temperature for hours. The heat produced in rotating Q slowly was so little as to be quite negligible. The ice in K was piled up so as to cover P completely, and I was unable to detect any difference between the temperatures within and without P by fine, properly protected thermal junctions introduced for the purpose. If while K revolved, Q was kept still, the amount of ice melted in Q became irregular, though the whole amount of drip in two or three hours was not very different from the amount of steady drip in an equal time when Q was rotating. Only selected lumps of ice were put into Q. The ice to be used was first broken up into pieces weighing something like fifteen grams each, by means of an ice-cracking machine, and these pieces were then put into ice-water so that their sharp edges might become slightly rounded. They were then drained and dropped into Q. In this way a slight amount of water attached to the ice was introduced, but with the method employed, the error due to this cause appeared to be of slight importance. In some experiments the ice to be used was carefully dried in cold blotting-paper, but this precaution did not seem to be necessary if the use of small bits of ice with sharp edges was avoided. Q's capacity was about two thousand cubic centimeters. After Q had been freshly filled in the course of any experiment while K was rotating, no record was kept for some time, perhaps fifteen minutes, of the amount of drip. Before the expiration of this interval the extra water introduced into Q with the ice had drained off, and the indications had become steady. The drip tube always contained a few drops of water, but this amount remained sensibly constant during the progress of an experiment. The drip was collected in a graduated vessel and the approximate amount was noted from time to time to see whether the flow was steady. The whole was then more accurately determined by weighing, at longer intervals. The regularity of drip seemed to be a far more sensitive test of the approximate attainment of the final state of the body experimented on and its surroundings than was a sensibly constant temperature gradient on the axis. In most experiments with the apparatus a sufficiently steady state was attained in about seven hours from the beginning of the heating. Z and K together weighed when filled with ice about three hundred kilograms, and many hundreds of pounds of cracked ice were needed for a single day's experiment. The area a of the bottom of the small ice-pot was assumed to be 126.70 sq. cm.: this includes a slight correction for the thickness of the brass walls. The latent heat h of melting ice was assumed to be 79.25.

If, then,

r = rate of melting of ice in the pot, in grams per second,
t = temperature of difference between the faces of the slab,
d = thickness of the slab,
k = mean thermal conductivity of the slab,

$$k = \frac{r \cdot d \cdot h}{t \cdot a}.$$

TABLE I.

Material.	Weight of the slab in kilograms.	Thickness of the slab in centimeters.	Density.	Temperature of the warm face of the slab.	Temperature of the cool face of the slab.	Rate of melting of ice in the pot in grams per second.	Thermal conductivity of the specimen.
Trap	97.53	8.90	2.90	83.6	23.3	0.0340	0.0031
Trap	104.80	9.70	2.82	83.7	22.1	0.0365	0.0036
Amygdaloid .	98.45	9.68	2.67	82.7	15.2	0.0385	0.0035
Amygdaloid .	95.26	9.30	2.71	83.3	22.1	0.0358	0.0034
Conglomerate	78.02	9.25	2.55	82.1	23.3	0.0480	0.0047
Conglomerate	97.50	9.85	2.64	81.1	24.7	0.0476	0.0052

Table I gives some of the data of my final determinations. It would have been easy to give the value of k with greater apparent accuracy, but, in view of the lack of homogeneity in some of the slabs, this would have been misleading. The trustworthiness of results obtained with the apparatus described above is discussed in these Proceedings for August, 1898.

Table II shows the results of other determinations of the thermal conductivities of different specimens of rock. Many of these, as will be seen, were made by Messrs. Herschel, Lebour, and Dunn for the British Association for the Advancement of Science. The names of the dif-

TABLE II.

Substances.	Observers.	Conductivities.
Fire brick	B. A. Committee	0.0017
Fine red brick	"	0.0015
Chalk	"	0.0020 to 0.0033
Tough dry clay	"	0.0022
Devonshire clay slate	"	0.0027
Whinstone trap rock	"	0.0028 to 0.0048
Caen stone	"	0.0043
Cornish red serpentine	"	0.0044
English marbles and limestones	"	0.0047 to 0.0056
Granite	"	0.0053
Sandstone and hard dry grit	"	0.0055
English plate glass	"	0.0023
American plate glass	Peirce and Willson	0.0026 to 0.0028
Carrara white marble	"	0.0051
American and Italian colored marbles	"	0.0062 to 0.0076
" Sugar-white marble "	Despretz	0.0077
" Fine-grained gray marble "	"	0.0097
Calton Hill trap (damp)	Kelvin	0.0042
Craigleith Quarry sandstone (damp)	"	0.0107
Greenwich Park gravel (damp)	Everett	0.0125
Sandy loam	Neumann	0.0136
Serpentine	"	0.0059

ferent substances are of course rather indefinite. My own experience shows that even thick sheets of fine plate glass bought at different times of the same maker may have conductivities which differ from each other by as much as eight per cent; while the conductivities of slabs of dry white marble from different quarries may differ as much as thirty per cent. In the table I have, in two or three cases, taken the average of nearly agreeing results.

It is evident that the conductivities of the specimens of dry rock which I have examined are relatively rather low.

The Jefferson Physical Laboratory,
April, 1903.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

ON FAMILIES OF CURVES WHICH ARE THE LINES OF CERTAIN PLANE VECTORS EITHER SOLENOIDAL OR LAMELLAR.

BY B. O. PEIRCE.

ON FAMILIES OF CURVES WHICH ARE THE LINES OF CERTAIN PLANE VECTORS EITHER SOLENOIDAL OR LAMELLAR.

BY B. O. PEIRCE.

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IF a vector function has no component parallel to the axis of z and if the tensors of its components taken parallel to the axes of x and y can be expressed by the scalar point functions $X = \phi_1 (x, y)$, $Y = \phi_2 (x, y)$, which are independent of z, every line of the vector is a curve parallel to the xy plane, defined by the equations $\frac{dx}{X} = \frac{dy}{Y} = \frac{dz}{0}$, and it is sometimes convenient to call the vector itself "plane" and to say that it is "coplanar with" $z = 0$. The projection on the xy plane of any line of such a vector is itself a line of the vector, and a survey of the whole field can be obtained by studying the lines which lie in this plane.

The "divergence" of a vector coplanar with the xy plane is the quantity $\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y}$, and the "curl" of the vector is a vector, directed parallel to the z axis, of intensity $\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}$. If the divergence is zero in any region, the vector is said to be "solenoidal" in that region; a vector the curl of which vanishes is said to be "lamellar."

Given any family of curves in the xy plane represented by the equation $u \equiv f_1 (x, y) = c_1$, it is possible to find an infinite number of plane vectors which have the u curves as lines, by assuming in each case X at pleasure, and then making

$$Y = -X \cdot \frac{\dfrac{\partial u}{\partial x}}{\dfrac{\partial u}{\partial y}}.$$

The vector (X_0, Y_0) and the vector $(R \cdot X_0, R \cdot Y_0)$, where R is any function of xy, evidently have the same lines, and, if (X_0, Y_0) has for lines the u curves, no other vector has the same lines unless it is of the form $(R \cdot X_0, R \cdot Y_0)$. Of all the vectors which have the u curves for lines some are lamellar, for, if v is any function orthogonal to u, defined by the equation

$$\frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial y} = 0,$$

so that the curves of the families $u = c_1$, $v = c_2$ cut one another at right angles, the vector which has the components $\left(\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}\right)$ has for its lines the u curves, and it is lamellar, since

$$\frac{\partial^2 v}{\partial x \cdot \partial y} = \frac{\partial^2 v}{\partial y \cdot \partial x}.$$

If (X_0, Y_0) which has the u curves for lines is lamellar, so is the vector $[X_0 \cdot F(v), Y_0 \cdot F(v)]$, where F represents any ordinary function; and no lamellar vector has the same lines unless it is of the form just given.

If (X_1, Y_1) is a solenoidal vector which has the u curves for its lines, the vector $[X_1 \cdot F(u), Y_1 \cdot F(u)]$ has the same lines and is also solenoidal; no solenoidal vector has these lines unless it can be written in this form. It will soon appear that of all the vectors the lines of which are the u curves, some are always solenoidal, but no vector which has these curves for lines can be both solenoidal and lamellar, unless u happens to satisfy Lamé's condition for isothermal parameters,* that is, unless $\frac{\nabla^2(u)}{h_u^2}$ is expressible as a function of u alone, where

$$\nabla^2(u) = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}, \quad h_u^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2.$$

If a set of orthogonal curvilinear coördinates in the xy plane be defined by the functions

$$u = f_1(x, y), \quad v = f_2(x, y);$$

and if

$$U = \xi(x, y), \quad V = \eta(x, y)$$

represent the magnitudes, at the point (x, y), of the components, taken in the directions in which u and v increase most rapidly, of a vector, Q, coplanar with $z = 0$; it is not difficult to prove, by direct transformation or otherwise, that the divergence of Q is given by the well known expression

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* Lamé, Leçons sur les coordonnées curvilignes, p. 31; Leçons sur les fonctions inverses, p. 5; Somoff Ziwet, Theoretische Mechanik, I. 113 and 128.

$$\text{Div. } Q \equiv \left(\frac{U}{h_u}\right) \nabla^2 (u) + h_u^2 \cdot \frac{\partial}{\partial u}\left(\frac{U}{h_u}\right) + \left(\frac{V}{h_v}\right) \nabla^2 (v) + h_v^2 \cdot \frac{\partial}{\partial v}\left(\frac{V}{h_v}\right) \tag{1}$$

and that

$$\text{Tensor curl } Q \equiv h_u \cdot h_v \left[\frac{\partial}{\partial u}\left(\frac{V}{h_v}\right) - \frac{\partial}{\partial v}\left(\frac{U}{h_u}\right)\right]. \tag{2}$$

If the lines of Q coincide with the u curves, the vector has no component perpendicular to these curves and U is everywhere equal to zero, so that

$$\text{Div. } Q \equiv \frac{V}{h_v} \cdot \nabla^2 (v) + h_v^2 \cdot \frac{\partial}{\partial v}\left(\frac{V}{h_v}\right), \tag{3}$$

$$\text{Tensor curl } Q \equiv h_u \cdot h_v \cdot \frac{\partial}{\partial u}\left(\frac{V}{h_v}\right), \tag{4}$$

where h_v is the gradient of v.

In applying these expressions it is convenient to remember that

$$\frac{\nabla^2(u)}{h_u^2} = \frac{\partial \log\left(\frac{h_u}{h_v}\right)}{\partial u}, \qquad \frac{\nabla^2(v)}{h_v^2} = \frac{\partial \log\left(\frac{h_v}{h_u}\right)}{\partial v}.$$

It is easy to see from (3) and (4) that the statements which follow are true:

(*a*) If V is to be solenoidal,* we must have

$$\frac{\partial}{\partial v}\left(\log \frac{V}{h_v}\right) = -\frac{\nabla^2 (v)}{h_v^2}. \tag{5}$$

The second member of this equation is expressible as a function of u and v; if it be integrated with respect to v while u is considered constant, and if the arbitrary function $\chi(u)$ be added to the result, we shall get $\psi(u, v) + \chi(u)$ the partial derivative of which with respect to v is

$$-\frac{\nabla^2 (v)}{h_v^2}\,; \quad \text{then } V \equiv h_v \cdot e^{\chi(u)} \cdot e^{\psi(u,v)}. \tag{6}$$

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* See equation (18).

(*b*) If V is to be lamellar, we may write

$$V \equiv h_v \cdot \tau\,(v), \tag{7}$$

where τ is any ordinary function. Its divergence is

$$\tau\,(v) \cdot \nabla^2(v) + h_v^{\,2} \cdot \tau'\,(v).$$

If V is to be solenoidal as well as lamellar, we may obtain Lamé's condition immediately by substituting the value of V from (7) in (5).

(*c*) If, like the vector which defines the field of electromagnetic force within an infinitely long cylinder of revolution which carries lengthwise a uniformly distributed, steady current of electricity, V is solenoidal and a function of u only, we must have

$$V\left[\frac{\nabla^2(v)}{h_v} + h_v^{\,2}\,\frac{\partial}{\partial v}\left(\frac{1}{h_v}\right)\right] = 0,$$

or

$$2 \cdot \nabla^2(v) = \frac{\partial\,(h_v^{\,2})}{\partial v} \tag{8}$$

(*d*) If, like the attraction within a homogeneous, infinitely long, cylinder of revolution, V is lamellar and a function of v only, the gradient of v cannot involve u, so that

$$h_v = f\,(v), \text{ where } f \text{ is arbitrary, or } \frac{\partial h_v}{\partial u} = 0. \tag{9}$$

(*e*) If V is lamellar and a function of u only, $\frac{V}{h_v}$ must be independent of u and

$$\frac{\;\frac{\partial h_v}{\partial u}\;}{h_v} \text{ is a function of } u \text{ only.} \tag{10}$$

In this case h_v is either a function of u only or is expressible as the product of a function of u and a function of v.

(*f*) If V is to be solenoidal* and a function of v only, the expression

$$\frac{\;\frac{\partial h_v}{\partial v}\;}{h_v} - \frac{\nabla^2(v)}{h_v^{\,2}} = \frac{\;\frac{dV}{dv}\;}{V} \tag{11}$$

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* See equation (27).

must be either constant or expressible in terms of v. If V is not lamellar, h_v must in this case involve u.

(g) If V is lamellar and if Ω is a scalar potential function of V, Ω must be expressible in terms of v and the divergence of V is equal to

$$\frac{d\Omega}{dv} \cdot \nabla^2(v) + \frac{d^2\Omega}{dv^2} \cdot h_v^2. \tag{12}$$

(h) If the tensor of V has the same value for all values of x and y, V is lamellar if, and only if, h_v is constant or expressible in terms of v; it is solenoidal if, and only if,

$$2\nabla^2(v) = \frac{\partial h_v^2}{\partial v}. \tag{8}$$

(i) Whatever u is, the vector which has the components

$$X_1 = \frac{f(u)}{h_v} \cdot \frac{\partial v}{\partial x}, \qquad Y_1 = \frac{f(u)}{h_v} \cdot \frac{\partial v}{\partial y},$$

and the vector which has the components

$$X_2 = \frac{\phi(v)}{h_v} \cdot \frac{\partial v}{\partial x}, \qquad Y_2 = \frac{\phi(v)}{h_v} \cdot \frac{\partial v}{\partial y},$$

have the u curves for lines. The tensor of the first is a function of u only, that of the second a function of v only.

(j) If a solenoidal vector has the u lines for curves, its curl must be of the form $\phi(u) \cdot \nabla^2(u) + \phi'(u) \cdot h_u^2$, where ϕ is arbitrary. If, for instance, the u curves are concentric circumferences, the curl of the vector must be expressible as a function of the distance from the centre.

(k) If the tensor of a vector V which has the u curves for lines is a function of u only, its divergence is of the form $V\left(\frac{\nabla^2(v)}{h_v} - \frac{\partial h_v}{\partial v}\right)$. If the u curves are concentric circumferences, V must be solenoidal.

(l) If the tensor of V is expressible in terms of v, the tensor of its curl is $-V\frac{h_u}{h_v} \cdot \frac{\partial h_v}{\partial u}$. If the u curves are straight lines emanating from a point, the curl is zero and the divergence a function of the distance from the point. The velocity in the case of a steady squirt* motion of a gas illustrates this.

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* Minchin, Uniplanar Kinematics, 178, Examples 21 and 22.

THE GRADIENTS OF FUNCTIONS OF TWO INDEPENDENT VARIABLES.

Before we consider briefly some of the equations of condition which have just been stated, it will be well to make a few simple statements concerning the gradients * of functions of x and y.

The gradient of a function may or may not be expressible in terms of the function itself. The gradients of the expressions $(x^2 + y^2)$, $(x^2 - y^2)$ illustrate these two cases.

If the gradient of a function v is equal to $f(v)$, it is possible to form a function of v, $a\int \frac{dv}{f(v)}$, the gradient of which is constant.

If the gradient of a function v is equal to the constant a, it is possible to form two functions of v, namely $\frac{bv}{a}$ and $\frac{1}{a}\int f(v)\,dv$, the gradients of which are equal, respectively, to the arbitrarily chosen constant b and to the arbitrary function $f(v)$.

If the gradient of a function v is either constant or expressible in terms of v, the gradient of any differentiable function of v is expressible as a function of v.

If h_v is neither constant nor expressible in terms of v, no function of v exists the gradient of which is expressible in terms of v.

Since the gradients of two conjugate functions are numerically equal, it is clear that if h_v is expressible in terms of v, not all other functions the gradients of which are functions of v, are themselves expressible in terms of v.

If, for x and y in the expression

$$h_v^2 = \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2,$$

the quantities $\lambda = G(x, y)$, $\mu = H(x, y)$ be substituted, we shall obtain the new expression

$$h_v^2 = h_\lambda^2\left(\frac{\partial u}{\partial \lambda}\right)^2 + h_\mu^2\left(\frac{\partial u}{\partial \mu}\right)^2 + 2\left(\frac{\partial \lambda}{\partial x}\cdot\frac{\partial \mu}{\partial x} + \frac{\partial \lambda}{\partial y}\cdot\frac{\partial \mu}{\partial y}\right)\left(\frac{\partial v}{\partial \lambda}\cdot\frac{\partial v}{\partial \mu}\right) \tag{13}$$

and if we write $\lambda = (x + y\,i)$, $\mu = (x - y\,i)$,

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* Lamé, Leçons sur les coordonnées curvilignes, p. 6; Maxwell, Treatise on Electricity and Magnetism, § 17.

$$h_v^{\,2} = 4\,\frac{\partial v}{\partial \lambda}\cdot\frac{\partial v}{\partial \mu}: \tag{14}$$

from this last equation it is evident that if the gradient of v vanishes, v is either a function of $x + y\,i$ or a function of $x - y\,i$.

It is often convenient in dealing with differential equations which involve the gradients of functions, to use the independent variables of equation (14) and we may note that u and v, two functions of λ and μ, are conjugate if, and only if,

$$\frac{\partial u}{\partial \lambda} = i\,\frac{\partial v}{\partial \lambda}, \qquad \frac{\partial u}{\partial \mu} = -\,i\,\frac{\partial v}{\partial \mu}. \tag{15}$$

If u and v are orthogonal functions,

$$\frac{\partial u}{\partial \lambda}\cdot\frac{\partial v}{\partial \mu} + \frac{\partial u}{\partial \mu}\cdot\frac{\partial v}{\partial \lambda} = 0. \tag{16}$$

If the gradients of u and v, two real functions of x and y, are everywhere equal while the directions of their gradient vectors are different,

$$\frac{\partial (u - v)}{\partial x}\cdot\frac{\partial (u + v)}{\partial x} + \frac{\partial (u - v)}{\partial y}\cdot\frac{\partial (u + v)}{\partial y} = 0 \tag{17}$$

and the functions $(u - v)$ and $(u + v)$ are orthogonal. The converse of this statement is true. If two orthogonal functions have equal gradients these functions are conjugate.

If the gradient vectors of two functions have the same direction at every point of the $x\,y$ plane, one of these functions is expressible in terms of the other.

The quantities $u = \cos\,(b\,x - y)$, $v = \sin\,(b\,y + x)$ illustrate the fact that the gradient of each of two orthogonal functions may be expressible in terms of the function itself.

The quantities $u = x^2 + y^2$, $v = \tan^{-1}\left(\frac{y}{x}\right)$ illustrate the fact that the gradients of both of two orthogonal functions may be expressible in terms of one of the functions.

If the gradient of v, one of two orthogonal functions (u, v) is expressible in terms of u, or is constant, no other but a linear function of v has a gradient expressible in terms of u.

If the gradient of each of two orthogonal functions (u, v) is expressible as a product of a function of u and a function of v, so that

$$h_u = f(u) \,.\, F(v), \quad h_v = \phi(u), \quad \psi(v),$$

it is possible to find two functions, $\int \frac{du}{f(u)}$, $\int \frac{dv}{\psi(v)}$, of u and v respectively, the gradient of each of which is expressible in terms of the other.

A solution of Laplace's Equation and any function of its conjugate are orthogonal functions the ratio of the gradients of which is a function of the second function.

VECTOR POTENTIAL FUNCTIONS OF PLANE SOLENOIDAL VECTORS.

If u, v define a system of orthogonal curvilinear coördinates in the xy plane, and if Q_u, Q_v, Q_z are the components of a vector Q, taken in the directions in which u, v, z increase most rapidly, the components of the curl of Q in these directions are

$$h_v \left[\frac{\partial Q_z}{\partial v} - \frac{\partial}{\partial z}\left(\frac{Q_v}{h_v}\right)\right], \qquad h_u \left[\frac{\partial}{\partial z}\left(\frac{Q_u}{h_u}\right) - \frac{\partial Q_z}{\partial u}\right],$$
$$h_u h_v \left[\frac{\partial}{\partial u}\left(\frac{Q_v}{h_v}\right) - \frac{\partial}{\partial v}\left(\frac{Q_u}{h_u}\right)\right].$$

We may denote these quantities by K_u, K_v, K_z, respectively.

If Q is to be a vector potential function of a given solenoidal plane vector $(0, V, 0)$, which has the u curves for lines, we may assume that the components of Q involve u and v only, and since in this case, $K_u = 0$, $K_v = V$, write $Q_z = F(u)$, where $V = -h_u \,.\, \frac{dF(u)}{du}$. Any vector of the form $[Q_u, Q_v, F(u)]$, where Q_u, Q_v, are any functions of u and v subject only to the condition $\frac{\partial}{\partial v}\left(\frac{Q_u}{h_u}\right) = \frac{\partial}{\partial u}\left(\frac{Q_v}{h_v}\right)$, is a vector potential function of a solenoidal vector which has the u curves as lines, and there is no vector of this latter kind which does not have as a vector potential a vector of the form just given. In most cases it is simplest to make $Q_u = Q_v = 0$.

If, now, we ask what condition must be satisfied by the function u in order that the curves of the family $u = c$ may be the lines of a vector the tensor of which involves u only, we learn that, since V is of the form $-h_u \cdot F'(u)$, it is necessary and sufficient that h_u be a function of u only. That is

$$\frac{\partial h_u}{\partial v} = 0 \tag{18}$$

Since the divergence of any vector V which has the u curves for lines may be written in the form $h_u \, . \, h_v \, . \, \frac{\partial}{\partial v}\left(\frac{V}{h_u}\right)$ as well as in the form (3), the condition stated in equation (18) is at once obtained.

If we denote the quantities

$$\frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial^2 u}{\partial x^2}, \ \frac{\partial^2 u}{\partial x \, . \, \partial y}, \ \frac{\partial^2 u}{\partial y^2}, \qquad \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y}, \ \frac{\partial^2 v}{\partial x^2}, \ \frac{\partial^2 v}{\partial x \, . \, \partial y}, \ \frac{\partial^2 v}{\partial y^2},$$

by p, q, r, s, t, p', q', r', s', t', respectively, we have, since u and v are orthogonal,

$$pp' + qq' = 0; \tag{19}$$

whence by differentiation we get

$$p' r + p r' + q' s + q s' = 0, \tag{20}$$

$$p' s + p s' + q' t + q t' = 0. \tag{21}$$

We have, moreover, $\frac{\partial x}{\partial u} = \frac{h_x}{h_u} \, . \cos(x, u) = \frac{1}{h_u} \, . \, \frac{p}{h_u} = \frac{p}{{h_u}^2}$,

and, similarly, $$\frac{\partial y}{\partial u} = \frac{q}{{h_u}^2}, \qquad \frac{\partial x}{\partial v} = \frac{p'}{{h_v}^2}, \qquad \frac{\partial y}{\partial v} = \frac{q'}{{h_v}^2}, \tag{22}$$

so that $$\frac{\partial h_u}{\partial u} = \frac{\partial h_u}{\partial x} \, . \, \frac{\partial x}{\partial u} + \frac{\partial h_u}{\partial y} \, . \, \frac{\partial y}{\partial u} = \frac{1}{{h_u}^2}\left(\frac{\partial h_u}{\partial x} \, . \, \frac{\partial u}{\partial x} + \frac{\partial h_u}{\partial y} \, . \, \frac{\partial u}{\partial y}\right)$$

$$= \frac{p^2 r + 2 p q s + q^2 t}{{h_u}^3}, \tag{23}$$

and $$\frac{\partial h_u}{\partial v} = \frac{\partial h_u}{\partial x} \, . \, \frac{\partial x}{\partial v} + \frac{\partial h_u}{\partial y} \, . \, \frac{\partial y}{\partial v} = \frac{1}{{h_v}^2}\left(\frac{\partial h_u}{\partial x} \, . \, \frac{\partial v}{\partial x} + \frac{\partial h_u}{\partial y} \, . \, \frac{dv}{\partial y}\right)$$

$$= \frac{p' \left[p q (r - t) + s (q^2 - p^2)\right]}{q \, h_u \, {h_v}^2}.$$

Since, however, ${h_u}^2 \equiv p^2 + q^2$, $\quad {h_v}^2 \equiv p'^2 + q'^2$; $\quad q^2 \, . \, {h_v}^2 = p'^2 \, . \, {h_u}^2$,

and $$\frac{\partial h_u}{\partial v} = \pm \frac{(p q r + (q^2 - p^2) s - p q t)}{h_v \, . \, {h_u}^2}. \tag{24}$$

Equation (18) is equivalent, therefore, to the equation

$$p q r + (q^2 - p^2) s - p q t = 0. \tag{25}$$

If for r and t in (25) we substitute their values as obtained from (20) and (21), we shall get the equation

$$q'^2 r' - 2 p' q' s' + p'^2 t' = 0. \tag{26}$$

and this is (8) in expanded form.

If equation (18) or its equivalent (26) is satisfied, it is evident that by choosing $F(u)$ at pleasure we may find an infinite number of solenoidal vectors which have the u curves as lines and have tensors which involve u only.

A comparison of equations (9) and (18) shows that the condition that the u curves be possible lines of a set of solenoidal vectors the tensors of which involve u only, is the condition that the v curves be possible lines of a set of lamellar vectors the tensors of which involve u only.

If Q is a vector potential function of a solenoidal vector which has the u curves for lines, and a tensor expressible in terms of v, $-h_u F'(u)$ is a function of v, and h_u must be expressible as the product of a function of u and a function of v, that is,

$$h_u = f(u) \cdot \phi(v). \tag{27}$$

If for u in this differential equation we substitute w, defined by the equation $w \equiv \int \frac{du}{f(u)}$, we get the simpler equation

$$h_w = \phi(v) \quad \text{or} \quad \frac{\partial h_w}{\partial w} = 0. \tag{28}$$

It is to be noticed that w has the same lines as u, and that (27) and (28) define the same curves; the equations (11) and (28) are evidently equivalent.

If u is such that a solenoidal vector, V, can be found which has the u curves for lines and a tensor expressible in terms of v, its x and y components are $\left(\frac{V}{h_v} \cdot \frac{\partial v}{\partial x}, \frac{V}{h_v} \cdot \frac{\partial v}{\partial y}\right)$. If we denote these components by X, Y, every other solenoidal vector which has the same lines has components of the form $X \cdot \psi(u)$, $Y \cdot \psi(u)$, and the vector is not a function of v alone unless the factor $\psi(u)$ degenerates into a constant, and the vector is a simple multiple of V.

A comparison of (10) and (27) shows that if the u curves are possible lines of a solenoidal vector the tensor of which is expressible in terms of

v, the v curves are the possible lines of a lamellar vector the tensor of which is a function of v only.

If $\dfrac{\partial h_u}{\partial v} = 0$ and $\dfrac{\partial h_v}{\partial u} = 0$, the u curves are the lines of a set of solenoidal vectors the curls of which are expressible in terms of u only; and the v curves are the lines of a set of solenoidal vectors the curls of which are expressible in terms of v only.

Possible Systems of Isothermal Straight Lines and Isothermal Circles in a Plane.

(1) Let $\alpha x + \beta y = 1$, where α and β are any functions of a single parameter u, represent a family of straight lines in the xy plane, then we may write

$$\frac{\partial u}{\partial x} = \frac{-\alpha}{\alpha' x + \beta' y}, \qquad \frac{\partial u}{\partial y} = \frac{-\beta}{\alpha' x + \beta' y}, \qquad h_u^2 = \frac{\alpha^2 + \beta^2}{(\alpha' x + \beta' y)^2},$$

$$\frac{\nabla^2(u)}{h_u^2} = \frac{2(\alpha\alpha' + \beta\beta')}{\alpha^2 + \beta^2} - \frac{\alpha'' x + \beta'' y}{\alpha' x + \beta' y}. \qquad (29)$$

If then $\dfrac{\nabla^2(u)}{h_u^2}$ is to be a function of u only, the last term in the second member of this last equation must be expressible in terms of u only, and we have $\alpha' = 0$, or $\beta' = 0$, or, in general, $\alpha'' : \alpha' = \beta'' : \beta'$, so that $\alpha = c\beta + d$, where c and d are constants of integration. The equation of the family of lines must be of the form $(c\beta + d)x + \beta y = 1$, and the lines all pass through the fixed point $\left(\dfrac{1}{d}, -\dfrac{c}{d}\right)$, which may be chosen at pleasure. If $d = 0$, the lines are parallel.

(2) Let $x^2 + y^2 - 2\alpha x - 2\beta y = \gamma$, where α, β, γ are functions of a single parameter u, represent a family of circumferences in the xy plane, then we may write

$$\frac{\partial u}{\partial x} = \frac{2(x - \alpha)}{2\alpha' x + 2\beta' y + \gamma'}, \qquad \frac{\partial u}{\partial y} = \frac{2(y - \beta)}{2\alpha' x + 2\beta' y + \gamma'},$$

$$h_u^2 = \frac{4(\alpha^2 + \beta^2 + \gamma)}{(2\alpha' x + 2\beta' y + \gamma')^2},$$

$$\frac{\nabla^2(u)}{h_u^2} = \frac{2\alpha\alpha' + 2\beta\beta' + \gamma'}{\alpha^2 + \beta^2 + \gamma} - \frac{1}{\alpha^2 + \beta^2 + \gamma} \cdot \frac{2\alpha'' x + 2\beta'' y + \gamma''}{2\alpha' x + 2\beta' y + \gamma'}. \qquad (30)$$

If $\frac{\nabla^2(u)}{h_u^2}$ is a function of u only, the last term in the second member of (30) must be expressible in terms of u only. If $\alpha' = \beta' = 0$, we have a family of concentric circumferences. In general we may write $\alpha'' : \alpha' = \beta'' : \beta' = \gamma'' : \gamma'$, or $\beta = m\,\alpha + n$, $\gamma = 2\,k\,\alpha + l$, so that the equation of the circles must be of the form

$$x^2 + y^2 - 2\,\alpha\,x - 2\,y\,(m\,\alpha + n) - 2\,k\,\alpha - l = 0, \qquad (31)$$

where α is the only parameter. If we represent the first member of this equation by S_α, the equation $S_{\alpha_1} - S_{\alpha_2} = 0$ represents the straight line through the points of intersection of the circles which correspond to the two values α_1, α_2 of the parameter. In this case the line is $x + my + k = 0$, whatever the values of α_1 and α_2, therefore, as is well known, the system of isothermal circles * must pass through two fixed, real or imaginary, points.

FUNCTIONS THE GRADIENTS OF WHICH ARE EXPRESSIBLE IN TERMS OF THE FUNCTIONS THEMSELVES.

Several of the conditions stated in the previous pages [see (d), (h), and equation (18)] require that the gradient of a function be expressible in terms of the function itself, so that the normal derivative of the function has the same numerical value at all points of any one of its curves of level. We may state this requirement in a somewhat simpler form, however, if we remember that since the gradient of any function, ϕ, of u is equal to $\phi'(u) \cdot h_u$, the lines of all functions which satisfy the equation $h_u = f(u)$, whatever f may be, are included in the lines of functions which satisfy the equation $h_u = k$, where k is any constant (for instance 1). Every such family of lines forms a set of parallel curves. We have to solve, then, the equation

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = k^2, \qquad (32)$$

one of the standard forms for partial differential equations of the first order.

Its complete integral is

$$u = a\,x + y\,\sqrt{k^2 - a^2} + c,$$

and its general integral,

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* Darboux, Leçons sur la théorie générale des surfaces.

$$u = a\,x + y\sqrt{k^2 - a^2} + \psi(a), \tag{33}$$

subject to the condition

$$0 = x - \frac{a\,y}{\sqrt{k^2 - a^2}} + \psi'(a).$$

The equation $h_u = k$ is also equivalent to the equation,

$$\frac{\partial u}{\partial \lambda} \cdot \frac{\partial u}{\partial \mu} = 1. \tag{34}$$

The complete integral of (35) is

$$u = a\,\lambda + \frac{\mu}{a} + c,$$

and its general integral * may be found by eliminating a between the equations,

$$u = a\lambda + \frac{\mu}{a} + \phi(a), \qquad 0 = \lambda - \frac{\mu}{a^2} + \phi'(a). \tag{35}$$

If u is to be harmonic while h_u is expressible in terms of u, u is of the form $\phi(\lambda) + \psi(\mu)$, where $\lambda = x + yi$, $\mu = x - yi$. Since

$$h_u{}^2 = 4\,\frac{\partial u}{\partial \lambda} \cdot \frac{\partial u}{\partial \mu},$$

we must have

$$4\,\phi'(\lambda) \cdot \psi'(\mu) = 4\,f\,[\phi(\lambda) + \psi(\mu)],] \tag{36}$$

and if we differentiate both sides of this equation with respect to λ and μ we shall get

$$\phi''(\lambda) \cdot \psi'(\mu) = \phi'(\lambda) \cdot f'\,[\phi(\lambda) + \psi(\mu)],$$

$$\phi'(\lambda) \cdot \psi''(\mu) = \psi'(\mu) \cdot f'\,[\phi(\lambda) + \psi(\mu)],$$

whence

$$\frac{\phi''(\lambda)}{[\phi'(\lambda)]^2} = \frac{\psi''(\mu)}{[\psi'(\mu)]^2}. \tag{37}$$

Since the first member of (37) involves λ only, and the second member μ only, we may equate each member to a constant, $-\,k^2$, and consider separately the cases where k is or is not zero.

---

* Forsyth, Differential Equations, p. 307.

(1) If $k = 0$, $\phi''(\lambda) = 0$, $\psi''(\mu) = 0$, $\phi(\lambda) = c\lambda + m$,

$$\psi(\mu) = d\mu + n, \quad \text{and} \quad u = c\lambda + d\mu + g.$$

If c and $\pm d$ are either real and equal, or conjugate complex quantities, the u curves are a set of real parallel straight lines.

(2) If $k \neq 0$, $\phi'(\lambda) = \dfrac{1}{k^2\lambda + m}$, $\psi'(\mu) = \dfrac{1}{k^2\mu + n}$,

$$\phi(\lambda) = \frac{1}{2^2} \cdot \log(k^2\lambda + m) + a, \quad \psi(\mu) = \frac{1}{k^2} \cdot \log(k^2\mu + n) + b,$$

and if the constants of integration are so chosen as to make u real or purely imaginary, the u curves are a set of parallel, that is, concentric, circumferences.

Every family of isothermal lines which are the curves of a function u which satisfies (18) is either a set of parallel straight lines or a set of concentric circumferences. No other families of parallel curves are isothermal.

THE EQUATION $h_u \cdot \dfrac{\partial h_u}{\partial u} = \nabla^2(u)$.

We have seen that equations (18) and (26) are equivalent; this equation, therefore, defines the families of straight lines which form the orthogonal trajectories of the families of parallel curves defined by the equation $h_v = f(v)$, and we may write

$$q^2 r - 2pqs + p^2 t = 0. \tag{38}$$

Monge's method yields the first integral $u = F\left(\dfrac{p}{q}\right)$, (39)

and of this equation

$$u = \phi\left(\frac{a + y}{c - x}\right) \tag{40}$$

is the complete integral and

$$u = \phi\left(\frac{a + y}{\psi(a) - x}\right), \tag{41}$$

where $\psi(a) - x = (a + y)\,\psi'(a)$, the general integral.

Every family of straight lines in the xy plane, that is every set of lines defined by the equation $\alpha x + py = 1$, where α and p are arbitrary functions of a single parameter, are contained as, of course, they should be, in this general integral.

It is evident that every family of isothermal lines which are the curves of a function u which satisfies (38) is a set of straight lines which pass through a point.

TRANSFORMATION OF THE EQUATION $h_u = f(v)$.

Given a function which satisfies (10) or (27), there always exists a function which has the same lines, and a gradient expressible in terms of the orthogonal function alone. The lines of all functions which satisfy these equations are therefore those of functions which satisfy an equation of the form

$$\frac{\partial h_u}{\partial u} = 0 \tag{42}$$

or

$$p^2 r + 2\, p\, q\, s + q^2 t = 0. \tag{43}$$

If we take advantage of the Principle of Duality and make $p = x'$, $q = y'$, $p\,x + q\,y - z = z'$, we shall get the transformed equation

$$x'^2 \cdot t' - 2\, x' y' \cdot s' + y'^2 \cdot r' = 0, \tag{44}$$

and if then we put, $m = -2 \log (x^2 + y^2)$, $n = \tan^{-1}\left(\frac{y}{x}\right)$, the result is

$$\frac{\partial u}{\partial m} = \frac{\partial^2 u}{\partial n^2}, \tag{45}$$

which is equivalent to Fourier's familiar equation for the linear flow of heat.

If u is to be harmonic, while $\frac{\partial h_u}{\partial u} = 0$, we may write

$$u = \phi\,(x + yi) + \psi\,(x - yi),$$

and substitute this value in equation (43).

The resulting equation is

$$[\phi'\,(x + yi)]^2 \cdot \psi''\,(x - yi) + [\psi'\,(x - yi)]^2 \cdot \phi''\,(x + yi) = 0, \tag{46}$$

or

$$\frac{\psi''\,(x - yi)}{[\psi'\,(x - yi)]^2} = -\frac{\phi''\,(x + yi)}{[\phi'\,(x + yi)]^2}. \tag{47}$$

This last equation is possible only if each member is constant, $(-k^2)$, whence $\psi = \frac{1}{k^2} \cdot \log(k^2 \mu + m) + a$, $\phi = -\frac{1}{k^2} \cdot \log(k^2 \lambda + n) + b$, and the u curves are a family of straight lines meeting in some point. No other families of isothermal lines are possible curves of scalar functions, the gradients of which are expressible in terms of the corresponding orthogonal functions.

THE JEFFERSON LABORATORY, CAMBRIDGE, MASS.,
April, 1903.

Proceedings of the American Academy of Arts and Sciences.

Vol. XXXVIII. No. 25. — May, 1903.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

THE SPECTRA OF GASES AND METALS AT HIGH TEMPERATURES.

By John Trowbridge

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

THE SPECTRA OF GASES AND METALS AT HIGH TEMPERATURES.

By John Trowbridge.

Presented April 8, 1903. Received May 2, 1903.

In previous papers I have described certain phenomena which arise in the employment of photography in spectrum analysis — especially the phenomenon of the appearance of dark lines instead of bright lines in the spectra of electrical discharges in Geissler tubes.

The application of photography to spectrum analysis has the great advantage of giving an impersonal record of certain phenomena by substituting a chemical method of investigation for eye observations.

We thus obviate the personal equation of the observer; but unfortunately we bring in vagaries of the photographic plate.

The photographic plate can be called an instrument with an infinite number of adjustments. The molecular movements of the silver molecules under different degrees of electric stimulation can give us a great number of combinations. In short, the photographic plate does not afford a simple method of observing the effects of different waves of light on the molecules of matter. Its complicated nature is well shown by the records it gives of the intensity of light which provokes its action. It has long been recognized that the blackness of a negative is no criterion of relative intensity of light; in other words, that the negative cannot be employed as a photometer except in the crudest way. A method of illustrating this fact is given in this paper.

The photographic method of observation in spectrum analysis, however, is naturally of great use in preserving records of a great multiplicity of phenomena; but we have to be on our guard in interpreting these phenomena; for, unlike the galvanometer or the bolometer, our recording instrument is complex. Furthermore, when we use an electric spark to agitate the molecules of a gas or the molecules of a metal we use one complicated means to study a still more complicated phenomenon. When one asks what is the spectrum of water vapor, one must define the conditions of the electrical stimulus; for one can, by increasing the range of this electrical stimulus, run through a gamut of dissociations and recombinations. One never deals with a strictly pure vapor or gas.

The spectra of metals in atmospheric air are the visible evidence of extremely complicated chemical reactions due to the reaction of the metallic vapor and the gases of the atmosphere. The spectra of gases also in narrow containing vessels of glass or of quartz, are modified by the walls of these vessels when the temperature of the gases is very high; moreover, the ordinary method of obtaining photographic spectra either of metals in air or rarefied gases by long continued discharges produced by the Ruhmkorf coil of transformers masks certain fundamental reactions.

It is therefore desirable to study the effect of known quantities of energy successively applied to produce spectra either of metals or gases. This can best be accomplished by charging a condenser to a known amount by a known electromotive force, and by discharging the condenser between terminals of metals either in air or in gases. If the spectra produced in this manner by discharges varying from one to any desired number are photographed on the same plate and treated alike in the same developer, the ground may be prepared for some generalization of the extremely complicated reactions I have mentioned; I believe that this method is a fundamental one to use if order is to be brought out of the chaos of spark spectra.

I have applied this method in the following manner: A storage battery of from ten thousand to twenty thousand cells is employed to charge a condenser — .1 to .3 microfarad. By a simple mechanical appliance the condenser is detached from the poles of the battery, and is discharged between suitable terminals. Although it is impossible to avoid a slight spark at the moment of making contact with the receiving system, an approximately equal quantity of electricity is communicated to this system at each discharge. The method also permits of a definite control.

In order to photograph on the same plate the spectra produced by successive discharges the photographic plate must be slid vertically from one position to another, at the focus of a Rowland grating. The most convenient arrangement for the study of gaseous spectra is to employ a grating of short focus, and to enclose it in a light, tight box. For compactness, and to dispense with a dark room, I have employed the method of mounting the grating in such a manner that the normal to the ruled surface passes through the slit. The camera swings on the arc of a circle described by an arm of half the radius of the grating. This arm is pivoted at a point half way between the ruled surface and the slit. In Figure 1, G is the grating, M the point midway between the grating and the slit, C the camera swinging on the arc described from M. Figure 2 gives a side view and elevation of the camera. P is the photographic plate, O an opening closed by a slide operated by a lever arm which engages with A. The plate holder closed by another slide S, can move up or down in parallel ways. In Figure 1 is shown a lever arm LM with fulcrum at F. This lever lifts another lever AM, which in turn lifts or closes the shutter A, Figure 2. The lever LM is outside the dark box, and the operation of exposing the plate and closing the camera can be performed without opening the box. The method which Rowland used in mounting the grating is undoubtedly preferable to the above, when an accuracy greater than one-tenth of an Ångstrom unit is aimed at. The advantage, however, of the method I have employed is in its compactness and in the possibility of working in a light room; moreover, in gas spectra an accuracy even to one Ångstrom unit is often respectable.

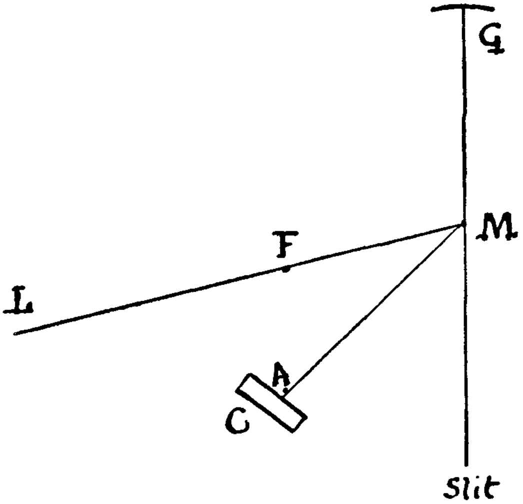


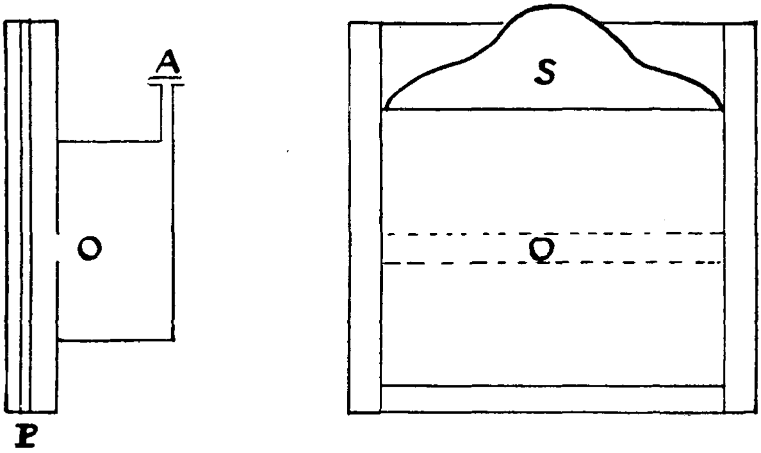
Fig I

The capillaries which I used in this investigation may be termed scientific electric furnaces for studying by photography the spectra of gases at high temperatures, and the spectra of the vapor of metals at such temperatures in rarefied media. These capillaries were from four to five inches long, with internal bore of from one millimeter to one and a half millimeters. The metallic terminals were approximately of the same diameter as the capillary, and were inserted in the capillaries until the distance between the ends of the terminals in the capillaries varied from one centimeter to two millimeters.

The capillaries were made of lead glass; of German glass which did not contain an appreciable amount of lead; and of quartz.

When an electric discharge occurs near a luted joint or surface a species of what may perhaps be termed electric distilling takes place. If silicate of soda is the luting agent, the vessel containing the rarefied gas becomes coated with a white film which shows the soda reaction; if a preparation of gums or shellac is used an organic film is obtained.

Fig 2



By the use of long capillaries in which the discharging metallic points are several inches from the points of luting this distilling action can be entirely avoided. It is therefore not necessary to use platinum for closing the ends of the capillaries in order to exhaust the latter. Quartz capillaries can be used without fear of introducing impurities from the luting agents, if long terminals are employed with the luting points at a distance from the discharge points.

In the use of capillaries with metallic terminals almost in contact with the walls of the containing vessels there must necessarily be spectra, one would suppose, arising from the substances of these walls and a combination of these substances with the rarefied gases. It is also possible that even if the walls do not show a characteristic spectrum they may influence by a species of katalytic action the reaction between the rarefied gases and the vapors of the metallic terminals.

It is interesting to observe the appearance of successive orders of spectra photographed by the camera I have described with progressively increasing strengths of electrical discharges as well as repeated discharges of the same strength. When, for instance, the capillaries are filled with rarefied water vapor one generally obtains with comparatively weak electrical discharges hydrocarbon spectra which possibly arise from traces of carbonic acid in the capillaries or from traces of grease in the stopcocks of the pump. These banded spectra decrease in strength as the strength of the electrical discharges increases, and finally disappear with powerful discharges. The hydrocarbon combinations are evidently broken up and new reactions and combinations enter. We therefore may not be able to recognize in these new reactions the presence of a simple component.

In the study of the spectra of heavenly bodies which may be considered from the point of view of furnaces, this fact I think should be borne closely in mind. Substances may be present which do not appear in the spectra observed at great distances; for the relative brightness of metallic lines and gaseous lines is much modified by the combinations which enter in an environment of high temperature.

Figure 3 (the photographs are reproductions of the negatives, and are not positives) is a reproduction of the spark spectrum of calcium in the neighborhood of the H.H. lines of the solar spectrum taken by the method of successive discharges of known amounts of electrical energy. The discharges ran from one to five. It is interesting to notice on the negatives that the photometric intensity of the lines estimated by the blackness is not directly proportional to the amount of energy. Thus the spectrum produced by four discharges is not twice as intense as that produced by two discharges. It is noticeable, also, that the calcium lines, wave lengths 3737 and 3706, are stronger on the negative than those which coincide with the H.H. lines, and always appear with these lines.

Figure 4 is a negative of discharges running from one to four in a Geissler tube of glass which did not contain an appreciable amount of lead in its composition. The capillary was two millimeters, internal diameter, and four inches in length. The pressure of hydrogen was one millimeter. It is noticeable that the lines which coincide with the H.H. lines are not accompanied by the lines, wave lengths 3737 and 3706;

although the intensity of the lines coinciding with the H.H. lines, if they were calcium lines, would require the presence of these lines.

A series of experiments was undertaken with metallic terminals one centimeter apart, in lead glass. The diameter of the capillary varied from one to two millimeters. A spectrum similar to that of Figure 4 was obtained with the addition of certain lead lines, several of which were reversed on the side toward the ultra violet. The portion of these lines not reversed broadened toward the red end of the spectrum; and this broadening increased with the intensity of the discharge (Figure 7). No calcium lines appeared. When cadmium terminals were used many of the cadmium lines were reversed, and here also the bright portion of these lines was much broadened. In the case of cadmium no other lines was observed (Figure 6). The gaseous ions contributed little or nothing to the photographic effect.

When iron terminals were employed no iron lines were obtained even when the terminals were only three millimeters apart; nevertheless the lines and bands usually attributed to silicon came out with great intensity. When, however, aluminum terminals were substituted for iron terminals, aluminum lines, together with the supposititious silicon lines, were obtained. It was noticeable that the two lines coinciding with H.H. lines of the solar spectrum did not appear, while the two characteristic aluminum lines between the H.H lines came out reversed. The lines corresponding with the H.H. lines always appeared when a discharge of like intensity produced the spectrum of aluminum in air.

The iron of which the terminals were made was ordinary soft iron, with a melting point not far from 1100°, while the melting point of aluminum is between 700° and 800°. If the silicon is volatilized it is difficult to see why the iron gave no spectrum, while the aluminum yielded one, for there is not a very great difference between their melting points.

Another series of experiments then were made with metallic terminals in quartz capillaries varying in internal diameter from two to three millimeters, the terminals being one centimeter apart. The same spectra were observed as are represented in Figure 3, with an absence of the lines corresponding with the solar H.H. lines. This absence was noticeable, also, when metallic terminals were one centimeter apart in lead glass. Iron terminals gave no iron lines in the quartz tubes, while aluminum lines appeared when aluminum terminals were used instead of iron terminals.

When the metallic terminals were placed three millimeters apart in the quartz tubes the light from the tubes was very feeble, traces of metallic spectra appeared, and the walls of the quartz capillaries were speedily covered with thin films of the metals; even at the first discharge, before there was a sensible obscuration due to the formation of the films, there was no evidence of gaseous spectra. The main discharge appeared to be carried over by the metallic vapor and no dissociation of the gas was evident.

Measured by definite amounts of electrical energy, the rating of the intensity of spectral lines differs totally from existing eye estimates. The lines which coincide with the H.H. lines in the spectra of the metals with high melting points generally came out first on the photographic plate when the method of successive discharges was employed.

Rarefied nitrogen gave far less light than hydrogen, water vapor, or oxygen. When oxygen was employed, characteristic groups of doublets were obtained like the A and B groups in the solar spectrum. The heads of these groups apparently coincided with the middle of broad lines shown in Figure 4. The middle of these broad bands or broad lines coincides also with narrow lines usually attributed to silicon. Are certain lines attributed to silicon really oxygen lines? Salet, and also Rowland, assign the photometric intensity of 4 to the lines 4131.5 and 4126.5, 3 to the lines 3905 to 3855.7, and 10 to the line 2881. When the lines given on plate 4 and those on plate 7 are photographed on the same plate by the same number of discharges and are compared in regard to intensity, the rating is completely reversed, the lines at 4131.5 and 4126.5 and 3905 to 3855.7 being 10, and the line 2881 being three or four.

The broadening of what have been considered metallic lines in rarefied cases I consider a most interesting phenomenon. Only the strong lines of the spark spectrum of the metal in air seem to be reversed under the effect of powerful discharges in rarefied gases. This broadening appears to be the evidence of reactions between the vapor of the metal and the surrounding gases. In this connection it is well to bear in mind the fact that metals continue to give off gases for a long time when submitted to powerful electrical discharges in vacuum tubes. This has been shown by Dr. Rollins of Boston in his researches on X-rays.

These nascent gases are in condition to exhibit complicated reactions with the strongly heated metallic terminals.

My experiments lead me to strongly doubt conclusions drawn from the apparent absence of this or that element in the spectra of stars; for there is a strong possibility that reactions enter which may mask the presence of this gas or that metal.

The conclusions of my work thus far are as follows:

1. The metallic lines due to terminals in rarefied hydrogen, and rarefied air, when these terminals are one centimeter apart in glass or quartz capillaries, exhibit a reversed action. When this takes place it is generally coincident with the position of the line when the spectrum is taken in air, while the spectrum of the line on the least refrangible side is much broadened. This seems to indicate a gaseous product; an oxidization or hydration due to the dissociation of the air and water vapor present.

2. Highly heated rarefied hydrogen and rarefied air passing over containing walls of glass or amorphous silica give broad bands which apparently coincide with narrow silicon lines of far lesser intensity. These also I attribute to the dissociation of air and water vapor. The brilliancy of the light produced in this reaction is far greater when there is an excess of hydrogen in the tubes than when rarefied air fills them. It is a question whether lines produced by metals like silicon in their difficulty of volatilization in air are really due to the metals. I am inclined to attribute some of them to the environment; that is, to a reaction between the metal and the gases present.

3. Spark spectra of metals appear to represent complicated reactions of gases with the metallic vapor.

4. Metallic vapor carries the main portion of an electric discharge when these terminals are within three millimeters of each other in rarefied hydrogen or rarefied air. The gaseous ions, if the dissociation occurs, give little light.

5. The broadening of the light accompanying the reversed lines, if unsuspected, might lead one to conclude that a shift of the bright portion had occurred.

6. Since the iron lines do not appear under what seems favorable conditions while aluminum lines appear; while in other cases gaseous lines mask metallic spectra, it seems desirable to be cautious in regard to speculations in regard to types of stars.

7. Whatever may be the cause of the reversals of lines observed in narrow capillaries of glass or of quartz, it seems to me that it is a fact which should be reckoned with in photographic study of stars, especially in the case of sudden changes of light.

Jefferson Physical Laboratory,
Harvard University.

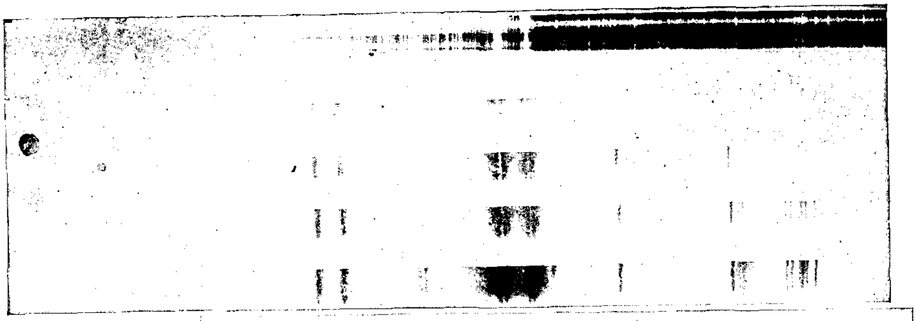


FIGURE 3.

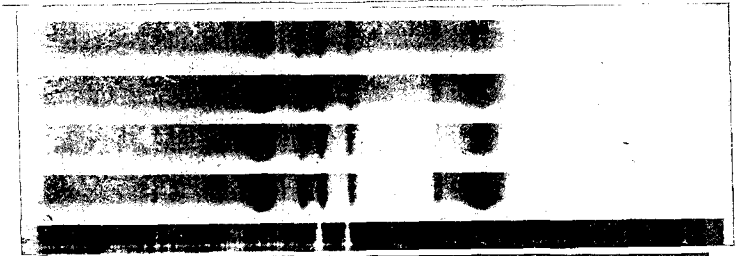


FIGURE 4.

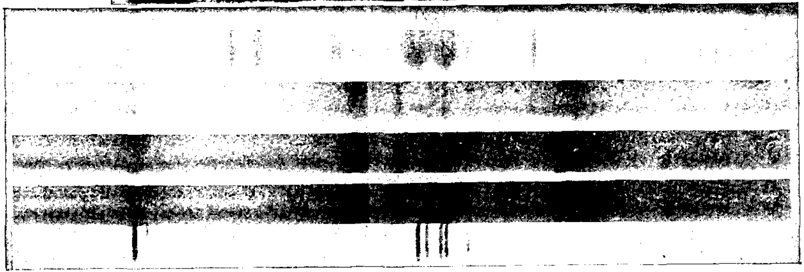


FIGURE 5.

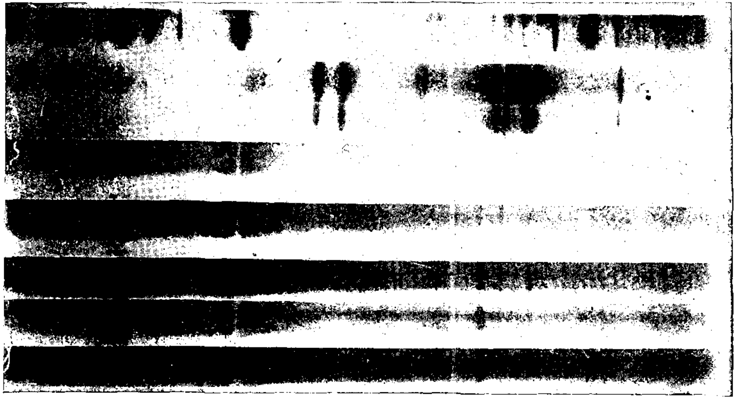


FIGURE 6.

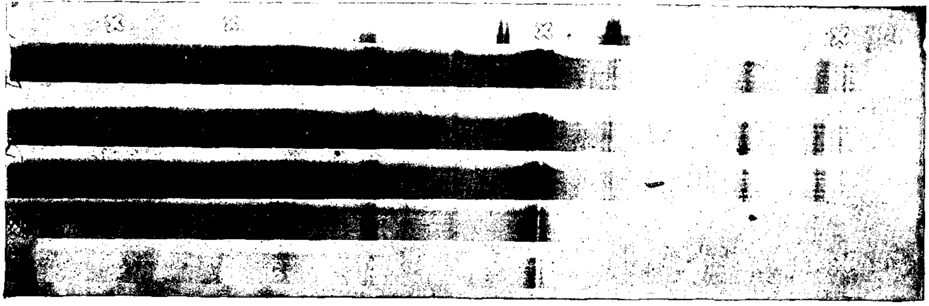


FIGURE 7.

Proceedings of the American Academy of Arts and Sciences.

VOL. XXXVIII. NO. 26. — JULY, 1903.

RECORDS OF MEETINGS, 1902–1903.

A TABLE OF ATOMIC WEIGHTS. BY THEODORE WILLIAM RICHARDS.

REPORT OF THE COUNCIL: BIOGRAPHICAL NOTICES.

ALPHEUS HYATT. BY ALPHEUS S. PACKARD.

JOHN DANIEL RUNKLE. BY HARRY W. TYLER.

OFFICERS AND COMMITTEES FOR 1903–1904.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM.

INDEX.

(TITLE PAGE AND TABLE OF CONTENTS.)

RECORDS OF MEETINGS.

Nine hundred and thirty-third Meeting.

OCTOBER 8, 1902. — STATED MEETING.

The PRESIDENT in the chair.

The Corresponding Secretary announced that letters had been received from Arthur James Balfour and W. E. H. Lecky, acknowledging their election as Foreign Honorary Members; from the Chief of the Department of Liberal Arts, World's Fair, St. Louis, inviting the Academy to make an exhibit of the work of its members; and from the International Botanical Congress, regarding the organization of the session at Vienna in 1905. Letters from the Trustees of Princeton University, inviting the Academy to be represented by a delegate at the inauguration of Woodrow Wilson as President of the University, and from the Trustees and Faculties of Northwestern University, requesting the Academy to appoint one or more delegates to the installation of Edmund Janes James as President of the University, were referred to the Executive Committee.

Letters were also received announcing the death of H. von Wild and Gaetano Negri; the institution of a section of terrestrial magnetism and seismology at the Royal Meteorological Institute of the Netherlands; the reorganization of the National Museum at Buenos Ayres; inviting the President and Secretary to the Jubilee Meeting of the North of England Institute of Mining and Mechanical Engineers.

The Chair announced the following deaths: —

John Daniel Runkle, Class I., Section 1; Horatio Hollis Hunnewell, Class II., Section 2; Horace Gray, Class III., Section 1; Charles Greely Loring, Class III., Section 4, Resident Fellows.

John Wesley Powell, Class II., Section 1, Associate Fellow.

Hervé Auguste Etienne Albans Faye, Class I., Section 1; Heinrich von Wild, Class II., Section 1; Rudolf Virchow, Class II., Section 4, Foreign Honorary Members.

On the motion of the Corresponding Secretary, it was

Voted, To meet, on adjournment, on the 12th of November.

The following gentlemen were elected members of the Academy: —

Harry Walter Tyler, of Boston, to be a Resident Fellow in Class I., Section 1 (Mathematics and Astronomy).

Alfred Edgar Burton, of Boston, to be a Resident Fellow in Class I., Section 4 (Technology and Engineering).

Hugo Münsterberg, of Cambridge, to be a Resident Fellow in Class III., Section 1 (Philosophy and Jurisprudence).

Frederic Jesup Stimson, of Boston, to be a Resident Fellow in Class III., Section 1.

Arthur Twining Hadley, of New Haven, to be an Associate Fellow in Class III., Section 3 (Political Economy and History).

Luigi Cremona, of Rome, to be a Foreign Honorary Member in Class I., Section 1 (Mathematics and Astronomy), in place of the late Charles Hermite.

Joseph John Thomson, of Cambridge, to be a Foreign Honorary Member in Class I., Section 2 (Physics), in place of the late Marie Alfred Cornu.

Emil von Behring, of Marburg, to be a Foreign Honorary Member in Class II., Section 4 (Medicine and Surgery).

John Morley, of London, to be a Foreign Honorary Member in Class III., Section 3 (Political Economy and History).

The President gave an account of his explorations of the coral reefs of the Maldives.

H. Helm Clayton read a paper entitled "On the Observed Movements of the Dust from the Volcanic Eruptions in the West Indies."

Edward Atkinson, in presenting the first two copies of a complete report issued by the Experiment Station in Insurance Engineering on Diffusion of Light and Corrosion of Steel, said:

"In our practice as underwriters we are called upon to deal with heat and light from a very different point of view as compared to that of pure science. In order to be prepared for questions in the future on so-called fire-proof construction, corrosion, and other matters, I have called upon my clients in the Factory Mutual Companies to assess themselves at the rate of one cent per hundred dollars of their insurance and to place in my hands a sum of money to be expended in what I have named the Insurance Engineering Experiment Station. In response I have received between fifteen thousand dollars ($15,000) and sixteen thousand dollars ($16,000), in spending which I have a free hand: but I have placed myself under an Advisory Committee consisting of the Directors of the Insurance Company of which I am President; chiefly under the advice of the Executive Committee, Messrs. Arthur T. Lyman, Howard Stockton, and Theophilus Parsons.

"We are about to have a sufficient area of land placed at our disposal on a long lease, and the representatives of various types of fire-proof construction are each to put up a building which shall be tested by fire and water and lapse of time, but which will also serve more than ample for our laboratories and experiments.

"All this is preliminary to laying out a course of instruction in Insurance Engineering and a year hence transferring the control of the Station to the Institute of Technology.

"In the interval if any members of the Academy want an opportunity to apply high temperature at or above the melting-point of cast-iron for a considerable period of time, we expect to have buildings of ten or twelve foot cube constructed for that purpose, and shall place them at their disposal.

"I will send to any member any reasonable number of copies of the report on the Diffusion of Light and the Corrosion of Steel on application by card addressed to my office, as we have issued a very large edition in order to interest men of Science, architects, and engineers in the general subject."

The following papers were presented by title: —

Contributions from the Harvard Mineralogical Museum.—XII. (1) "Babingtonite from Somerville, Mass." (2) "Babingtonite from Athol, Mass." By C. Palache and F. A. Fraprie. Presented by J. E. Wolff.

"On the Thermal Development of the Spark Spectrum of Carbon." By Henry Crew and John C. Baker. Presented by Charles R. Cross.

Nine hundred and thirty-fourth Meeting.

NOVEMBER 12, 1902. — ADJOURNED STATED MEETING.

VICE-PRESIDENT TROWBRIDGE in the chair.

The Corresponding Secretary announced that letters had been received from Hugo Münsterberg, F. J. Stimson, H. W. Tyler, accepting Resident Fellowship; Arthur T. Hadley, acknowledging election as Associate Fellow; E. v. Behring, John Morley, acknowledging election as Foreign Honorary Members; C. A. Young, accepting his appointment as representative of the Academy at the inauguration of the President of Princeton University; T. C. Chamberlin, accepting his appointment as delegate of the Academy at the installation of the President of Northwestern University; the Royal Academy of Sciences of Turin, announcing the death of its President, Alfonso Cossa; the Central Meteorological Institute of Sweden, announcing the death of its former Director, Robert Rubenson; and the Nobel Committee for Chemistry and the Nobel Committee for Physics of the Royal Academy of Sciences of Sweden, inviting competition for the Nobel prizes for 1903.

The following papers were presented: —

"The Fall of the Campanile of San Marco." By Arlo Bates.

"Recent Studies of the Lunar Surface." By William H. Pickering.

The Committee of Publication reported the acceptance of the following articles since the Annual Meeting: —

Contributions from the Chemical Laboratory of Case School of Applied Science. — XLII. "An Apparatus for Continuous Vacuum Distillation." By Charles F. Mabery.

Contributions from the Gray Herbarium of Harvard University. New Series. — No. XXIV. (Papers from the Hopkins-Stanford Expedition to the Galapagos Islands.) By B. L. Robinson. I. — "Flora of the Galapagos Islands."

Contributions from the Chemical Laboratory of Harvard College. "Concerning Gas-Analysis by Measurement in Constant Volume under Changing Pressure." By Theodore William Richards.

Contributions from the Chemical Laboratory of Case School of Applied Science. — XLIV. "A Method for Determining the Index of Refraction of Solid Hydrocarbons with the Pulfrich Refractometer. Index of Refraction of the Solid Hydrocarbons in Petroleum." By Charles F. Mabery and Lee Shepherd.

Contributions from the Chemical Laboratory of Harvard College. "The Significance of Changing Atomic Volume. III. The Relation of Changing Heat Capacity to Change of Free Energy, Heat of Reaction, Change of Volume, and Chemical Affinity." By Theodore William Richards.

Contributions from the Chemical Laboratory of Harvard College. "The Speed and Nature of the Reaction of Bromine upon Oxalic Acid." By Theodore William Richards and Wilfred Newsome Stull.

Contributions from the Chemical Laboratory of Harvard College. "The Range and Invariability of Faraday's Law." By Theodore William Richards and Wilfred Newsome Stull.

Contributions from the Chemical Laboratory of Harvard College. "An Apparatus for the Measurement of the Expansion of Gases by Heat under Constant Pressure." By Theodore William Richards and Kenneth Lamartine Mark.

Contributions from the Chemical Laboratory of Harvard College. "The Transition Temperature of Sodic Sulphate referred anew to the International Standard." By Theodore William Richards and Roger Clark Wells.

Nine hundred and thirty-fifth Meeting.

DECEMBER 10, 1902.

The PRESIDENT in the chair.

The Corresponding Secretary read letters from L. Cremona and J. J. Thomson, acknowledging their election as Foreign Honorary Members; and from the Entomological Society of Belgium, announcing the death of its President, Pierre-Jules Tosquinet.

The President announced the following deaths: —

Henry Mitchell, Resident Fellow in Class I., Section 1.

Ogden Nicholas Rood, Associate Fellow in Class I., Section 2.

Alfred Richard Cecil Selwyn, Associate Fellow in Class II., Section 1.

The following papers were presented: —

"Results obtained from the Use of Quartz Geissler Tubes in Spectrum Analysis." By John Trowbridge.

"A Theory of Tone-Relations — Colors, Intensities, Neutrality, Values." By Denman W. Ross.

The following paper was presented by title: —

"The Pressure due to Radiation." By E. F. Nichols and G. F. Hull. *Presented by* C. R. Cross.

Nine hundred and thirty-sixth Meeting.

JANUARY 14, 1903. — STATED MEETING.

VICE-PRESIDENT TROWBRIDGE in the chair.

On the motion of the Recording Secretary, it was

Voted, To meet on adjournment on the second Wednesday in February.

On the recommendation of the Rumford Committee, it was

Voted, To appropriate from the income of the Rumford Fund one hundred dollars ($100) for the preparation of a catalogue of the books on light and heat in the library of the Academy.

Voted, To transfer the appropriation of seven hundred and fifty dollars ($750), granted to Theodore W. Richards at the annual meeting of 1902, toward the prosecution of researches on the thermodynamical properties of chemical substances involved in chemical reactions.

Voted, To appropriate three hundred dollars ($300), from the income of the Rumford Fund to George E. Hale, of the Yerkes Observatory, for the purchase of a Rowland concave grating to be used in the photographic study of the spectra of the brightest stars.

On the motion of E. L. Mark, with the written approval of the Committee of Finance, it was

Voted, To appropriate from the income of the General Fund two hundred and fifty dollars ($250) for publishing.

The following papers were read: —

"Account of some Lunar phenomena." By N. S. Shaler.

"Some Recent Studies on Immunity." By W. T. Councilman.

The following papers were presented by title: —

"Mendel's Law of Heredity." By W. E. Castle.

"Synopsis of the Genus Lonicera." By Alfred Rehder. Presented by E. L. Mark.

"On the Temperature Coefficient of Chilled and Seasoned Cast Iron Magnets." By B. O. Peirce.

Nine hundred and thirty-seventh Meeting.

FEBRUARY 11, 1903. — ADJOURNED STATED MEETING.

VICE-PRESIDENT WALCOTT in the chair.

The Chair announced the following deaths: —

Morrill Wyman, of Class I., Section 4; James Elliot Cabot, of Class III., Section 4, Resident Fellows.

Sir George Gabriel Stokes, Baronet, of Class I., Section 2, Foreign Honorary Member.

Harold C. Ernst read a paper entitled, "A New Method of Stating Ehrlich's Theory of Immunity."

The following paper was presented by title: —

"Diffusion, Supersaturation, and the Metastable Limit." By Harry W. Morse and George W. Pierce. Presented by John Trowbridge.

Nine hundred and thirty-eighth Meeting.

MARCH 11, 1903. — STATED MEETING.

The RECORDING SECRETARY in the chair.

The Chair announced the death of Gaston Bruno Paulin Paris, Foreign Honorary Member in Class III., Section 4.

The Chair appointed the following councillors to serve as Nominating Committee: —

Charles R. Sanger, of Class I., Theobald Smith, of Class II., A. Lawrence Lowell, of Class III.

The following persons were elected members of the Academy:

George Ashley Campbell, of Boston, to be a Resident Fellow in Class I., Section 2 (Physics).

Albert Sauveur, of Cambridge, to be a Resident Fellow in Class I., Section 4 (Technology and Engineering).

Samuel Franklin Emmons, of Washington, to be an Associate Fellow in Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe), in place of the late Clarence King.

Alfred Thayer Mahan, of New York, to be an Associate Fellow in Class III., Section 3 (Political Economy and History).

Karl Alfred Ritter von Zittel, of Munich, to be a Foreign Honorary Member in Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe), in place of the late Friherre Adolf Erik Nordenskiöld.

The following papers were read: —

"The Pompeian Fresco called 'The Judgment of Solomon.'" By Morris H. Morgan.

"Origin of the Great Mountain Ranges." By William M. Davis.

Nine hundred and thirty-ninth Meeting.

APRIL 8, 1903.

The Academy met in the Geological Lecture Room of the Harvard University Museum, Cambridge.

The PRESIDENT in the chair.

In the absence of the Recording Secretary, W. M. Davis was appointed Secretary *pro tempore.*

The Chair announced the death of Henry Barker Hill, Resident Fellow in Class I., Section 3.

The following papers were presented by title: —

"On Families of Curves which are the Lines of Certain Plane Vectors either Solenoidal or Lamellar. By B. O. Peirce.

"On the Thermal Conductivities of Certain Pieces of Rock from the Calumet and Hecla Mine." By B. O. Peirce.

Contributions from the Gray Herbarium. New Series, No. XXV. "A Revision of the Genus Flaveria." By J. R. Johnston. Presented by B. L. Robinson.

"Spectra of Gases and Metals at High Temperatures." By John Trowbridge.

The grounds for the award of the Rumford Premium to George E. Hale, of the Yerkes Observatory, were stated by Charles R. Cross, chairman of the Rumford Committee. The medals were presented by the President, and Professor Hale responded, illustrating his recent researches in solar and stellar physics with projections on the screen.

The following exhibits in the adjoining halls of the Museum were then inspected: —

"Demonstration of some Phenomena of Optical Mineralogy." By J. E. Wolff.

"Photomicrographic Illustrations of North American Woods." By E. C. Jeffrey.

"The Soltwedel Plates of Sugar Cane." By G. L. Goodale.

"Plant Dissections, Illustrating the Study of Herbarium Specimens." By B. L. Robinson.

"Some Recent Discoveries in the New England Flora." By M. L. Fernald.

"A Method of Keeping Ornithological Notes." By William Brewster.

"Types of Indian Basketry." By F. W. Putnam and W. C. Willoughby.

"Phonograph Records of Indian Songs and Note Books of Indian Languages." By R. B. Dixon.

"Model of the Cahokia Mound Group." By F. W. Putnam and D. I. Bushnell, Jr.

"Illustrations of the Effects of the Earth's Rotation." By W. M. Davis.

"Selected Views from the Gardner Collection of Geological and Geographical Photographs." By P. S. Smith.

"Specimens of Fossil Brachiopods showing Details of Structure." By R. T. Jackson.

"A Two-Circle Goniometer, and a Collection of Cleaved Mineral Forms." By C. Palache.

"Rocks and Coals from the Rhode Island Coal Field, and other Specimens." By J. B. Woodworth.

"Recent Meteorological Charts and Atlases." By R. DeC. Ward.

"Cuspate Capes of our Atlantic Coast." By M. A. Read.

"Recent Folios of the Geologic Atlas of the United States." By L. Laforge.

"Thin Sections and Microphotographs of North American Woods." By E. C. Jeffrey.

"Contributions from the Cryptogamic Laboratories." By R. Thaxter.

Nine hundred and fortieth Meeting.

ANNUAL MEETING. — MAY 13, 1903.

VICE-PRESIDENT WALCOTT in the chair.

The following letters were read: —

From Albert Sauveur, accepting Fellowship; S. F. Emmons, A. T. Mahan, acknowledging election as Associate Fellows; Ed. Mazelle, announcing his appointment as Director of the Astronomical-Meteorological Observatory of Trieste.

The Chair announced the following deaths: —

William Sumner Appleton, Resident Fellow in Class III., Section 2; Josiah Willard Gibbs, Associate Fellow in Class I., Section 2.

The annual report of the Council was read by the Recording Secretary.

In the unavoidable absence of the Treasurer, and at his request, the Recording Secretary read the annual report of the Treasurer, of which the following is an abstract: —

General Fund.

Receipts.

Balance, April 30, 1902		$272.42
Investments	$5,824.68	
Assessments	985.00	
Admission fees	50.00	
Sales of publications	81.84	6,941.52
		$7,213.94

Expenditures.

General expenses		$2,916.42
Publishing	$2,962.86	
Library	1,200.83	
Catalogue	104.70	4,268.39
Balance, April 30, 1903		29.13
		$7,213.94

Rumford Fund.

Receipts.

Balance, April 30, 1902		$ 381.96
Investments	$2,651.31	
Sale of publications	4.07	2,655.38
		$3,037.34

Expenditures.

Researches	$2,015.00	
Medals	331.50	
Publishing	323.11	
Library	154.07	
Miscellaneous	15.87	$2,839.55
Balance, April 30, 1903		197.79
		$3,037.34

WARREN FUND.

Receipts.

Balance, April 30, 1902		$ 760.72
Investments	$1,030.19	
Appropriation refunded	130.66	1,160.85
		$1,921.57

Expenditures.

Investigations	$ 760.00
Balance, April 30, 1903	1,161.57
	$1,921.57

BUILDING FUND.

Receipts.

Balance, April 30, 1902	$ 689.55
Investments	665.37
	$1,354.92

Expenditures.

Income invested and transferred to capital account	$1,250.00
Balance, April 30, 1903	104.92
	$1,354.92

The following reports were also presented: —

REPORT OF THE LIBRARIAN.

The card-catalogue has been continued by the Assistant Librarian, who has type-written 2615 cards, making the total number of cards 6300. All the books on light and heat, constituting with periodicals not yet catalogued, the Rumford Library, have been catalogued, besides the books on chemistry, engineering, the useful arts and general natural history, and some 200 books on miscellaneous subjects. The usual appropriation of $100 from the income of the General Fund is requested for continuing this work, and $50 is requested from the income of the Rumford Fund for the same purpose.

The accessions during the year have been as follows: —

	Vols.	Parts of vols.	Pams.	Maps.	Total.
By gift and exchange . . .	507	2155	249	24	2935
By purchase — General Fund	35	512			547
By purchase — Rumford Fund	22	251	2		272
Total	564	2918	251	24	3757

21 volumes and 2 pamphlets on light and heat, recommended by the Rumford Committee, were bought at an expense of $68.96.

. . . .

The expenses charged to the Library are as follows: — Miscellaneous, which includes expenses in no way related to the Library, $602.27; Binding, $324.02, General, and $40.23, Rumford Funds; Subscriptions, $274.54, General, and $113.94, Rumford Funds; making a total of $1200.83 for the General and $154.07 for the Rumford Funds. An appropriation of $1400 from the income of the General Fund is requested in addition to the customary appropriation of $150 from the income of the Rumford Fund for the purchase of books and periodicals.

A. LAWRENCE ROTCH, *Librarian.*

REPORT OF THE RUMFORD COMMITTEE.

At the last Annual Meeting of the Academy, May 4, 1902, the sum of $1000 was placed at the immediate disposition of the Committee for use in furtherance of research. During the year which has elapsed since that date, the following grants have been made from this appropriation.

Dec. 10, 1902. One hundred and fifty dollars to Dr. Ralph S. Minor of Little Falls, N. Y., in aid of his research on the dispersion and absorption of substances for ultra-violet radiation.

Jan. 14, 1903. One hundred dollars to Dr. Sidney D. Townley of Berkeley, California, for the construction of a stellar photometer of the type devised by Professor Pickering.

Jan. 14, 1903. Two hundred dollars to Professor E. B. Frost for the coustruction of a special lens for use in connection with the stellar spectograph of the Yerkes Observatory to aid in the study of the radial velocities of faint stars.

Jan. 14, 1903. Two hundred and fifty dollars to Professors E. F. Nichols and G. F. Hull of Dartmouth College, in aid of their investigation on the relative motion of the earth and the ether by the method of Fizeau's polarization experiment. It was subsequently voted (April 8, 1903), to allow the grantees to change the application of this grant to an investigation of the effect of the motion of the earth on the intensity of radiation.

Feb. 11, 1903. One hundred and fifty dollars to Professor E. C. Pickering for the construction of two stellar photometers to be placed at the disposal of the Committee. It was furthermore voted that one of these be placed at the disposal of the Rev. Edmund Goetz, Director of the Southern Station of the Georgetown College Observatory.

The following votes have also been passed by the Committee: —

Dec. 10, 1902. That the Academy be recommended to appropriate one hundred dollars from the income of the Rumford Fund for the preparation of a catalogue of the books on light and heat in the library of the Academy, this being the second appropriation for the purpose.

Jan. 14, 1903. To recommend to the Academy the transfer of the appropriation of seven hundred and fifty dollars voted to Professor Theodore W. Richards at the Annual Meeting of 1902, toward the prosecution of an extended series of researches on the thermo-dynamics of substances involved in chemical reactions.

Jan. 14, 1903. To recommend to the Academy the appropriation of three hundred dollars from the income of the Rumford Fund to Professor George E. Hale of the Yerkes Observatory for the purchase of a Rowland concave grating to be used in the photographic study of the spectra of the brighter stars.

These recommendations were acted upon favorably by the Academy.

April 8, 1903. To request the Academy to appropriate from the income of the Rumford Fund the sum of seventy-five dollars to Mr. F. L. Bishop of the Bradley Polytechnic Institute in aid of his investigation on the thermal conductivity of lead; and also to appropriate the sum of two hundred dollars to Professor Frederick A. Saunders of the Syracuse University in aid of an investigation on the characteristics of the spectra produced under varying conditions, as specified in his application.

Feb. 11, 1903. To ask the Academy to appropriate from the income of the Rumford Fund the sum needed to purchase and bind certain books upon light and heat.

April 8, 1903. To ask the Academy to appropriate from the income of the Rumford Fund the sum of one hundred and fifty dollars for the purchase and binding of periodicals.

April 8, 1903. To request the Academy to appropriate the sum of one thousand dollars from the income of the Rumford Fund for the immediate needs of the Committee in furtherance of research.

May 13, 1903. To request the Academy to appropriate from the income of the Rumford Fund the sum of fifty dollars for cataloguing books and periodicals in the library of the Academy relating to light and heat.

Reports upon the progress of investigations aided by grants from the Rumford Fund have been received from Messrs. A. L. Clark, E. B. Frost, G. E. Hale, F. A. Laws, R. S. Minor, E. F. Nichols, A. A. Noyes, E. C. Pickering, T. W. Richards, S. D. Townley, R. W. Wood.

The Committee has considered the claims of a number of persons for the Rumford Premium but has decided to make no recommendation for an award this year.

CHAS. R. CROSS, *Chairman.*

REPORT OF THE C. M. WARREN COMMITTEE.

The C. M. Warren Committee has the honor to report that Professor Mabery and Professor A. A. Noyes have reported satisfactory progress in the work assisted by grants from the Warren Fund.

They regret to be obliged to announce that Professor Charles L. Jackson, owing to pressure of work, felt obliged to resign the Chairmanship of the Committee.

Professor Leonard P. Kinnicutt was elected Chairman to fill the vacancy caused by Professor Jackson's resignation.

The Committee recommends the following appropriations from the income of the C. M. Warren Fund: —

To Professor C. F. Mabery, Case School of Applied Science, Cleveland, O., to complete his researches on petroleum, for which work he has already received various grants from the C. M. Warren Fund, three hundred dollars.

To Professor H. O. Hofman, Massachusetts Institute of Technology, Boston, Mass., to complete his research on the decomposition of zinc sulphate, for which work one hundred and sixty dollars was granted last year, three hundred and fifty dollars.

To Dr. Samuel P. Mulliken, Massachusetts Institute of Technology, Boston, Mass., to assist him in continuing his investigation on a systematic procedure for the identification of compounds of carbon containing hydrogen, nitrogen, and oxygen; Professor C. L. Jackson dissenting, five hundred dollars.

LEONARD P. KINNICUTT, *Chairman.*

Report of the Committee of Publication.

The Committee of Publication begs leave to report that there have been published during the academic year 1902–3, one Memoir, Vol. V., No. 12; two numbers of Volume XXXVII. of the Proceedings, and the first twenty-three numbers of Volume XXXVIII., aggregating 812 pages and 7 plates.

Eight numbers of Volume XXXVIII. (Nos. 7, 10, 12, 14, 15, 19, 20, and 23), in all 138 pages, were printed at the cost of the Rumford Fund ($487.52). The total expenditure for printing — including $12.00 for binding — paid from the General Fund was $2962.86.

The unexpended balance May 14, 1902, was $443.00. The appropriation made at the annual meeting on that date was $2400.00. An additional appropriation of $250.00 was made Jan. 14, 1903, and the return from sales was $81.84. The total amount from the General Fund available for publication was therefore $3174.84. The unexpended balance is $211.98.

The Committee recommends for the year 1903–4 an appropriation of $2600.00.

For the Committee,

E. L. Mark, *Chairman.*

May 13, 1903.

On the recommendation of the Committee of Finance, it was

Voted, To make the following appropriations from the income of the General Fund for the expenditures of the Academy during the ensuing year: —

For general expenses		$3000
For publishing		2400
For the library		
Books and binding	$900	
Miscellaneous	500	
Catalogue	100	1500
		$6900

On the recommendation of the Rumford Committee, it was

Voted, To make the following appropriations from the income of the Rumford Fund: —

For the immediate needs of the Rumford Committee in furtherance of research, $1000.

For the purchase and binding of periodicals, $150.

For the purchase and binding of certain specified books on light and heat, $100.

For cataloguing books and periodicals in the library relating to light and heat, $50.

To F. L. Bishop, in aid of his investigation on the thermal conductivity of lead, $75.

To F. A. Saunders, in aid of his investigation on the characteristics of spectra produced under varying conditions, $200.

On the recommendation of the C. M. Warren Committee, it was

Voted, To make the following appropriations from the income of the C. M. Warren Fund: —

To C. F. Mabery, to complete his researches on petroleum, $300.

To H. O. Hofman, to complete his research on the decomposition of zinc sulphate, $350.

To S. P. Mulliken, to assist him in continuing his investigation on a systematic procedure for the identification of compounds of carbon containing hydrogen, nitrogen, and oxygen, $500.

On the motion of the Recording Secretary, it was

Voted, That the annual assessment for the ensuing year be five dollars ($5).

The annual election resulted in the choice of the following officers and committees: —

WILLIAM W. GOODWIN, *President.*
JOHN TROWBRIDGE, *Vice-President for Class I.*
HENRY P. WALCOTT, *Vice-President for Class II.*
JOHN C. GRAY, *Vice-President for Class III.*
WILLIAM M. DAVIS, *Corresponding Secretary.*
WILLIAM WATSON, *Recording Secretary.*
SAMUEL CABOT, *Treasurer.*
A. LAWRENCE ROTCH, *Librarian.*

Councillors for Three Years.

LEWIS J. JOHNSON, of Class I.
EDWARD H. BRADFORD, of Class II.
MORRIS H. MORGAN, of Class III.

Member of Committee of Finance.

ELIOT C. CLARKE.

Rumford Committee.

ERASMUS D. LEAVITT,	AMOS E. DOLBEAR,
EDWARD C. PICKERING,	ARTHUR G. WEBSTER,
CHARLES R. CROSS,	THEODORE W. RICHARDS,

ELIHU THOMSON.

C. M. Warren Committee.

CHARLES L. JACKSON,	ARTHUR M. COMEY,
SAMUEL CABOT,	ROBERT H. RICHARDS,
LEONARD P. KINNICUTT,	HENRY P. TALBOT.

ARTHUR MICHAEL.

The Chair appointed the following standing committees: —

Committee of Publication.

SETH C. CHANDLER, of Class I., EDWARD L. MARK, of Class II.,
CRAWFORD H. TOY, of Class III.

Committee on the Library.

WILLIAM F. OSGOOD, of Class I., SAMUEL HENSHAW, of Class II.,
HENRY W. HAYNES, of Class III.

Auditing Committee.

HENRY G. DENNY, WILLIAM L. RICHARDSON.

The following gentlemen were elected members of the Academy: —

Charles Palache, of Cambridge, to be a Resident Fellow in Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe).

William Francis Ganong, of Northampton, to be a Resident Fellow in Class II., Section 2 (Botany).

Charles R. Lanman gave an account of "The Completion of Whitney's Critical Commentary on the Atharva-Veda and the Continuity of Scientific Progress."

The following papers were presented by title: —

Contributions from the Gray Herbarium of Harvard University. New Series. No. XXVI. "New and otherwise Noteworthy Angiosperms from New Mexico and Central America." By J. M. Greenman. Presented by B. L. Robinson.

"Binary Families in a Triply Connected Region, with Especial Reference to Hypergeometric Families." By D. R. Curtiss. Presented by M. Bôcher.

"An Explanation of the False Spectra from Diffraction Gratings. By Theodore Lyman.

"Viscosity of Gases." By J. L. Hogg. Presented by John Trowbridge.

"The Spectra of Gases and Metals at High Temperatures." By John Trowbridge.

"The Changeable Hydrolytic Equilibrium of Dissolved Chromic Sulphate." By Theodore William Richards and Frederic Bonnet, Jr.

"The Anomalous Dispersion, Absorption, and Surface Color of Nitroso-dimethyl-aniline." By R. W. Wood. Presented by C. R. Cross.

On the motion of the Recording Secretary, the following resolution was unanimously adopted: —

Resolved, That the Fellows of the American Academy desire to place upon record their grateful appreciation of the services of their retiring President, Alexander Agassiz, during the nine years in which he has presided over their deliberations.

A TABLE OF ATOMIC WEIGHTS

OF SEVENTY-SEVEN ELEMENTS.

Compiled in April, 1902, *from the most Recent Data.*

BY THEODORE WILLIAM RICHARDS.

Name.	Symbol.	Atomic Weight.	Name.	Symbol.	Atomic Weight.
Aluminium	Al	27.1	Molybdenum	Mo	96.0
Antimony	Sb	120.0	Neodymium	Nd	143.6
Argon	A	39.92	Neon	Ne	19.94
Arsenic	As	75.0	Nickel	Ni	58.71
Barium	Ba	137.43	Niobium	Nb = Cb	94.
Beryllium	Be = Gl	9.1	Nitrogen	N	14.04
Bismuth	Bi	208.	Osmium	Os	190.8
Boron	B	11.0	Oxygen (standard)	O	16.000
Bromine	Br	79.955	*Palladium*	Pd	106.5
Cadmium	Cd	112.3	Phosphorus	P	31.0
Cæsium	Cs	132.88	Platinum	Pt	195.2
Calcium	Ca	40.13	Potassium	K	39.14
Carbon	C	12.001	*Praseodymium*	*Pr*	140.5
Cerium	Ce	140.	Rhodium	Rh	103.0
Chlorine	Cl	35.455	Rubidium	Rb	85.44
Chromium	Cr	52.14	Ruthenium	Ru	101.7
Cobalt	Co	59.00	Samarium ?	Sm	150.
Columbium	Cb = Nb	94.	Scandium	Sc	44.
Copper	Cu	63.60	Selenium	Se	79.2
"Didymium"	Nd + Pr	142.±	Silicon	Si	28.4
Erbium	*Er*	166.	*Silver*	Ag	107.93
Fluorine	F	19.05	Sodium	Na	23.05
Gadolinium	Gd	156. ?	Strontium	Sr	87.68
Gallium	Ga	70.0	Sulphur	S	32.065
Germanium	Ge	72.5	Tantalum	Ta	183.
Glucinum	Gl = Be	9.1	Tellurium	Te	127.5 ?
Gold	Au	197.3	Terbium ?	Tb	160.
Helium	He	3.96	Thallium	Tl	204.15
Hydrogen	H	1.0076	Thorium ?	Th	233. ?
Indium	In	114.	Thulium ?	Tu	171. ?
Iodine	I	126.85	Tin	Sn	119.0
Iridium	Ir	193.0	Titanium	Ti	48.17
Iron	Fe	55.88	Tungsten	W	184.
Krypton	Kr	81.7	Uranium	U	238.5
Lanthanum	La	138.5	Vanadium	V	51.4
Lead	Pb	206.92	Xenon	X	128.
Lithium	Li	7.03	Ytterbium	Yb	173.
Magnesium	Mg	24.36	Yttrium	Yt	89.0
Manganese	Mn	55.02	Zinc	Zn	65.40
Mercury	Hg	200.0	Zirconium	Zr	90.6

NOTE.

The accompanying table of atomic weights is but little changed since last year. Cæsium is made 132.88 instead of 132.9; calcium, 40.13 instead of 40.1; iron, 55.88 instead of 55.9; hydrogen, 1.0076 instead of 1.0075; and nickel, 58.71 instead of 56.70. The value for cæsium is due to some work, as yet unpublished, of Richards and Archibald, and that for calcium is increased in accuracy because the recent investigation of Hinrichsen[*] supports the less recent Harvard value.[†] The other very small changes are due simply to slight differences in the interpretation of data already well known. The decimal might have been omitted from palladium, because this element may still be a whole unit in doubt; but it has been retained as a compromise.

The differences between the present table, that of the German Committee,[‡] and that of F. W. Clarke,[§] are diminishing year by year. Nevertheless to as many as twenty-eight elements out of the seventy-seven are given values in these three tables differing among themselves by over one tenth of a per cent; namely, the atomic weights of antimony, bismuth, cerium, columbium, fluorine, gadolinium, germanium, helium, hydrogen, lanthanum, magnesium, mercury, neon, osmium, palladium, platinum, potassium, samarium, scandium, selenium, tantalum, tellurium, thorium, thulium, tin, titanium, uranium, and zirconium. To this list of uncertain elements should be added erbium, gallium, glucinum, indium, terbium, tungsten, ytterbium, upon which the three tables agree only because of lack of data upon which to base a disagreement. Thus nearly half of the elements are still in doubt by at least one part in a thousand. This circumstance is not so much a reproof to the many earnest workers upon the subject, as an evidence of the great difficulty of some of the problems involved.

Three of the elements given in the list above should not properly be included among the uncertain values, namely, hydrogen, magnesium, and potassium. The first finds its way into the list because of the disregard of significant figures by the German Committee, and the second chiefly because Clarke has included in his calculation work upon magnesic oxide undoubtedly erroneous on account of the presence of included gases.[‖] The case of potassium is somewhat peculiar; for in spite of the great wealth of data concerning this element, Clarke assigns to it the value 39.11, while the German Committee chooses 39.15. The low value is chiefly due to very unsatisfactory data concerning potassic iodide. To me it seems that the most recent work of Stas is far more satisfactory than his earlier work or than the work of any one else, hence the value 39.14 has been assigned to potassium in the present table since its first publication. Careful analyses by E. H. Archibald and myself confirm this conclusion.

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[*] Hinrichsen, Zeitschr. phys. Chem., **39**, 311 (1901).

[†] Richards, Journ. Am. Chem. Soc., **22**, 72 (1900), also **24**, 374 (1902).

[‡] Landolt, Ostwald, and Seubert, Extra insertion in Berichte d. d. ch. Ges. 1902, Heft 1.

[§] F. W. Clarke, Journ. Am. Chem. Soc., **24**, 201 (1902).

[‖] Richards and Rogers, These Proceedings, **28**, 209 (1893).

AMERICAN ACADEMY OF ARTS AND SCIENCES.

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REPORT OF THE COUNCIL. — PRESENTED MAY 13, 1903.

BIOGRAPHICAL NOTICES.

Alpheus Hyatt . . . . Alpheus S. Packard.
John Daniel Runkle . . . . Harry W. Tyler.

REPORT OF THE COUNCIL.

The Academy has lost seventeen members by death since the annual meeting of May 14, 1902: eight Resident Fellows,—William Sumner Appleton, James Elliot Cabot, Horace Gray, Henry Barker Hill, Charles Greely Loring, Henry Mitchell, John Daniel Runkle, Morrill Wyman; four Associate Fellows,—Josiah Willard Gibbs, John Wesley Powell, Ogden Nicholas Rood, Alfred Richard Cecil Selwyn; five Foreign Honorary Members,—Hervé Auguste Etienne Albans Faye, Gaston Bruno Paulin Paris, Sir George Gabriel Stokes, Bart., Rudolph Virchow, Heinrich von Wild.

ALPHEUS HYATT.

Our beloved and highly honored associate, who, in the ripeness of his intellectual powers, has been so suddenly snatched from us, was cast in no ordinary mould. Whether we regard him as a man, a patriot, a fellow student, a scientific investigator, an organizer of societies, of museums, or of methods of science-teaching, his many-sided life was a rare one. We come together to pay tribute to the memory of Alpheus Hyatt as a promoter of scientific enterprises, as one of the founders of a new school in the philosophy of biology, as a master in paleontological methods, endowed as he was with rare powers of mental absorption and concentration, and an unusual capacity for sound generalization.

The nineteenth century, as regards natural science in the United States of America, was a period of pioneer effort, and has been characterized by the careers of several great men. Their lives, unlike those of European savants, whose museums, laboratories, and methods of research had often been founded by a previous generation, had to be devoted, so to speak, to opening and laying out roads, to founding and building institutions, and making the way straight for the generations to come.

Such men were Henry, Dana, Agassiz, Wyman, Rogers, Hall, Baird, and others, all full of love for original research, but who unselfishly gave up much of their time, so dearly valued for private studies, to developing and expanding the educational and scientific resources of a young people.

Those who were pupils of that large-hearted, enthusiastic son of genius, Louis Agassiz, well know the sacrifices he made in the country of his adoption, in devoting the later years of his life to the foundation of the Museum of Comparative Zoölogy, to popularizing science, to illustrating the dignity of scientific studies, and enforcing the value of research in our colleges and young universities.

The lines thus drawn were followed by Hyatt; and it is safe to say that the impress he has made on zoölogical and geological science is deep and lasting.

Born in Washington, D. C., April 5, 1838, of Maryland parentage, Alpheus Hyatt was sent to the Maryland Military Academy, but afterwards entered Yale College, completing the freshman year, class of 1860. He then travelled a year in Europe, and as he once told us, strong family and clerical influence were brought to bear upon him while in Rome to induce him to enter the Church, his family being of the Roman Catholic faith.

He entered the Lawrence Scientific School in 1858; our acquaintance began in 1861. The pupils of Agassiz were then domiciled in Zoölogical Hall, a small two-story wooden building which stood on the site of the Peabody Museum of American Archaeology. We were soon interested and attracted by young Hyatt. Although he originally elected to make engineering his profession, he, with for that period a rather large number of other young men, was attracted by the fame and charming personality of Agassiz, as well as by the zoölogical treasures of the already rich and carefully selected museum. Hyatt's patience and his dogged perseverance, his powers of concentration, his philosophical tendencies, attracted our attention, while his open, frank, sunny disposition, his companionable, jovial, unselfish, pure spirit and scholarly aims, at once secured our love and respect. He was then and through life an all-round man, though very early in his studies specializing on the fossil Cephalopods.

Tyndall's "Heat as a Mode of Motion" had just appeared, and debates on the higher physics, in which he took a leading part, alternated with communications on the position of the Tunicata, of the Polyzoa, Brachiopods, and other types at the meetings of our Zoölogical Club. Hyatt was artistic in his tastes, drawing well on the blackboard, and handling the pencil with ease and facility. He read good literature, was a regular attendant on the lectures of Professor Lowell, and constantly present at the courses of that model of scientific lecturers, Professor Jeffries Wyman.

He was graduated with high honors under Professor Agassiz in 1862. The Civil War was then raging, and the young patriot, having already received the elements of military training, enlisted, though naturally not without opposition from his family in Maryland, and was active in raising a company in Cambridge. He received a commission in 1862 as a lieutenant, but soon rose to the rank of captain in the Forty-seventh Massachusetts Regiment, serving as aide-de-camp nine months, for the most part in New Orleans.

After being mustered out of the army he lived in Boston some time during 1865-66, and it was then, and afterwards in Salem, that we came to know him still more intimately than at Cambridge, as we were at times room-mates. Over our friendship, our mutual love and respect, a cloud never passed. The purity and unaffected goodness, nobility of character, sturdy honesty and reliability as a friend, were as conspicuous then as in his last years. His filial devotion was marked, and in later life the kindness and chivalric courtesy, acts of kind-heartedness and thoughtfulness which in some instances it gave him some trouble to perform, to an elderly lady as well as to other friends in need, will be treasured up in our memory. Of his delightful domestic life, the warm-heartedness of his welcome to his hospitable home, his friends will ever retain the most agreeable recollections.

In the foundation and organization of scientific and educational enterprises, societies, journals, and museums, Hyatt always lent a willing hand. Gifted with a fair amount of executive ability, with clear, persuasive powers of expression, a ready debater, often a powerful speaker, and excellent in planning, besides having a somewhat wide knowledge of men, he was most useful in promoting such undertakings.

In 1867 he, with three other pupils of Agassiz, became one of the curators of the Essex Institute, and in 1869 he took an active and most useful part in the foundation of the Peabody Academy of Science at Salem, Mass., where he served as one of the curators.

As Custodian of the Boston Society of Natural History from 1870 to the end of his life, he planned an arrangement of the museum in accordance, so far as was possible, with the phylogeny of the animal kingdom, beginning with the Protozoa. He was one of the two founders of the Teachers' School of Science, becoming its manager, and in this way accomplished a vast amount of good in training the teachers of Boston and its vicinity in the elements of natural history. While living in Salem he was one of the founders and editors of the "American Naturalist," contributing frequently to its pages.

The principal founder of the American Society of Naturalists, we well remember the zeal and interest he took in organizing this at present influential body, the preliminary meeting being held by a few at Springfield, Mass., in April, 1883. If we mistake not, he gave the name to the society. He was its first president, and a few years ago was elected an honorary member in recognition of his services.

He also founded and organized a seaside laboratory at Annisquam, Mass., under the auspices of the Woman's Education Association of Boston. He took personal charge of it, as his summer home was there. His interest in this school, and in marine zoölogy, led him to take part in the foundation of the Biological Laboratory at Wood's Holl. As an indication of the value of his services as an expert in zoölogy and his ability as an organizer, he was elected the first president of its board of trustees. He was elected a fellow of this Academy in 1869.

Throughout his life, after graduation at Harvard University, he was an honorary Assistant Curator in the Museum of Comparative Zoölogy, in charge of the collection of fossil Cephalopods. In 1888 he was offered the position of United States Commissioner of Fish and Fisheries, but preferred to live the almost ideal life, with its rich opportunities for research, which he rounded out at his home in Cambridge.

Professor Hyatt was not only a specialist, but a generalizer, a philosopher. His special, detailed, observational work was continually leading him to broad, sound generalizations, and not only in the field of embryology, taxonomy, and phylogeny, but of general organic evolution. He was a slow worker, very patient, cautious, constantly reviewing his work and conclusions. He was not always luminous in exposition; he sometimes, especially early in his life, failed from want of training and practice in writing, to state clearly and pointedly the views that crowded his mind. But this defect was largely outgrown. For this reason his first essay on "Parallelism," etc., was not understood by Mr. Darwin, as well as others who had not had experience in similar studies, but this defect of style was overcome in his later memoirs.

As the result of his manner of investigation, Hyatt became an acknowledged master in the methods of paleontology, in a mode of treatment of fossil forms then comparatively new to paleontology, due to his long and thorough training in zoölogy, comparative anatomy, and embryology. Before his time paleontologists, with the exception, of course, of Cuvier, Owen, Huxley, and Agassiz, had had but little training in anatomy and taxonomy. Hyatt's patient and beautiful studies on the fresh water Polyzoa, carried on in the sixties; his later studies on the sponges, on the molluscs, other than Cephalopods; his experience as a teacher of zoölogy and paleontology in the Massachusetts Institute of Technology from 1870 to 1881, and of zoölogy in Boston University from 1877 to the time of his death, in addition to his museum work, kept him informed of biological methods and results; while his field work in the paleozoic rocks of southern Labrador, of Newfoundland, and his work about Salem, but more especially his work in 1889 and succeeding years as paleontologist in charge for the U. S. Geological Survey of the lower Mesozoic (Trias and Jura), carried on in Texas and in California, besides his earlier studies at Steinheim, Germany, afforded him the means of observing and accumulating many facts, and of forming broad conclusions from many points of view.

His prolonged and life-long studies on the Cephalopods were thorough and exhaustive, and from them were wrung the basal principles of evolution, — work which for thoroughness and far-reaching results has seldom been surpassed, and which not only is of the highest value and interest to students of molluscs, but has already exerted and will continue to exert a wide influence on the progress of general zoölogy.

The permanent fame of our deceased associate will, we venture to claim, be based on his contributions to the embryology, laws of growth of the shells, and the phylogeny of the Cephalopod molluscs; and on his profound studies on the modifications of the tertiary shells at Steinheim near Stuttgart, Germany.

He will also be remembered for his conclusions on the position of the sponges. As far as we are aware he was the first one after McAllister, in a paper published in 1876, entitled "Sponges considered as a distinct Sub-kingdom of Animals"* to refer these organisms to a separate, independent branch or phylum of the animal kingdom.

The outcome of these studies resulted in his valuable contributions to the philosophy of biology. He was one of the founders and upholders of Neolamarckism, and even if the value of that phase of the evolution theory be called in question, he will be remembered as having been the discoverer of a series of facts of astonishing number and variety, all converging to one grand result, that of affording a true, solid basis for the theory of organic evolution.

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* Proceedings Boston Society of Natural History, XIX. Nov. 1, 1876. Hyatt claimed in this paper that during the previous year he regarded the sponges as distinct from the rest of the animal kingdom. l. c., p. 14. In this paper also, he rejected the position assigned them by Haeckel in the Coelenterata.

For a number of years Dr. Hyatt was engaged in a study of the origin and lines of evolution of the land shells peculiar to the Hawaiian Islands, on which the Rev. John T. Gulick had already (1887–1890) made such interesting and suggestive studies. Hyatt had presented several communications to scientific societies, showing the progress made in his work. His beautiful model of the islands, and arrangement of the actual shells fastened to the surface of the model, with cords of different colors showing the lines of migration and corresponding segregations and consequent differentiation of the specific forms, gave promise of the most valuable and fruitful results. He was planning to make a journey to the Hawaiian Islands in March of the present year, when death overtook him. But the results of his long continued labors will, it is hoped, not be lost to science, as arrangements were made previous to his death for their completion by a capable hand.

The first paper of a general nature which young Hyatt published * contained the germ of his chief life-work. It gave some of the results of six years' study on fossil Cephalopods, and was on the parallelism existing between the different stages of life in the individual and those of the tetrabranchiate Cephalopods as a group. It was published in 1866, the same year in which appeared Haeckel's " Generelle Morphologie," which, he (Haeckel) says, constituted the first attempt to apply the general doctrine of development to the whole range of organic morphology (anatomy and biogenesis). Although both of the Haeckelian principles of palingenesis and cenogenesis, with all their evolutional implications, were stated with considerable fullness by Fritz Müller in his " Für Darwin," published in 1864, Hyatt attempted to show that the life of the individual displays, to use his own words, "during its rise and decline, phenomena correlative with the rise and decline of the collective life of the group to which it immediately belongs." In this memoir he carried out and greatly amplified D'Orbigny's views, to which he gives the fullest credit, as to the changes of the larger number of Ammonites from larval, or to use his own term, nepionic, to adult, and from adult to senile stages. The theme, so often discussed by Agassiz before his students, that the development of the individual is an epitome of that of the order or class to which it belongs, with the later vital addition by F. Müller and by Haeckel of the evolution by actual descent, the principle now being

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* "On the Parallelism between the Different Stages of Life in the Individual and those in the Entire Group of the Molluscous Order Tetrabranchiata." Memoirs Boston Soc. Nat. Hist., I. 1866.

called Haeckel's "biogenetic law," runs through and is the *motif* of Hyatt's life-long work.

Some of us in our maturer years return to or continue to work along the lines taken up in youth. Hyatt for a period of forty years, in a series of profound studies on the Cephalopods, in publications rich in exact observations and with ample illustrations, many of the drawings beautifully executed by his own hand, independently expanded and illustrated this fundamental phase of evolution, which so impressed him in his student days.

His remarkable memoir on the "Genesis of the Arietidae," published in 1889 in the Smithsonian "Contributions to Knowledge," and jointly with the Museum of Comparative Zoölogy, is a storehouse of facts on which his generalizations are based. When he presented his views to the National Academy of Science at a Washington meeting, Professor Henry expressed his approbation of the value and profoundity of this research. In this work he insists on his law of Morphogenesis, i. e., he attempts to demonstrate that "a natural classification may be made by means of a system of analysis in which the individual is the unit of comparison, because its life in all its phases, morphological and physiological, healthy or pathological, embryo, larva, adolescent, adult, and old (ontogeny), correlates with the morphological and physiological history of the group to which it belongs (phylogeny)."

In the beginning of his studies, contemporaneously with Cope, he insisted on the fact and results of a process of acceleration and retardation and the growth of the individual as well as the evolution of the family, order, or class of which it was a member.

"All modifications and variations," he says, "in progressive series tend to appear first in the adolescent or adult stages of growth, and then to be inherited in successive descendants at earlier and earlier stages according to the law of acceleration, until they either become embryonic, or are crowded out of the organization, and replaced in the development by characteristics of later origin."

Another of the nineteen conclusions prefacing this essay is No. 14: —

"The law of acceleration in development seems, therefore, to express an invariable mode of action of heredity, in the earlier reproduction of hereditary characteristics of all kinds and under all conditions. In progressive series it acts upon healthy characteristics, and appears to be an adaptation to favorable surroundings, and in retrogressive series upon pathological characteristics, and is probably an adaptation to unfavorable surroundings, usually leading to the extinction of the series or types."

This law of acceleration was, after the publication of Cope's and also of Hyatt's first paper (1870), arrived at independently by Würtenberger,* and adopted by the distinguished German paleontologist Neumayr, though he, apparently in ignorance of Hyatt's much earlier results, gives the credit for its discovery to Würtenberger, as does Weismann.

These changes Professor Hyatt insisted were primarily due to changes in the environment acting mechanically on the organism at different ages, the Lamarckian factors of use and disuse as well as environmental changes being constantly operative.

To explain the facts of retardation or abbreviated metamorphosis Dr. Hyatt formulates the law or process concerned in these phenomena, and which explain the mechanism of gradation, whether progressive or retrogressive, as follows: —

"Changes in environment, which introduce new adaptive characteristics in the nealogic or adult stages, necessarily add these to the hereditary stages of the younger periods of growth, and thus shorten the development of the latter by direct development." He goes on to illustrate this process by citing the changes of insects, of Taenia, and the loss of progressive characters correlated with a highly accelerated mode of development in man, due to a change from a horizontal to an upright position, and which, he says, were first pointed out nearly a century ago by Lamarck.

We will now, as briefly as the subject admits, trace the evolution of Professor Hyatt's views on evolution.

It has been objected to both Cope and Hyatt's theory that their law or process of acceleration and retardation are merely statements of facts. But both of these observers very soon after working out, each independently of the other, and in very different groups, these facts and processes, arrived at the conclusion that the changes they formulated were primarily due to the Lamarckian factors of change of environment, and to use and disuse.

They, with others of their contemporaries, rehabilitated and extended the Lamarckian factors and became the founders of the Neolamarckian school of evolutionists.

Hyatt's first public avowal of evolutional views was in a paper read in 1870,† though for several years he had practically adopted the theory of descent. His 1866 paper on "Parallelism," etc., was in this same vein of thought, but in this paper, conceived along evolutional lines, he did not explicitly state what were the actual dynamic agencies at work to produce the transformation from one form to another. His studies on the geological succession of the Ammonites, and the obvious genetic relations of the series he examined afterwards, naturally led him to adopt such views.

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* Ausland, 1873.

† On Reversions among the Ammonites. Proc. Boston Soc. Nat. Hist., XIV. June, 1871.

He claimed in this paper that the reversionary species he examined "all descended from one." In 1873 he showed that in the Ammonoids there are everywhere instances of the slow accumulation of differences, according with the Darwinian method, and of their quick and sudden production, "according to the law of acceleration as explained by Cope and the writer, and subsequently by Mivart." *

His Lamarckian leanings, however, so far as his published works show, crop out in his "Abstract of a Memoir on the Biological Relations of the Jurassic Ammonites." † The stages of growth he describes are here directly attributed to the favorable nature of the physical surroundings, primarily producing characteristic changes which become perpetuated and increased by inheritance within the groups. The production of retrogressive, senile forms he attributes to "the action of unfavorable surroundings." He carefully guards against "attributing the origin of these differences to the law of natural selection," limiting the action of this law "strictly to the modification of the structural differences which tend to appear first in the varieties and then by inheritance in larger and larger groups and at earlier and earlier stages in the life of the individual."

Hyatt's beautiful research on the problems suggested by the fossil pond-snails of Steinheim near Stuttgart, Germany, is an important contribution both to paleontology and to evolutional data. Hilgendorf had previously (in 1866) described the conditions, and his results were regarded in Germany as amounting to a demonstration of the truth of the evolution theory. Hyatt, during a year's residence in 1872 near Stuttgart, spent five weeks examining the pits and made careful collections.

The discussion is a most valuable contribution to the theory of descent. Sir Richard Owen wrote him in 1881: "I cannot say more than that I deem it a model of the way and aim in and by which such researches should be conducted in the present phase of Biology."

Hyatt's patient and long sustained studies led him to the following conclusions:

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* Proceedings Boston Soc. Nat. Hist., XVI. p. 167. Dec. 3, 1873.

† Ibid., XVII. pp. 236–241. Dec. 16, 1874.

1. The extraordinary modifications and series of shells found at Steinheim are in one way exceptional, and owe their existence to exceptional conditions.

2. These conditions appear to be the isolation of the modified descendants of *Planorbis laevis*, due to the absence of competing types and the character of the environment.

Besides other conclusions, he points out that in these and other series of animals which he had studied, "in a given number of generations inherited characteristics of every kind tend to appear in the descendants at earlier stages than that at which they first occurred in the ancestral forms."

These studies led him to explain the frequent occurrence of parallel forms, or, as they are now called, cases of convergence, also to examine into the causes of senescence, of geological extinction, etc. He maintained that there was no such thing as indefinite variation, and took strong ground, based on extended series of paleontological data, as to the reality of use-inheritance. In this as well as his other memoirs he insists on the inadequacy of natural selection in causing variation and the formation of new groups.

In a later work, entitled "Phylogeny of an Acquired Characteristic,"* Hyatt is at his best. The results of a life-long study of the genetic series of Cephalopods, of their rise, culmination, decay or senile features, and their final extinction, are portrayed in a masterly way. His conclusions are based on the careful detailed survey of a multitude of facts, and the story he unravels from the series upon series of fossil forms which passed in review before eyes so well trained, and a mind so skilled in induction and deduction, forms a chapter in the history of organic evolution which will remain a classic.

It was only after long years of research, and the most patient, thoughtful reflection on the series of facts he independently worked out, by his study of the embryology of the protoconch and conch of the Orthoceratites, and their successors the Nautiloids and Ammonoids, that he thus describes the gross results in a few sentences which show the mode of origin of new orders through the Lamarckian factors of effort and use. In this essay Hyatt struck the keynote to the cause of the origin of new types, or class and ordinal forms, i. e. by the changes in habits, and efforts of the organism to adapt itself to such new conditions of existence.

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* Phylogeny of an Acquired Characteristic. Proc. Amer. Phil. Soc., XXXII. p. 371 (1894).

These are the Lamarckian or dynamic agencies of use and disuse, brought into action by environmental changes.

"The *efforts*," he says, "of the Orthoceratite to adapt itself fully to the requirements of a mixed habitat of swimming and crawling gave rise to the Nautiloidea; the efforts of the same type to become completely a littoral crawler evolved the Ammonoidea. The successive forms of the Belemnoidea arose in the same way. But here the ground-swimming habitat and complete fitness for that was the object. The Sepioidea, on the other hand, represent the highest aims as well as the highest attainments of the Cephalopods in their evolution into surface-swimming and rapacious forms. We cannot seriously imagine these changes to have resulted from intelligent effort; but we can, with Lamarck and Cope, picture them as due to efforts on the part of the animal to take up new quarters in its environment and thus acquire habits and structures suitable to the changed physical requirements of its surroundings, and this position is better supported by facts than any other hypothesis."

Here are some examples of the complete and thoroughgoing way in which Hyatt thought out, during his painfully slow but sure investigation of facts, broad generalizations which we feel confident will stand the test of time and farther research.

After speaking of "the effort to change of habitat and consequently change of habits," due to change of environment, and of changes in structure resulting from the efforts on the part of the organism to meet the changes in the physical surroundings, he adds: "That this process should end in the production of structures suited to the environment is inevitable. *With these factors at work, both without and within the organism, the evolution of their structures obeys a physical law which acts amid a thousand disturbing forces, perhaps, but nevertheless must act with predominating force in one mean path or direction, the resultant determined by the environment and the inherited structures of the organism.*"

Another beautiful research was his attempt to account by gravity for the spiral shell of molluscs. He points out the fact of an obvious correlation between the coiling of the shell and the habit of crawling. He shows that those gastropod shells which degenerate and tend to lose the spiral mode of growth and become irregularly straightened out in their older stages of growth, are forms which become attached or which lead sedentary lives, i. e. Vermetus attached late in life, and Magilus which lives buried in coral. He points out the tendency in the descendants of straight shells (Orthoceratites, etc.) to become, as the result of assuming reptant habits, first arcuate and then coiled; these being acquired characters which have been "introduced late in the ontogeny and gradually forced back to younger and younger stages in successive generations, or species, or genera." He also accounts for the peculiar horizontal growth of the oyster, pecten, etc., by their fixed mode of life.

These observations, modes of investigations, and laws or principles so carefully thought out, are the work of a master in biology. They already have been most fruitful in their results, and have been found to apply in other groups of organized beings. Hyatt was thus the founder of a new school in paleontology, and the brilliant results of the work of a younger generation of paleontologists, viz., Beecher, Jackson, Suchert, Smith, and others, all of whom acknowledge him as their master, afford the best of proof of our claim. Certainly the same principles we think will apply to Crustacea, and also to insects, as Hyatt has claimed, and they were applied to vertebrates by Cope and his successors in vertebrate paleontology. For all this work Hyatt's name will forever be associated with the names of Agassiz, Barrande, Neumayr, Waagen, Mojsisovics, and others who have worked along these lines.

Hyatt's systematic work on the Cephalopods was very extensive. It will be remembered that there are estimated to be about twenty-five hundred species of Nautiloids and five thousand species of Ammonoids. The results of his labors may be seen in the portion he contributed to the translation by Dr. Eastman of the condensed American edition of Zittel's Paleontology.

In systematic work on the Ammonites Hyatt followed hard after Suess, who *in 1865 inaugurated the subdivision of the group into genera.* As stated by Zittel: * "A similar reform was advocated by Alpheus Hyatt in his memoir on the Liassic Ammonites (1869). The previous nomenclature of families was discarded by Hyatt, and numerous new genera were erected, whose limits were much more narrowly defined than had been customary. As one might have expected, the new tendency met at first with strong opposition, but it was supported and followed by Laube, Zittel, Mojsisovics, Waagen, and Neumayr."

Dr. Hyatt was not a voluminous writer, but his works are solid, original, independent contributions to science, and will stand, if we mistake not, the test of time.

He was elected a member of the National Academy of Science in 1875, and since 1897 has been a corresponding member of the Geological

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* Zittel's History of Geology and Palaeontology. Translated by M. M. Ogilvie-Gordon. 1891. p. 403.

Society of London. In 1898 he received from Brown University the degree of Doctor of Laws. At the time of his death (Feb. 15, 1902) he was the Vice-president of Class II. of this Academy.

Endowed by nature with talents of a high order, he cultivated and, to use his own favorite phrase, accelerated their development and increase through his life. His strength of character is evinced by the forceful influence he exerted both in scientific and educational channels.

" *Talent*," says Baron Osten Sacken, the venerable diplomatist and naturalist, "is a gift of nature, and does not, for that reason, constitute in itself a merit; the merit lies in the *character* which makes talent fruitful." And that profound genius and master in biology — Von Baer — has said : "In the domain of Science, talent alone, coupled with diligence and *the power of self-control*, is of any value." *

We close this notice of our departed friend, who endeared himself to his associates so closely by his amiable and manly qualities, feeling sure that posterity will confirm the estimate here given of his worth as a man, and of the secure place he will hold as a master in science.

A. S. PACKARD.

JOHN DANIEL RUNKLE.

JOHN DANIEL RUNKLE was born at Root, N. Y., October 11, 1822, and died at Southwest Harbor, Me., July 8, 1902, near the close of his eightieth year.

The early years of life on the farm offered little opportunity for study, and he was already twenty-five when he entered the newly established Lawrence Scientific School of Harvard University. His name stands alone in the catalogue of 1848–49 as "student in mathematics." John W. Draper and James E. Oliver were fellow students; Josiah P. Cooke and William T. Harris, resident graduates. He was a member of the first graduating class, of 1851, with Joseph Le Conte and David A. Wells, receiving the degree of Bachelor of Science, and at the same time, for high scholarship, the honorary degree of Master of Arts.

The work of computation for the Nautical Almanac was carried on at this time in Cambridge by a staff including, among other men of subsequent eminence, Simon Newcomb, Asaph Hall, George W. Hill, T. H.

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* Quoted from "An Introduction to the Record of my Life-work in Entomology." By C. R. Von Osten Sacken. 1901.

Safford, and J. M. Van Vleck. Mr. Runkle's connection with the Almanac began in 1849, and continued in some form as late as 1884.

In 1852 he contributed to the "Astronomical Journal" papers on the "Elements of Thetis" and on the "Elements of Psyche."

In 1855 his "New tables for determining the values of the coefficients, in the perturbative function of planetary motion, which depend upon the ratio of the mean distances," were published as one of the Smithsonian Contributions to Knowledge.

In 1858 Mr. Runkle founded the "Mathematical Monthly." Encouragement was received and formal indorsement given by the American Association for the Advancement of Science and by several educational bodies. The list of contributors included many distinguished names, among others Arthur Cayley, William Chauvenet, George W. Hill, Simon Newcomb, Benjamin Pierce, John Herschel. The time for the publication of a long-lived mathematical journal was not, however, ripe, and only three volumes appeared.

From 1860 until his death Professor Runkle's time and strength were almost continuously and exclusively devoted to the establishment and upbuilding of the Massachusetts Institute of Technology. He was first Secretary of the Institute, and at the opening of the school became professor of mathematics. In October, 1868, he became Acting President in consequence of President Rogers' serious illness, and in 1870 he was made President, holding the office for the following eight years.

The situation was a most exacting one, making altogether exceptional demands. The school, only five years old, was in no condition to lose the guidance of its founder. It had not yet gathered the momentum necessary for steady, straightforward progress. Opinions did and will differ as to President Runkle's judgment on the difficult questions that, as time passed, pressed overwhelmingly upon him for solution. No man could have been more devotedly loyal to the school or to its founder, his predecessor and ultimately his successor. None could have shown more steadfast courage, not only against heavy odds, but too often with but feeble support.

The more notable events of the Runkle presidency were: the fruitless negotiations with Harvard University for a union; the establishment of the laboratories of mining engineering and metallurgy; the introduction of shop instruction and the foundation of the School of Mechanic Arts; the development of professional summer schools in the field; the beginnings of an engineering laboratory; the increased efficiency of military instruction and the summer encampment at Philadelphia in 1876; the erection of a gymnasium, including a lunch-room; the admission of women as students.

In 1878 Dr. Runkle resigned the presidency of the Institute and spent the following two years in Europe.

It had been President Runkle's merit to be the first to appreciate the American need of mechanic arts instruction based on principles already successfully applied in Russia. He was primarily interested in it as an invaluable addition to existing engineering courses, but he also saw clearly its great potential significance for general secondary education, and so far as possible, under pressure of other needs, demonstrated this by the inauguration of the School of Mechanic Arts, in which boys of high-school age were offered a two years' course, including mathematics, English, French, history, mechanical and freehand drawing, and shop-work. His visit to Europe enabled him to make a study of Continental schools of similar purpose; and the results of this study are embodied in a paper presented to the Society of Arts in April, 1881, on "Technical and Industrial Education Abroad," in an extended contribution to the Report of the Massachusetts Board of Education for 1880–81, and in a "Report on Industrial Education" in 1884. Others have taken a more directly prominent share in the introduction and extension of mechanic arts or manual training in primary and secondary schools, but the actual experiment initiated by him in Boston had in its time wide influence and imitation.

As a teacher of mathematics, Professor Runkle found his highest usefulness and most congenial vocation, — a vocation to be happily continued for not less than twenty-one years. His teaching was characterized by stimulating, luminous, unconventional exposition, by quick incisive questioning, by warm personal interest in his students, and by a constant substratum of uplifting earnestness and dignity. None of his students could fail to acquire admiring affection; very few could withstand the incentive to work.

Professor Runkle was a man of much intellectual quickness and strength, of ardent, but in later years serene, temperament, of warm and generous affections, of cordial, unaffected courtesy, in all the relations of life a sincere and loyal gentleman. Throughout his early and middle life he was a pioneer, first in the struggle for his own education and that of his brothers, next in the establishment and continuance of a much-needed, but, as it turned out, premature mathematical journal, then and for many years in the development of the Massachusetts Institute of Technology, and the introduction of education in the mechanic arts. In all these undertakings his insight and courage were invaluable. He made President Rogers' plans for the Institute his own. He held steadfastly to its fundamental ideals, and, taking account of his scanty resources, made remarkable progress toward their fulfilment. The main changes he initiated have been abundantly justified by time, and he lived to see their fulfilment.

He was elected a Fellow of the Academy on the 26th of May, 1857, and served one year (1877–78) as Councillor.

New members elected are: Resident Fellows, 6; Associate Fellows, 3; Foreign Honorary Members, 5. The roll of the Academy now includes 197 Resident Fellows, 98 Associate Fellows, and 72 Foreign Honorary Members.*

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* By the resignation of a Resident Fellow, the death of an Associate Fellow and a Foreign Honorary Member, and the election of new members at the annual meeting of May 13, 1903, the roll stands at date of publication, 198 Resident Fellows, 98 Associate Fellows, 72 Foreign Honorary Members.

American Academy of Arts and Sciences.

OFFICERS AND COMMITTEES FOR 1903-04.

PRESIDENT.

William W. Goodwin.

VICE-PRESIDENT.

Class I.	**Class II.**	**Class III.**
John Trowbridge,	Henry P. Walcott,	John C. Gray.

CORRESPONDING SECRETARY.

William M. Davis.

RECORDING SECRETARY

William Watson.

TREASURER.

Francis Blake.

LIBRARIAN.

A. Lawrence Rotch.

COUNCILLORS.

Class I.	**Class II.**	**Class III.**
George F. Swain,	Robert De C. Ward,	Denman W. Ross,
	Terms expire 1904.	
Arthur G. Webster,	Edward L. Mark,	Arlo Bates,
	Terms expire 1905.	
Lewis J. Johnson,	Edward H. Bradford,	Morris H. Morgan,
	Terms expire 1906.	

COMMITTEE OF FINANCE.

William W. Goodwin, Francis Blake, Eliot C. Clarke.

RUMFORD COMMITTEE.

Charles R. Cross, *Chairman*,
Erasmus D. Leavitt, Edward C. Pickering, Amos E. Dolbear,
Arthur G. Webster, Theodore W. Richards, Elihu Thomson.

C. M. WARREN COMMITTEE.

Leonard P. Kinnicutt, *Chairman*,
Charles L. Jackson, Samuel Cabot, Arthur M. Comey,
Robert H. Richards, Henry P. Talbot, Arthur Michael.

COMMITTEE OF PUBLICATION.

Edward L. Mark, of Class II., *Chairman*,
Seth C. Chandler, of Class I., Crawford H. Toy, of Class III.

COMMITTEE ON THE LIBRARY.

A. Lawrence Rotch, *Chairman*,
Harry M. Goodwin, of Class I., Samuel Henshaw, of Class II.,
Henry W. Haynes, of Class III.

AUDITING COMMITTEE.

Henry G. Denny, William L. Richardson.

LIST
OF THE
FELLOWS AND FOREIGN HONORARY MEMBERS.

(Corrected to June 30, 1903.)

RESIDENT FELLOWS. — 198.

(Number limited to two hundred.)

Class I. — *Mathematical and Physical Sciences.* — 81.

Section I. — 18.

Mathematics and Astronomy.

Solon I. Bailey,	Cambridge.
Maxime Bôcher,	Cambridge.
William E. Byerly,	Cambridge.
Seth C. Chandler,	Cambridge.
Gustavus Hay,	Boston.
Percival Lowell,	Boston.
James Mills Peirce,	Cambridge.
Edward C. Pickering,	Cambridge.
William H. Pickering,	Cambridge.
Henry S. Pritchett,	Boston.
John Ritchie, Jr.,	Roxbury.
Edwin F. Sawyer,	Brighton.
Arthur Searle,	Cambridge.
William E. Story,	Worcester.
Henry Taber,	Worcester.
Harry W. Tyler,	Boston.
O. C. Wendell,	Cambridge.
P. S. Yendell,	Dorchester.

Section II. — 24.

Physics.

A. Graham Bell,	Washington, D.C.
Clarence J. Blake,	Boston.
Francis Blake,	Weston.
George A. Campbell,	Boston.
Harry E. Clifford,	Newton.
Charles R. Cross,	Brookline.
Amos E. Dolbear,	Somerville.
A. W. Duff,	Worcester.
H. M. Goodwin,	Roxbury.
Edwin H. Hall,	Cambridge.
Hammond V. Hayes,	Cambridge.
William L. Hooper,	Somerville.
William W. Jacques,	Newton.
Frank A. Laws,	Boston.
Henry Lefavour,	Boston.
Theodore Lyman,	Brookline.
Benjamin O. Peirce,	Cambridge.
A. Lawrence Rotch,	Boston.
Wallace C. Sabine,	Boston.
John S. Stone,	Boston.
Elihu Thomson,	Swampscott.
John Trowbridge,	Cambridge.
A. G. Webster,	Worcester.
Robert W. Willson,	Cambridge.

Section III. — 21.

Chemistry.

Samuel Cabot,	Boston.

Arthur M. Comey,	Cambridge.
James M. Crafts,	Boston.
Charles W. Eliot,	Cambridge.
Charles L. Jackson,	Cambridge.
Walter L. Jennings,	Worcester.
Leonard P. Kinnicutt,	Worcester.
Charles F. Mabery,	Cleveland, O.
Arthur Michael,	Boston.
George D. Moore,	Worcester.
Charles E. Munroe,	Wash'gton, D.C.
John U. Nef,	Chicago, Ill.
Arthur A. Noyes,	Boston.
Robert H. Richards,	Jamaica Plain.
Theodore W. Richards,	Cambridge.
Charles R. Sanger,	Cambridge.
Stephen P. Sharples,	Cambridge.
Francis H. Storer,	Boston.
Henry P. Talbot,	Newton.
Charles H. Wing,	Ledger, N. C.
Edward S. Wood,	Boston.

SECTION IV. — 18.

Technology and Engineering.

Alfred E. Burton,	Boston.
Eliot C. Clarke,	Boston.
Heinrich O. Hofman,	Jamaica Plain.
Ira N. Hollis,	Cambridge.
L. J. Johnson,	Cambridge.
Gaetano Lanza,	Boston.
E. D. Leavitt,	Cambridge.
William R. Livermore,	New York.
Hiram F. Mills,	Lowell.
Cecil H. Peabody,	Brookline.
Alfred P. Rockwell,	Manchester.
Andrew H. Russell,	Manila.
Albert Sauveur,	Cambridge.
Peter Schwamb,	Arlington.
H. L. Smyth,	Cambridge.
Charles S. Storrow,	Boston.
George F. Swain,	Boston.
William Watson,	Boston.

CLASS II. — *Natural and Physiological Sciences.* — 68.

SECTION I. — 15.

Geology, Mineralogy, and Physics of the Globe.

H. H. Clayton,	Milton.
Algernon Coolidge,	Boston.
William O. Crosby,	Jamaica Plain.
William M. Davis,	Cambridge.
Benj. K. Emerson,	Amherst.
O. W. Huntington,	Newport, R. I.
Robert T. Jackson,	Cambridge.
T. A. Jaggar, Jr.,	Cambridge.
William H. Niles,	Cambridge.
Charles Palache,	Cambridge.
John E. Pillsbury,	Boston.
Nathaniel S. Shaler,	Cambridge.
Robert DeC. Ward,	Cambridge.
John E. Wolff,	Cambridge.
J. B. Woodworth,	Cambridge.

SECTION II. — 12.

Botany.

F. S. Collins,	Malden.
Geo. E. Davenport,	Medford.
William G. Farlow,	Cambridge.
Charles E. Faxon,	Jamaica Plain.
Merritt L. Fernald,	Cambridge.
William F. Ganong,	Northampton.
George L. Goodale,	Cambridge.
John G. Jack,	Jamaica Plain.
B. L. Robinson,	Cambridge.
Charles S. Sargent,	Brookline.
Arthur B. Seymour,	Cambridge.
Roland Thaxter,	Cambridge.

SECTION III. — 25.

Zoölogy and Physiology.

Alexander Agassiz,	Cambridge.
Robert Amory,	Boston.

James M. Barnard,	Milton.
Henry P. Bowditch,	Jamaica Plain.
William Brewster,	Cambridge.
Louis Cabot,	Brookline.
William E. Castle,	Cambridge.
Samuel F. Clarke,	Williamstown.
W. T. Councilman,	Boston.
Charles B. Davenport,	Chicago, Ill.
Harold C. Ernst,	Jamaica Plain.
Edward G. Gardiner,	Boston.
Samuel Henshaw,	Cambridge.
Theodore Hough,	Boston.
John S. Kingsley,	Somerville.
Edward L. Mark,	Cambridge.
Charles S. Minot,	Milton.
Edward S. Morse,	Salem.
George H. Parker,	Cambridge.
William T. Porter,	Boston.
James J. Putnam,	Boston.
Samuel H. Scudder,	Cambridge.
William T. Sedgwick,	Boston.
James C. White,	Boston.
William M. Woodworth,	Cambridge.

SECTION IV. — 16.

Medicine and Surgery.

Samuel L. Abbot,	Boston.
Edward H. Bradford,	Boston.
Arthur T. Cabot,	Boston.
David W. Cheever,	Boston.
Frank W. Draper,	Boston.
Thomas Dwight,	Nahant.
Reginald H. Fitz,	Boston.
Charles F. Folsom,	Boston.
Frederick I. Knight,	Boston.
Samuel J. Mixter,	Boston.
W. L. Richardson,	Boston.
Theobald Smith,	Jamaica Plain.
O. F. Wadsworth,	Boston.
Henry P. Walcott,	Cambridge.
John C. Warren,	Boston.
Francis H. Williams,	Boston.

CLASS III. — *Moral and Political Sciences.* — 49.

SECTION I. — 10.

Philosophy and Jurisprudence.

James B. Ames,	Cambridge.
John C. Gray,	Boston.
G. Stanley Hall,	Worcester.
Geo. F. Hoar,	Worcester.
Francis C. Lowell,	Boston.
Hugo Münsterberg,	Cambridge.
Josiah Royce,	Cambridge.
Jeremiah Smith,	Cambridge.
Frederic J. Stimson,	Dedham.
Edward H. Strobel,	Cambridge.

SECTION II. — 20.

Philology and Archæology.

Charles P. Bowditch,	Jamaica Plain.
Lucien Carr,	Cambridge.
Franklin Carter,	Williamstown.
Joseph T. Clarke,	Boston.
Henry G. Denny,	Roxbury.
William Everett,	Quincy.
J. W. Fewkes,	Washington, D.C.
William W. Goodwin,	Cambridge.
Henry W. Haynes,	Boston.
Charles R. Lanman,	Cambridge.
David G. Lyon,	Cambridge.
Morris H. Morgan,	Cambridge.
Bennett H. Nash,	Boston.
Frederick W. Putnam,	Cambridge.
Edward Robinson,	Boston.
F. B. Stephenson,	Boston.
Crawford H. Toy,	Cambridge.
John W. White,	Cambridge.
John H. Wright,	Cambridge.
Edward J. Young,	Waltham.

SECTION III. — 10.

Political Economy and History.

Charles F. Adams,	Lincoln.
Edward Atkinson,	Brookline.
Andrew McF. Davis,	Cambridge.
Ephraim Emerton,	Cambridge.
A. C. Goodell,	Salem.
Henry C. Lodge,	Nahant.
A. Lawrence Lowell,	Boston.
James F. Rhodes,	Boston.
Charles C. Smith,	Boston.
F. W. Taussig,	Cambridge.

SECTION IV. — 9.

Literature and the Fine Arts.

Francis Bartlett,	Boston.
John Bartlett,	Cambridge.
Arlo Bates,	Boston.
George S. Boutwell,	Groton.
T. W. Higginson,	Cambridge.
George L. Kittredge,	Cambridge.
Charles Eliot Norton,	Cambridge.
Denman W. Ross,	Cambridge.
Barrett Wendell,	Boston.

ASSOCIATE FELLOWS. — 98.

(Number limited to one hundred. Elected as vacancies occur.)

Class I. — *Mathematical and Physical Sciences.* — 36.

Section I. — 14.

Mathematics and Astronomy.

Edward E. Barnard,	Williams Bay, Wis.
S. W. Burnham,	Chicago.
George Davidson,	San Francisco.
Fabian Franklin,	Baltimore.
Asaph Hall,	Goshen, Conn.
George W. Hill,	W. Nyack, N.Y.
E. S. Holden,	New York.
Emory McClintock,	Morristown, N.J.
E. H. Moore,	Chicago.
Simon Newcomb,	Washington.
Charles L. Poor,	New York.
George M. Searle,	Washington.
J. N. Stockwell,	Cleveland, O.
Chas. A. Young,	Princeton, N. J.

Section II. — 6.

Physics.

Carl Barus,	Providence, R.I.
G. E. Hale,	Williams Bay, Wis.
S. P. Langley,	Washington.
T. C. Mendenhall,	
A. A. Michelson,	Chicago.
E. L. Nichols,	Ithaca, N. Y.

Section III. — 8.

Chemistry.

T. M. Drown,	So. Bethlehem, Pa.
Wolcott Gibbs,	Newport, R.I.
Frank A. Gooch,	New Haven.
S. W. Johnson,	New Haven.
J. W. Mallet,	Charlottesville, Va.
E. W. Morley,	Cleveland, O.
J. M. Ordway,	New Orleans.
Ira Remsen,	Baltimore.

Section IV. — 8.

Technology and Engineering.

Henry L. Abbot,	Cambridge.
Cyrus B. Comstock,	New York.
W. P. Craighill,	Charlestown, W. Va.
John Fritz,	Bethlehem, Pa.
F. R. Hutton,	New York.
George S. Morison,	New York.
William Sellers,	Edge Moor, Del.
Robt. S. Woodward,	New York.

Class II. — *Natural and Physiological Sciences.* — 31.

Section I. — 10.

Geology, Mineralogy, and Physics of the Globe.

Cleveland Abbe,	Washington.
George J. Brush,	New Haven.
T. C. Chamberlin,	Chicago.
Edward S. Dana,	New Haven.
Walter G. Davis,	Cordova, Arg.
Samuel F. Emmons,	Washington.
G. K. Gilbert,	Washington.
S. L. Penfield,	New Haven.
R. Pumpelly,	Newport, R.I.
Charles D. Walcott,	Washington.

SECTION II. — 6.

Botany.

L. H. Bailey,	Ithaca, N. Y.
D. H. Campbell,	Palo Alto, Cal.
J. M. Coulter,	Chicago.
C. G. Pringle,	Charlotte, Vt.
John D. Smith,	Baltimore.
W. Trelease,	St. Louis.

SECTION III. — 9.

Zoölogy and Physiology.

Joel A. Allen,	New York.
W. K. Brooks,	Lake Roland, Md.
F. P. Mall,	Baltimore.
S. Weir Mitchell,	Philadelphia.
H. F. Osborn,	New York.
A. S. Packard,	Providence, R.I.
A. E. Verrill,	New Haven.
C. O. Whitman,	Chicago.
E. B. Wilson,	New York.

SECTION IV. — 6.

Medicine and Surgery.

John S. Billings,	New York.
W. S. Halsted,	Baltimore.
W. W. Keen,	Philadelphia.
William Osler,	Baltimore.
Wm. H. Welch,	Baltimore.
H. C. Wood,	Philadelphia.

CLASS III. — *Moral and Political Sciences.* — 31.

SECTION I. — 7.

Philosophy and Jurisprudence.

James C. Carter,	New York.
Joseph H. Choate,	New York.
Melville W. Fuller,	Washington.
William W. Howe,	New Orleans.
Charles S. Peirce,	Milford, Pa.
G. W. Pepper,	Philadelphia.
T. R. Pynchon,	Hartford, Conn.

SECTION II. — 7.

Philology and Archæology.

Timothy Dwight,	New Haven.
B. L. Gildersleeve,	Baltimore.
D. C. Gilman,	Baltimore.
T. R. Lounsbury,	New Haven.
Rufus B. Richardson,	Athens.
Thomas D. Seymour,	New Haven.
A. D. White,	Ithaca, N.Y.

SECTION III. — 8.

Political Economy and History.

Henry Adams,	Washington.
G. P. Fisher,	New Haven.
Arthur T. Hadley,	New Haven.
H. E. von Holst,	Chicago.
Henry C. Lea,	Philadelphia.
Alfred T. Mahan,	New York.
H. Morse Stephens,	Ithaca.
W. G. Sumner,	New Haven.

SECTION IV. — 9.

Literature and the Fine Arts.

James B. Angell,	Ann Arbor, Mich.
L. P. di Cesnola,	New York.
H. H. Furness,	Wallingford, Pa.
R. S. Greenough,	Florence.
Herbert Putnam,	Washington.
Augustus St. Gaudens,	Windsor, Vt.
John S. Sargent,	London.
E. C. Stedman,	Bronxville, N. Y.
W. R. Ware,	New York.

FOREIGN HONORARY MEMBERS.—72.

(Number limited to seventy-five. Elected as vacancies occur.)

CLASS I. — *Mathematical and Physical Sciences.* — 23.

SECTION I. — 7.

Mathematics and Astronomy.

Arthur Auwers,	Berlin.
Luigi Cremona,	Rome.
George H. Darwin,	Cambridge.
Sir William Huggins,	London.
H. Poincaré,	Paris.
Otto Struve,	Karlsruhe.
H. C. Vogel,	Potsdam.

SECTION II. — 5.

Physics.

Ludwig Boltzmann,	Leipsic.
Oliver Heaviside,	Newton Abbot.
F. Kohlrausch,	Berlin.
Lord Rayleigh,	Witham.
Joseph J. Thomson,	Cambridge.

SECTION III. — 6.

Chemistry.

Adolf Ritter von Baeyer,	Munich.
Marcellin Berthelot,	Paris.
J. H. van't Hoff,	Berlin.
D. Mendeleeff,	St. Petersburg.
Sir H. E. Roscoe,	London.
Julius Thomsen,	Copenhagen.

SECTION IV. — 5.

Technology and Engineering.

Sir Benjamin Baker,	London.
Lord Kelvin,	Largs.
Maurice Lévy,	Paris.
H. Müller-Breslau,	Berlin.
W. Cawthorne Unwin,	London.

CLASS II. — *Natural and Physiological Sciences.* — 26.

SECTION I. — 7.

Geology, Mineralogy, and Physics of the Globe.

Sir Archibald Geikie,	London.
Julius Hann,	Vienna.
Albert Heim,	Zurich.
Sir John Murray,	Edinburgh.
Freih. v. Richthofen,	Berlin.
Henry C. Sorby,	Sheffield.
K. A. Ritter von Zittel,	Munich.

SECTION II. — 6.

Botany.

E. Bornet,	Paris.
A. Engler,	Berlin.
Sir Joseph D. Hooker,	Sunningdale.
W. Pfeffer,	Leipsic.
H. Graf zu Solms-Laubach,	Strassburg.
Eduard Strasburger,	Bonn.

SECTION III. — 6.

Zoölogy and Physiology.

Sir Michael Foster,	Cambridge.
Ludimar Hermann,	Königsberg.
A. von Kölliker,	Würzburg.
H. Kronecker,	Bern.
E. Ray Lankester,	London.
Elias Metschnikoff,	Paris.

SECTION IV. — 7.

Medicine and Surgery.

Emil von Behring,	Marburg.
Sir T. L. Brunton,	London.
A. Celli,	Rome.
Sir V. A. H. Horsley,	London.
R. Koch,	Berlin.
Lord Lister,	London.
F. v. Recklinghausen,	Strassburg.

CLASS III. — *Moral and Political Sciences.* — 23.

SECTION I. — 5.

Philosophy and Jurisprudence.

A. J. Balfour,	Prestonkirk.
Heinrich Brunner,	Berlin.
A. V. Dicey,	Oxford.
F. W. Maitland,	Cambridge.
Sir Frederick Pollock, Bart.,	London.

SECTION II. — 7.

Philology and Archæology.

Ingram Bywater,	Oxford.
F. Delitzsch,	Berlin.
W. Dörpfeld,	Athens.
Sir John Evans,	Hemel Hempstead.
H. Jackson,	Cambridge.
J. W. A. Kirchhoff,	Berlin.
G. C. C. Maspero,	Paris.

SECTION III. — 5.

Political Economy and History.

James Bryce.	London.
Theodor Mommsen,	Berlin.
Sir G. O. Trevelyan, Bart.,	London.
W. E. H. Lecky,	London.
John Morley,	London.

SECTION IV. — 6.

Literature and the Fine Arts.

E. de Amicis,	Florence.
Georg Brandes,	Copenhagen.
F. Brunetière,	Paris.
Jean Léon Gérôme,	Paris.
Rudyard Kipling,	Burwash.
Sir Leslie Stephen,	London.

STATUTES AND STANDING VOTES.

STATUTES.

Adopted May 30, 1854: *amended September* 8, 1857, *November* 12, 1862, *May* 24, 1864, *November* 9, 1870, *May* 27, 1873, *January* 26, 1876, *June* 16, 1886, *October* 8, 1890, *January* 11 *and May* 10, 1893, *May* 9 *and October* 10, 1894, *March* 13, April 10 *and May* 8, 1895, *May* 8, 1901, *and January* 8, 1902.

CHAPTER I.

OF FELLOWS AND FOREIGN HONORARY MEMBERS.

1. The Academy consists of Resident Fellows, Associate Fellows and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, viz.: Class I. The Mathematical and Physical Sciences; — Class II. The Natural and Physiological Sciences; — Class III. The Moral and Political Sciences. Each Class is divided into four Sections, viz.: Class I., Section 1. Mathematics and Astronomy; — Section 2. Physics; — Section 3. Chemistry; — Section 4. Technology and Engineering. Class II., Section 1. Geology, Mineralogy, and Physics of the Globe; — Section 2. Botany; — Section 3. Zoölogy and Physiology; — Section 4. Medicine and Surgery. Class III., Section 1. Philosophy and Jurisprudence: — Section 2. Philology and Archæology; — Section 3. Political Economy and History; — Section 4. Literature and the Fine Arts.

2. The number of Resident Fellows shall not exceed two hundred. Only residents in the Commonwealth of Massachusetts shall be eligible to election as Resident Fellows, but resident fellowship may be retained after removal from the Commonwealth. Each Resident Fellow shall pay an admission fee of ten dollars and such annual assessment, not exceeding ten dollars, as shall be voted by the Academy at each annual

meeting. Resident Fellows only may vote at the meetings of the Academy.

3. The number of Associate Fellows shall not exceed one hundred, of whom there shall not be more than forty in either of the three classes of the Academy. Associate Fellows shall be chosen from persons residing outside of the Commonwealth of Massachusetts. They shall not be liable to the payment of any fees or annual dues, but on removing within the Commonwealth they may be transferred by the Council to resident fellowship as vacancies there occur.

4. The number of Foreign Honorary Members shall not exceed seventy-five; and they shall be chosen from among persons most eminent in foreign countries for their discoveries and attainments in either of the three departments of knowledge above enumerated. There shall not be more than thirty Foreign Members in either of these departments.

CHAPTER II.

OF OFFICERS.

1. There shall be a President, three Vice-Presidents, one for each Class, a Corresponding Secretary, a Recording Secretary, a Treasurer, and a Librarian, which officers shall be annually elected, by ballot, at the Annual Meeting, on the second Wednesday in May.

2. At the Annual Meeting of 1901, nine Councillors shall be elected by ballot, one from each Class of the Academy to serve for one year, one from each Class for two years, and one from each Class for three years; *and at annual meetings thereafter three Councillors shall be* elected in the same manner, one from each Class, to serve for three years; but the same Fellow shall not be eligible for two successive terms. The nine Councillors, with the President, the three Vice-Presidents, the two Secretaries, the Treasurer, and the Librarian, shall constitute the Council. Five members shall constitute a quorum. It shall be the duty of this Council to exercise a discreet supervision over all nominations and elections. With the consent of the Fellow interested, they shall have power to make transfers between the several Sections of the same Class, reporting their action to the Academy.

3. If any office shall become vacant during the year, the vacancy shall be filled by a new election, and at the next stated meeting, or at a meeting called for this purpose.

CHAPTER III.

OF NOMINATIONS OF OFFICERS.

1. At the stated meeting in March, the President shall appoint from the next retiring Councillors a Nominating Committee of three Fellows, one for each class.

2. It shall be the duty of this Nominating Committee to prepare a list of candidates for the offices of President, Vice-Presidents, Corresponding Secretary, Recording Secretary, Treasurer, Librarian, Councillors, and the Standing Committees which are chosen by ballot; and to cause this list to be sent by mail to all the Resident Fellows of the Academy not later than four weeks before the Annual Meeting.

3. Independent nominations for any office, signed by at least five Resident Fellows and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call for the Annual Meeting, which shall then be issued not later than one week before that meeting.

4. The Recording Secretary shall prepare for use, in voting at the Annual Meeting, a ballot containing the names of all persons nominated for office under the conditions given above.

5. When an office is to be filled at any other time than at the Annual Meeting, the President shall appoint a Nominating Committee in accordance with the provisions of Section 1, which shall announce its nomination in the manner prescribed in Section 2 at least two weeks before the time of election. Independent nominations, signed by at least five Resident Fellows and received by the Recording Secretary not later than one week before the meeting for election, shall be inserted in the call for that meeting.

CHAPTER IV.

OF THE PRESIDENT.

1. It shall be the duty of the President, and, in his absence, of the senior Vice-President present, or next officer in order as above enumerated, to preside at the meetings of the Academy; to summon extraordinary meetings, upon any urgent occasion; and to execute or see to the execution of the Statutes of the Academy. Length of continuous membership in the Academy shall determine the seniority of the Vice-Presidents.

2. The President, or, in his absence, the next officer as above enumerated, is empowered to draw upon the Treasurer for such sums of money as the Academy shall direct. Bills presented on account of the Library, or the Publications of the Academy, must be previously approved by the respective committees on these departments.

3. The President, or, in his absence, the next officer as above enumerated, shall nominate members to serve on the different committees of the Academy which are not chosen by ballot.

4. Any deed or writing to which the common seal is to be affixed shall be signed and sealed by the President, when thereto authorized by the Academy.

CHAPTER V.

OF STANDING COMMITTEES.

1. At the Annual Meeting there shall be chosen the following Standing Committees, to serve for the year ensuing, viz.: —

2. The Committee of Finance, to consist of the President, Treasurer, and one Fellow chosen by ballot, who shall have full control and management of the funds and trusts of the Academy, with the power of investing or changing the investment of the same at their discretion. The general appropriations for the expenditures of the Academy shall be moved by this Committee at the Annual Meeting, and all special appropriations from the general and publication funds shall be referred to or proposed by this Committee.

3. The Rumford Committee, of seven Fellows, to be chosen by ballot, who shall consider and report on all applications and claims for the Rumford Premium, also on all appropriations from the income of the Rumford Fund, and generally see to the due and proper execution of this trust.

4. The C. M. Warren Committee, of seven Fellows, to be chosen by ballot, who shall consider and report on all applications for appropriations from the income of the C. M. Warren Fund, and generally see to the due and proper execution of this trust.

5. The Committee of Publication, of three Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Memoirs and the Proceedings shall be intrusted.

6. The Committee on the Library, of the Librarian ex officio and three other Fellows, one from each class, who shall examine the Library, and make an annual report on its condition and management.

7. An Auditing Committee of two Fellows, for auditing the accounts of the Treasurer.

CHAPTER VI.

OF THE SECRETARIES.

1. The Corresponding Secretary shall conduct the correspondence of the Academy, recording or making an entry of all letters written in its name, and preserving on file all letters which are received; and at each meeting he shall present the letters which have been addressed to the Academy since the last meeting. Under the direction of the Council for Nomination, he shall keep a list of the Resident Fellows, Associate Fellows, and Foreign Honorary Members, arranged in their Classes and in Sections in respect to the special sciences in which they are severally proficient; and he shall act as secretary to the Council.

2. The Recording Secretary shall have charge of the Charter and Statute-book, journals, and all literary papers belonging to the Academy. He shall record the proceedings of the Academy at its meetings; and after each meeting is duly opened, he shall read the record of the preceding meeting. He shall notify the meetings of the Academy, apprise officers and committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy. He shall post up in the Hall a list of the persons nominated for election into the Academy; and when any individual is chosen, he shall insert in the record the names of the Fellows by whom he was nominated.

3. The two Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them calculated to promote its interests.

CHAPTER VII.

OF THE TREASURER.

1. The Treasurer shall give such security for the trust reposed in him as the Academy shall require.

2. He shall receive officially all moneys due or payable, and all bequests or donations made to the Academy, and shall pay such sums as the Academy may direct. He shall keep an account of all receipts and expenditures; shall submit his accounts to the Auditing Committee; and shall report the same at the expiration of his term of office.

3. The Treasurer shall keep separate accounts of the income and appropriation of the Rumford Fund and of other special funds, and report the same annually.

4. All moneys which there shall not be present occasion to expend shall be invested by the Treasurer, under the direction of the Finance Committee.

CHAPTER VIII.

Of the Librarian and Library.

1. It shall be the duty of the Librarian to take charge of the books, to keep a correct catalogue of them, to provide for the delivery of books from the Library, and to appoint such agents for these purposes as he may think necessary. He shall make an annual report on the condition of the Library.

2. The Librarian, in conjunction with the Committee on the Library, shall have authority to expend such sums as may be appropriated, either from the General, Rumford or other special Funds of the Academy, for the purchase of books, and for defraying other necessary expenses connected with the Library.

3. To all books in the Library procured from the income of the Rumford Fund, or other special funds, the Librarian shall cause a stamp or label to be affixed, expressing the fact that they were so procured.

4. Every person who takes a book from the Library shall give a receipt for the same to the Librarian or his assistant.

5. Every book shall be returned in good order, regard being had to the necessary wear of the book with good usage. If any book shall be lost or injured, the person to whom it stands charged shall replace it by a new volume or set, if it belongs to a set, or pay the current price of the volume or set to the Librarian; and thereupon the remainder of the set, if the volume belonged to a set, shall be delivered to the person so paying for the same.

6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

7. The Librarian shall have custody of the Publications of the Academy and shall distribute copies among the Associate Fellows and Foreign Honorary Members, at their request. With the advice and consent of the President, he may effect exchanges with other associations.

CHAPTER IX.

OF MEETINGS.

1. There shall be annually four stated meetings of the Academy; namely, on the second Wednesday in May (the Annual Meeting), on the second Wednesday in October, on the second Wednesday in January, and on the second Wednesday in March. At these meetings only, or at meetings adjourned from these and regularly notified, shall appropriations of money be made, or alterations of the statutes or standing votes of the Academy be effected.

2. Fifteen Fellows shall constitute a quorum for the transaction of business at a stated meeting. Seven Fellows shall be sufficient to constitute a meeting for scientific communications and discussions.

3. The Recording Secretary shall notify the meetings of the Academy to each Fellow residing in Boston and the vicinity; and he may cause the meetings to be advertised, whenever he deems such further notice to be needful.

CHAPTER X.

OF THE ELECTION OF FELLOWS AND HONORARY MEMBERS.

1. Elections shall be made by ballot, and only at stated meetings.

2. Candidates for election as Resident Fellows must be proposed by two Resident Fellows of the section to which the proposal is made, in a recommendation signed by them, and this recommendation shall be transmitted to the Corresponding Secretary, and by him referred to the Council for nomination. No person recommended shall be reported by the Council as a candidate for election, unless he shall have received a written approval, signed at a meeting of the Council by at least five of its members. All nominations thus approved shall be read to the Academy at a stated meeting, and shall then stand on the nomination list during the interval between two stated meetings, and until the balloting. No person shall be elected a Resident Fellow, unless he shall have been resident in this Commonwealth one year next preceding his election. If any person elected a Resident Fellow shall neglect for one year to pay his admission fee, his election shall be void; and if any Resident Fellow shall neglect to pay his annual assessments

for two years, provided that his attention shall have been called to this article, he shall be deemed to have abandoned his Fellowship; but it shall be in the power of the Treasurer, with the consent of the Council, to dispense (*sub silentio*) with the payment both of the admission fee and of the assessments, whenever in any special instance he shall think it advisable so to do.

3. The nomination of Associate Fellows may take place in the manner prescribed in reference to Resident Fellows. The Council may in like manner originate nominations of Associate Fellows, which must be read at a stated meeting previous to the election, and be exposed on the nomination list during the interval.

4. Foreign Honorary Members shall be chosen only after a nomination made at a meeting of the Council, signed at the time by at least seven of its members, and read at a stated meeting previous to that on which the balloting takes place.

5. Three fourths of the ballots cast must be affirmative, and the number of affirmative ballots must amount to eleven to effect an election of Fellows or Foreign Honorary Members.

6. A majority of any section of the Academy is empowered to present lists of persons deemed best qualified to fill vacancies occurring in the number of Foreign Honorary Members or Associate Fellows allotted to it; and such lists, after being read at a stated meeting, shall be referred to the Council for Nomination.

7. If, in the opinion of a majority of the entire Council, any Fellow — Resident or Associate — shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his Fellowship; and provided that a majority of two thirds of the Fellows at a stated meeting, consisting of not less than fifty Fellows, shall adopt this recommendation, his name shall be stricken off the roll of Fellows.

CHAPTER XI.

OF AMENDMENTS OF THE STATUTES.

1. All proposed alterations of the Statutes or additions to them, shall be referred to a committee, and, on their report at a subsequent meeting, shall require for enactment a majority of two thirds of the members present, and at least eighteen affirmative votes.

2. Standing votes may be passed, amended, or rescinded, at any

stated meeting, by a majority of two thirds of the members present. They may be suspended by a unanimous vote.

CHAPTER XII.

OF LITERARY PERFORMANCES.

1. The Academy will not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its publications.

STANDING VOTES.

1. Communications of which notice had been given to the Secretary shall take precedence of those not so notified.

2. Resident Fellows who have paid all fees and dues chargeable to them are entitled to receive one copy of each volume or article printed by the Academy, on application to the Librarian personally or by written order, within two years from the date of publication. And the current issues of the Proceedings shall be supplied, when ready for publication, free of charge, to all the Fellows and members of the Academy who desire to receive them.

3. The Committee of Publication shall fix from time to time the price at which the publications of the Academy may be sold. But members may be supplied at half this price with volumes which they are not entitled to receive free, and which are needed to complete their sets.

4. Two hundred extra copies of each paper accepted for publication in the Memoirs or Proceedings of the Academy shall be placed at the disposal of the author, free of charge.

5. Resident Fellows may borrow and have out from the Library six volumes at any one time, and may retain the same for three months, and no longer.

6. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

7. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy, except by special leave of the Librarian.

8. Books, publications, or apparatus shall be procured from the income of the Rumford Fund only on the certificate of the Rumford Committee that they, in their opinion, will best facilitate and encourage the making of discoveries and improvements which may merit the Rumford Premium.

9. A meeting for receiving and discussing scientific communications may be held on the second Wednesday of each month not appointed for stated meetings, excepting July, August, and September.

RUMFORD PREMIUM.

In conformity with the terms of the gift of Benjamin, Count Rumford, granting a certain fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of said fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a premium to the author of any important discovery or useful improvement in light or in heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference being always given to such discoveries as shall, in the opinion of the Academy, tend most to promote the good of mankind; and to add to such medals, as a further premium for such discovery and improvement, if the Academy see fit so to do, a sum of money not exceeding three hundred dollars.

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